COPERNICUS—NEWTON—EINSTEIN.*

By T. Arnold Brown.

The remarkable coincidence that the fourth centenary of the death of Nicholas Copernicus in May, 1543, as well as the third centenaries of the death of Galileo in January, 1642, and the birth of Isaac Newton in December, 1642, all fall within the present academic session has induced me to present a rapid survey of the history of mathematical life and thought, with particular emphasis on cosmology, from the first stirrings of the Renaissance down to the present day.

Copernicus belongs to that rare class of creative thinkers who, combining a high degree of moral courage with a superb integrity of intellect, are impelled to call in question the conventional beliefs of their day, sanctified though they be by the full weight of established authority and by long years of tradition, prejudice or superstition. He is therefore well worthy to rank with those inspired seekers after truth who appear with unfailing regularity in the history of mathematics and who—if we may borrow a striking phrase of Einstein's—achieve immortality by "challenging an axiom". Moreover, the work of Copernicus in his day was destined to pave the way for the dazzling achievements of Newton in the century which followed.

Nicholas Copernicus was born in February, 1473, in the little trading centre of Thorn (or should it be Torun?) on the Vistula; the town lay in a region over which the King of Poland exercised some form of authority; but, even at that early stage in the chequered history of Europe, it did not go unchallenged by the Order of Teutonic Knights.

Though he completed early in life the first draft of the book which was to make his name so famous, Copernicus shrank from publication and devoted himself to revising and rewriting the manuscript, partly from a complete indifference to personal fame and even more from a distaste for the controversy to which the publication of such original ideas was bound to give rise. He was a loyal son of Mother Church, and the extent to which he challenged the prevailing ideas of his age was exceeded only by the discretion which he displayed in avoiding polemics and dispute during his life-time.

At the age of thirty-nine he entered upon a Canonry at Frauenburg, to which he had been earlier appointed, and while continuing his work in mathematics and astronomy he became a busy man of affairs, much preoccupied with the business of the Chapter. In 1521 he was commissioned to draw up a statement of the grievances of the Chapter against the Teutonic Knights for presentation to the Prussian Estates, and in the following year wrote a memorandum on the debased and confused state of the coinage in the district. In these respects, as in his reluctance to publish and aversion to controversy, there may be traced a curious parallel between his life and that of Newton who, in his later years, became a Member of Parliament, a man of affairs and, as Master of the Mint, the guardian of his country's coinage.

In the year 1539 Copernicus was visited by an enthusiastic young astronomer generally known as Rheticus, who held one of the mathematical chairs at the Protestant University at Wittenberg. The visit extended over nearly two years, during which time Rheticus set himself to study the manuscript of Copernicus. When he eventually returned to Wittenberg, it had probably been already settled that he was to perform the service which Roger Cotes was later to undertake in regard to the second edition of Newton's *Principia*, namely, superintend the printing of the complete book itself.

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* Being the substance of a lecture delivered to the Plymouth Branch of the Association at University College, Exeter, on Saturday, May 1st, 1943.

But Rheticus was not able to see it all through the press himself and the work was entrusted to a Lutheran preacher—Andreas Osiander—who did what he could to mitigate the heresy which characterised the book by adding a preface in which he asserted that the fundamental ideas laid down in it were merely abstract hypotheses convenient for purposes of calculation. He also gave it the deceptively innocent title *De Revolutionibus Orbium Celestium* —the last two words being probably his own addition.

It was not until the last days of Copernicus' life that a printed copy of the precious volume was placed in his hands.

The central idea with which the memory of Copernicus is associated is that all velocities are relative to some observer and that the apparent motions of the celestial bodies are to a great extent an illusion due to the motion of this mortal globe carrying the observer with it. Before Copernicus, there was one chief " System of the Physical World "—the Ptolemaic—in which the earth was accorded a privileged position at the centre of the universe. Henceforth there were two. More than a century elapsed before the new idea received general acceptance, and more than three centuries were to pass before men realised that not only was the derivative $\frac{ds}{dt}$ a purely relative conception, but likewise the s and the t themselves.

Isaac Newton was born on Christmas Day, 1642, in the first sad winter of the Civil War, at Woolsthorpe Manor House, which is situated near the Great North Road some six miles south of Grantham and a few minutes from the Parish Church of Colsterworth, where the entry of his baptism on January 1, 1643, may still be seen. The room in which the birth took place bears a tablet over the mantelpiece recording the event and quoting the famous lines of Pope.

His early education was received at the King's School, Grantham, where, after an interruption of about three years devoted to work on the family farm, he eventually prepared for entry to Trinity College, Cambridge.

He was not expected to walk the six miles to school each day, and so he lodged at the house of an apothecary in High Street, Grantham. There he unearthed a parcel of old books, including a number on alchemy, which remained a subject of interest throughout his life. He also fell in love with the apothecary's pretty daughter ; but Newton never married and the lady was wed to another.

Newton was nurtured in an atmosphere of political and religious strife during a period in which the English people were driving painfully towards " the ideology of ordered freedom, which has been the greatest contribution of English thought and experience to the civilisation of the human race ". The Journals of the House of Commons had long been closed to the public, and it was not until Newton was eleven years old that Parliament proclaimed itself the authentic dispenser of its own news through *Mercurius Politicus*, which received its material at the whim of the Clerk of the House. England had to wait a further forty years for that stroke which freed the Press and printing from legalised official interference except in time of war, and which has been by Macaulay so vividly described. Newton represented as truly as any of his contemporaries the spirit of the new age of free enquiry and expression, and it is no occasion for surprise that we should find him in middle life stoutly confronting the notorious Judge Jeffreys himself, resisting the insidious aggression which the Crown sought to practise in University affairs and consenting to represent his University in the Convention Parliament of 1689. He was a very perfect Christian democrat.

Hundreds of years later we have witnessed the spectacle of Albert Einstein, next in the honoured line of succession to build an entirely new " System of the Physical World ", spurned and rejected by those who had basked in the glory of his greatest discoveries, taking up the cudgels once more in the defence of freedom and acting as an itinerant ambassador of peace and goodwill.

In June, 1661, Newton entered Cambridge, and three years later he was elected Scholar of Trinity College. In 1665 he became B.A., but the place which he reached in the final list is shrouded in mystery, for the order of seniority is provokingly omitted from the Grace Book of that year.

Within a few months he had invented his method of " fluxions " and turned his attention to the investigation of the properties of colours and light. He had obviously assimilated the ideas of his great contemporaries, Descartes and Pascal, in algebra and was probably led to the differential calculus by Fermat's method of drawing tangents. It seems evident that he was preparing a plan of campaign against those outstanding problems of astronomy which had been handed on from Kepler and Galileo and which must have proved a tempting target for so vigorous and original a mind. At all events he began to consider ways and means of improving existing telescopes, experimented with the task of grinding optic glasses to a figure other than spherical, and procuring a triangular prism discovered the " unequal refrangibility " of light.

This last discovery has led to the most spectacular applications in modern science. It has enabled the modern physicist not only to explore the innermost recesses of the atom, but also to extend his experiments to the sun and distant stars, and to investigate the properties of matter under physical conditions which it would be totally impossible to reproduce in any terrestrial laboratory.

As a graduate of one year's standing he had elucidated the complex problem of chromatic aberration and, being thus logically led to abandon his attempts to improve the refracting telescope, he took up the development of the reflecting type, which had been proposed by James Gregory of Aberdeen.

But when the great plague intervened and the University was dispersed he had perforce to abandon his " glass-works ", and he retired to the quiet seclusion of Woolsthorpe Manor to meditate upon the two other topics which were to bring him even greater renown, namely, the calculus and celestial mechanics. Of this period he wrote: " In the beginning of the year 1665 I found the method of approximating Series and the Rule for reducing any dignity (power) of any Binomial into such a series. . . . The next year . . . in May . . . I had entrance into the inverse method of Fluxions. And the same year I began to think of gravity extending to the orb of the Moon, and . . . from Kepler's Rule of the periodical times of the Planets. . . . I deduced that the forces which keep the Planets in their orbs must be reciprocally as the squares of their distances from the centres about which they revolve; and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the earth, and found them answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded Mathematics and Philosophy more than at any time since."

Twenty years were to elapse before the publication of these ideas, and much has been made of this delay. Some have maintained that he adopted an erroneous estimate of the earth's radius in seeking to verify the theory in relation to the motion of the moon; others that he was unable to integrate the separate attractions at an external point of the multitude of particles which go to make up the massive bulk of the earth, but possibly it was due to his reluctance to publish his results before the complete edifice could be exhibited in all its grandeur.

But the intrinsic value and importance of his work on optics and dynamics alike were becoming recognised, and when he returned to Cambridge in 1667 he was elected a Fellow of his College.

In the following year he constructed with his own hands a reflecting telescope with the object of testing out his law of gravitation on the satellites of Jupiter, and in the following year Isaac Barrow resigned the Lucasian Chair of Mathematics in favour of his brilliant disciple.

Under the Statutes attaching to the Chair, he was required to lecture for about an hour at least once a week during term, and also to conduct tutorials on two days per week during term and one day per week during vacations, if in residence.

He lectured on Optics, Algebra and Mechanics to a select group, and, if no one turned up to his lecture, he returned without complaint to his private experiments and meditations.

At a later date his notes on Algebra were written up by William Whiston, one of his pupils, and published under the title of *Arithmetica Universalis.*

Three years later he was elected a Fellow of the Royal Society.

Never having taken holy orders Newton was afraid lest his Fellowship at Trinity might be withdrawn. This made him anxious about his financial position, and he actually offered to resign from the Royal Society. How different were the circumstances under which Einstein in our day relinquished in exile his membership of the Berlin Academy of Science ! The situation in Newton's case was saved by his being allowed to remit the weekly payment of one shilling to the Society, and presently he received a Patent from the Crown allowing the Lucasian Professor to retain his Fellowship without becoming a priest.

This act of royal grace, taken in conjunction with the building of the Royal Observatory at Greenwich about the same time, goes a long way to retrieve the reputation of the " Merry Monarch ", who had also chartered the Royal Society.

One happy result of this easing of his financial position was his ability immediately to contribute to the expense of the building of a new library in Trinity College. In later life, when he had left Cambridge and become comparatively well off, he displayed great generosity not only towards his old College but also towards the Royal Society and the parish church at Colsterworth.

During the period under review Newton communicated several valuable results in optics to the Royal Society, but the next twenty years of his life were spent for the most part in quiet study at Cambridge. He was devoid of purely personal ambition and reluctant to publish his work, for he was touchy and sensitive in face of opposition and criticism.

But skilfully coaxed by Edmund Halley, the Secretary of the Royal Society, Newton at last consented to write up for publication his astronomical and dynamical researches. There was probably never a man who concentrated so severely and so rigorously upon the task to which he had set his hand. All material comforts were forsaken in the throes of mathematical composition. At a Council Meeting of the Royal Society held in 1686, the President—none other than the famous diarist, Samuel Pepys—was desired to licence for publication the *Philosophiae Naturalis Principia Mathematica*, and the cost of publication was most generously guaranteed by Edmund Halley. This masterpiece, which has not extravagantly been described as the most original creation of the human spirit ever to be produced, was published in the following year.

No attempt at a full description of the contents is possible here. The First and Second Books built up, on foundations laid by Galileo, the body of doctrine now known as the " Classical Mechanics ". The Third Book cut adrift from the limitations of traditional thought and created a new conception of the " System of the Physical World ". Never was so large a mass of natural phenomena brought within so comprehensive, so unified and so elegant a scheme.

The vindication of Newton's theory of universal gravitation was even more spectacular than the subsequent developments of his optical discoveries. A great comet had appeared in 1682 and had been carefully observed by Halley ; on referring to records of previous cometary appearances, he suspected that comets which had been noted in the years 1607 and 1531 were really one and the same body following their prescribed courses round the sun and reappearing at nearly regular intervals of seventy-six years. By the application of Newton's treatment of approximate parabolic orbits he looked forward into generations still to come, and predicted that the celestial wanderer would return in 1758 or 1759. Newton passed away in 1727 and Halley was left to maintain a solitary vigil. He too had been dead for seventeen years when the reappearance occurred within a month of the expected time. Halley's comet returned promptly to schedule in 1835 and 1910.

The theory of planetary perturbations led to the discovery of the planet Neptune in the nineteenth and to that of Pluto in the twentieth century. Without the guiding principle of gravitation to assist him, Einstein could not have developed his general principle of relativity.

Newton was more of an originator and a developer than a challenger. He gathered together all the scattered threads of scientific knowledge which were floating idly in the air, and by imparting to them a new strength and direction created a marvellous pattern, which has set the style and fashion for generation after generation of mathematical physicists right down to our own time.

He did not choose to regard himself as a pure mathematician, but rather as a humble searcher after Nature's laws. Yet he discovered the Binomial Theorem and the use of infinite series, invented the Calculus, made most valuable contributions to the theory of curvature, discussed the singularities of algebraic plane curves, developed a numerical method for the solution of algebraic and transcendental equations, and created the interpolation formula which bears his name. His hereditary enemy, Leibnitz, declared that of all the mathematics created up to his time, the better half was due to Newton.

During the years which followed the publication of the *Principia*, Newton's contacts with London life as a member of Parliament rendered him conscious of the limitations and anomalies of his own financial position. He was unable to yield to the generous impulses by which he was animated and, although most indignant at the idea of influential wires being pulled on his behalf behind his back, he eventually accepted the office of Warden and later Master of the Mint. To the English mind there is nothing grotesque in this spectacle of the greatest scientist which this country has ever produced being thus removed from the scene of his greatest triumphs and henceforth suffering the severest restrictions in the exercise of his unique powers. The appointment in 1696 was a bar to the further prosecution of his researches in physical astronomy. His later scientific work was considerable and retained its peculiar quality to the end, but it was carried out by snatches and in the intervals of business.

In 1703 he was elected President of the Royal Society, and re-elected annually for the remainder of his life. In the following year he published his book on Optics, which contained his theory of " fluxions ", and which like the *Principia* eventually ran to three editions.

In 1705 he was knighted by Queen Anne at Trinity College, but whether in recognition of his valuable services as Master of the Mint or in acknowledgment of his pre-eminence as a scientist is not recorded. The same year witnessed his failure to secure re-election for a third term to Parliament.

Newton's approach to matters of religion was rational, though devout. He realised the limitations of science and maintained his interest in theology throughout a long life, but he was not a regular attender at the College services while at Cambridge ; nor did he degenerate into a religious neurotic as Pascal did in the last years of his life.

Newton was not always amiable in his personal relationships. He was not on very good terms with Robert Hooke, who opposed the corpuscular theory of light and developed the engaging propensity of claiming as his own some of the more important discoveries of the great master. Nor were his personal relations with Flamsteed, the first Astronomer Royal, whom he bombarded with questions about the motion of the moon, always of the best. The famous and long-standing feud between Newton and Leibnitz reflects credit on neither of these great men. Each invented the calculus independently of the other, and the ridiculous squabble about priority has become a matter of small account now that the " relativity of simultaneity " has been established.

The progress of physical science depends upon the interplay of experiment and observation on the one hand and the mathematical analysis of the results obtained on the other, both activities being carried out at a high level of competence. New observations lead to new analysis, and this in turn suggests fresh experiments. Newton occupied a unique position inasmuch as he combined both qualities in the one personality.

The subsequent history of scientific thought has shown that practical achievement ever lags behind imaginative conception, and the mathematicians have never failed to anticipate the needs of the physicists by providing new weapons of attack upon the mysteries of nature.

In the Golden Age which followed upon the Newtonian era it gradually came to be recognised that, in spite of Newton's famous dictum, " non fingo hypotheses ", universal gravitation, with its implication of a perfect " aether " pervading the whole of a Euclidean space, remained a pure hypothesis, and that the ideas of absolute distance, time, velocity and all the rest, which had been accepted as axiomatic by Newton, were little more than figments of a disordered imagination. It is perhaps something of a paradox that this change of outlook should have been induced by the modern development of electrodynamics and optics—the fruition of seeds implanted by Newton.

One of the outstanding pioneers of this new age was Nikolas Ivanovitch Lobatchewsky who, having the audacity to challenge the validity of the geometry of Euclid and Pythagoras as applied to the physical universe, has not inaptly been described as the Copernicus of geometry. It took over 2000 years to emancipate the mind of man from the belief that Euclidean geometry represented *absolute* truth in its purest form, and it was this Russian professor in the University of Kazan who did it. Thus consider the famous Theorem of Pythagoras and suppose that, instead of drawing our triangle on the idealised plane imagined by Euclid, we draw it, as Archimedes was wont to do, upon the sandy surface of the earth, idealised to the extent of being supposed a " perfect " sphere. Since we cannot draw a straight line to lie within the surface of the sphere, we must replace straight lines by " geodesics " in the surface, *i.e.* lines of shortest (or longest) length joining any two points. The problem is now reduced to one in spherical trigonometry, and it is well known that, if $BC^2 = AB^2 + AC^2$, then either the angle at A is *not* a right angle or one of the sides at least is *not* " straight ".

It is important to realise just how this break-away from ancient geometry has been attained. The Euclidean plane has been distorted and endowed with the property of " curvature ". The sphere is obviously a surface of constant (total) curvature ; but it is equally easy to visualise a surface for which the curvature may vary not only with position but also with time. The smoothly rippling surface of a lake is an example of a non-Euclidean " space " possessing this property.

The appropriate parametric representation of such surfaces and the treatment of their curvature was the work of the celebrated Gauss, who was led by mainly practical considerations to investigate the deeper properties of surfaces, for he acted as scientific adviser to the Hanoverian and Danish governments in an extensive geodetic survey carried out by them during the years 1821–1848.

The non-Euclidean two-dimensional " space " which we have considered finds itself immersed in a three-dimensional " manifold " which is itself Euclidean, and we may employ the Theorem of Pythagoras in this manifold to obtain the conventional formula for an element of arc on the sphere, namely,

$$ds^2 = a^2 d\theta^2 + a^2 \sin^2 \theta \, d\phi^2.$$

It was left to Riemann entirely to dispense with this leg-up from Euclid and to create on a precisely similar pattern a non-Euclidean " space " of any number of dimensions, defined by the differential quadratic form :

$$ds^2 = g_{\mu\nu} \, d\theta_\mu \, d\theta_\nu.$$

The g's, which are neither more nor less mysterious than the dots which once puzzled a famous Chancellor of the Exchequer, depend upon the parameters θ. For each particular system of g's a corresponding type of space is defined and, if it should happen in three dimensions that $g_{\mu\mu} = 1$, and $g_{\mu\nu} = 0$, our old friend Euclid emerges as a particular case. Riemann went on to develop the theory and to generalise the idea of Gaussian curvature for such spaces.

While all this progress was being made in the mathematics of the nineteenth century, the physicists in their laboratories had been encountering complexities, which led to a re-examination of the fundamental bases of their beliefs. It gradually came to be understood, for example, that unless size were to be regarded as some metaphysical property of a material body, and not simply the result of measurement, then even the distance between two points of a rigid body was a relative conception and depended upon the motion of the particular observer who carried out the measurement. This debunking process was completed by Albert Einstein, who challenged the axiom that " two events can occur in different places at the same time ".

The resulting confusion was for a time considerable, until Minkowski and others came to the rescue by suggesting that perhaps the complexity was due to a too bigoted adherence to the geometry of Euclid and might be overcome by regarding the world of physical phenomena as a four-dimensional manifold in the space-time sense.

Einstein took up this idea and, by an ingenious selection of one of the infinite variety of four-dimensional geometries of Riemann, succeeded in creating his famous relativity theory of gravitation, which was first published in its complete form during the last war in November, 1915, when he was still a professor at the Prussian Academy of Science in Berlin.

This theory represents the space of everyday life rather as the three-dimensonal " surface " of a rippling lake, finite but unbounded, the ripples being matter and their movement representing the large-scale motions of the physical universe.

It is characterised by the following differential quadratic form in four dimensions:

$$ds^2 = (1 - 2m/r)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 - (1 - 2m/r) dt^2.$$

Here ds denotes not merely an element of distance but a generalised "interval" involving both distance and time, and m is the mass of the attracting particle in gravitational units.

Newton's formulation of the laws of motion for a particle in a gravitational field is now replaced by the single statement: "Every particle and light-pulse moves so that the integral of ds between two points of its track is stationary."

The theory has been subjected to three celebrated tests. Two are observational and cover speeds both of low and of high order. The third is experimental and very delicate. They are as follows:

(1) The famous large discordance in the observed position of the perihelion of the planet Mercury (8″ of angle per century) has been explained in an entirely natural manner.

(2) The deflection of a ray of light grazing the limb of the sun has been confirmed as 1″·75 as against 0″·87 or zero on previous theories.

(3) Physicists are generally agreed that the period of an atom vibrating in the photosphere of the sun appears longer than that for an identical atom in a terrestrial laboratory.

No one will pretend that the last word has been said. After the National Socialist revolution in Germany, Einstein, like many refugees before him, became an emigrant to the United States of America, and gratefully accepted sanctuary within the portals of Princeton University. The threat of a breakdown in civilisation itself, which seemed almost imminent in 1939, is now being averted, and no disaster of lesser magnitude would suffice to arrest the progress of mathematical research and discovery. T. A. B.

QUEENSLAND BRANCH.

REPORT FOR THE YEAR 1942–3.

THE Annual Meeting was held on 22nd May, 1942; the Annual Report and the Statement of Receipts and Expenses were presented and were adopted, after which the officers for the coming year were elected. The subject of the Presidential Address by Professor Simonds was "The Beginnings of Mathematics".

During the year two General Meetings were held: at the first of these, held on 28th August, Mr. E. W. Jones read a paper on "Computations", and at the second, on 30th October, Mr. J. P. McCarthy read a paper on "The Wallace (or Simson) line and the Wallace point".

The Statement of Receipts and Expenses shows a credit balance of £10 18s. 9d. The attendance at meetings has been affected by the fact that certain members are on duty with the Forces. The number of members is 26, of whom 8 are members of the Mathematical Association. In spite of acute difficulties, copies of the *Mathematical Gazette* reach us in due course and are circulated amongst Associate Members.

The Committee is as follows: *President*, Professor E. F. Simonds; *Vice-Presidents*, Messrs. S. Stephenson and I. Waddle; *Hon. Secretary and Treasurer*, Mr. J. P. McCarthy; *Members*, Miss E. H. Raybould, Messrs. R. A. Kerr, E. W. Jones, J. C. Deeney, P. B. McGovern.

J. P. MCCARTHY, *Hon. Secretary.*