Elementary Proof of the Potential Theorems regarding Uniform Spherical Shells.

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Let P be the point at which the potential has to be found (Fig. 7). Let the uniform surface density be σ, and take two radii PQ, PQ′ differing in length by a small quantity Q′K. Let QR be drawn perpendicular to BD, the diameter through P, and let CL be drawn perpendicular to PQ.

If we consider an elementary zone of the surface surrounding the diameter CP, we get

$$2\pi\sigma QR . QQ' = m,$$

where m is the mass of the zone.

Also, by similar triangles, we have

$$Q'K . CQ = QQ' . CL,$$

and $$CL . PQ = QR . CP.$$

Therefore $$m = 2\pi\sigma Q'K . CQ . PQ/CP.$$

Thus the mass per unit difference of radii is $2\pi a\sigma PQ/CP$, where $a = CQ$, the radius of the shell; and the potential, at P, of a zone corresponding to unit difference of radii is

$$2\pi a\sigma/CP,$$

which is constant.

Therefore, summing over the total difference of radii, the potential, at P, of the whole shell is

$$2\pi a\sigma \frac{PD - PB}{CP}.$$

At an inside point this becomes $4\pi a\sigma = M/a$, where M is the whole mass of the shell. At an outside point it becomes

$$4\pi a^2\sigma/CP = M/CP.$$

[*Note, added Jan. 1900.*—Mr Muirhead will communicate to the Society a neat modification of the above proof, which avoids any assumption of the expression for the surface of a sphere. It is interesting to note that the above proof can be used to give a *physical* determination of the value of the surface. For, since, at *any* inside point, the potential has the value $4\pi a\sigma$, at the centre the potential is $4\pi a^2\sigma/a$. Hence, from the definition of potential, the surface is $4\pi a^2$.—W. P.]

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