5. RADAR DETERMINATIONS OF PLANETARY MOTIONS

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ABSTRACT

Radar echoes have already been detected from Venus, Mercury, Mars, and Jupiter. Time delay measurements of high precision between Earth and Venus, as well as between Earth and Mercury, have disclosed significant deviations from the published ephemerides of these planets. Although new simultaneous solutions for the A.U., the relevant planetary masses, radii, and orbital elements have not yet been carried out, preliminary analysis indicates that the light-second equivalent of the A.U. is 499·0050 ± 0·0004 and that the corresponding kilometer equivalent is about 149 598 000 ± 100 (using c = 299 792·5 km/sec). The same analysis shows that, with respect to Earth, both Venus and Mercury seem to be ahead in their orbits relative to predictions. The radius of Venus appears to be about 6100 ± 50 km, in agreement with the most accurate optical determination. Doppler spread measurements show Venus to have a retrograde rotation with a period of about 247 ± 5 days. The celestial latitude of its axis is about −85 ± 2 degrees.

The development in the 1950's of sensitive radar systems and sophisticated data-processing techniques generated a realistic interest in obtaining radar echoes from near-by planets. Although echoes from the Moon had been detected in 1946, a scant 10 years after the invention of radar, the problem of detecting Venus is far more severe. Assuming that the radar reflection properties of both bodies are identical, one can show that radar echoes from Venus are always at least 10 million times weaker.

The first serious attempt to detect echoes from Venus was made at Lincoln Laboratory in 1958 (**1**). Although originally it was felt that a *bona fide* measurement of the distance to Venus had been achieved in that attempt, subsequent detections indicated quite conclusively that the 1958 measurement was not valid.

In the last few years, successful interplanetary radar experiments have been performed at many sites, with echoes having already been detected from Venus (**2–8**), Mercury (**9, 10**), Mars (**11, 12**) and Jupiter (**13, 14**).

The importance of these experiments lies in their addition of two dimensions to the space of Earth-based interplanetary measurements. Besides the conventional angular data, radar provides measurements of time delay (range) and Doppler shift (range rate). The relevance to solar-system celestial mechanics of these new types of measurements is best illustrated by the fact that, with essentially only one measurement a very significant improvement was made in relating the Astronomical Unit to terrestial units, despite the prior analysis of sophisticated optical observations that had been accumulated for centuries.

In the remainder of this paper, we shall discuss the results of interplanetary radar experiments, stressing those which are most pertinent to celestial mechanics.

The 1961 and 1962 radar experiments involving Venus were used mainly to deduce a more accurate value for the light-sec (or km) equivalent of the A.U. The results obtained from experiments performed at frequencies ranging from 440 MHz to 2388 MHz were in remarkable agreement—to within one part in 10^5—and yielded a value for the A.U. of about 499·005 light-sec, or equivalently, of about 149 598 000 km, under the assumption that the speed of light is 299 792·5 km/sec. This result for the A.U. is in disagreement with the determination by Rabe from optical observations of Eros by about ten times the standard error quoted by

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Rabe (**15**). The directness of the radar determination, coupled with the excellent agreement between the various observations, indicates quite conclusively that Rabe's result must be considerably less accurate than previously thought.

The actual results from these radar experiments (**16–19**) are shown in Fig. 1. The MIT

Facility	Frequency (MHz)	Astronomical Unit (light-sec)	Astronomical Unit (c = 299 792·5 km/sec)	Solar Parallax (sec) (R_o = 6 378·15 km)
MILLSTONE (M.I.T.) (1961 Conjunction)	440	499·005 ± 0·001	149 598 000 ± 300	8·79416 ± 0·00002
GOLDSTONE (JPL) (1961 Conjunction)	2388	499·007 ± 0·001	149 598 600 ± 250	8·79412 ± 0·00002
JODRELL BANK (1962 Conjunction)	410	499·000 ± 0·003	149 596 600 ± 900	8·79425 ± 0·00006
U.S.S.R. (1962 Conjunction)	~700	499·005 ± 0·001	149 597 900 ± 250	8·79415 ± 0·00002

FIG. 1. Radar determinations of the Astronomical Unit.

determination involved mainly time delays; the JPL result depended almost equally on time-delay and Doppler-shift data; the Jodrell Bank determination used Doppler-shift data exclusively; whereas the U.S.S.R. result depended on time-delay data. (A private communication indicated that the U.S.S.R. value from 1962 should be increased by 200 km.) Some of these results differ from one another by slightly more than the sum of the associated standard errors. It is not possible to determine whether the disagreement is intrinsic to the measurements or whether it stems from the use of different ephemerides by the different groups. Although the individual Millstone measurements were published, this practice has not been followed by the other groups. Independent analysis of all the available data is therefore not yet possible. As the accuracies of the measurements improve, slight differences in the ephemerides will cause more significant differences in the conclusions. The publication of actual measurement values will therefore become increasingly more important. Amending the standard national ephemerides to include the data necessary for reducing extremely accurate radar measurements should also prove very helpful. (Preparations for such additions are understood to be at an advanced stage.)

In the 1961 work, the ephemerides were assumed to be free of error, and the data were used to obtain a corresponding 'best fit' value for the A.U. Jet Propulsion Laboratory workers (**17**) noted, however, that the data were consistent with Venus being ahead in its orbit, relative to Earth, by about 0″5 of heliocentric arc more than provided for by the Duncombe corrections (**20**). They also found the radius of Venus to be consistent with their assumed value of 6100 km to within ±50 km.

More generally, radar data can be used to make a 'best fit' to a larger set of parameters. An attempt in this direction was made in the U.S.S.R. using time-delay data from the 1962 Venus inferior conjunction (**19**). In this work, variations in the A.U., in the relative longitude of Earth and Venus, and in the radius of Venus were allowed. In addition to the value for the A.U. shown in Fig. 1, the results also indicated that Venus is ahead in its orbit by about 0″5 of arc, in agreement with the JPL deduction, but that its radius is 6020 km.

In 1964, extensive series of Venus measurements have been performed at a number of observatories and much data accumulated. Some of the time-delay measurements made at Arecibo and at Millstone Hill are shown in Fig. 2 in comparison with the corresponding predictions. The predictions were deduced from Volumes 14 and 15, Part III, of the *Astr. Pap. Amer. Eph.*, amended to include Duncombe's corrections to the orbits of both Earth and Venus (**21**). The value of 499·005 light-sec was assumed for the A.U. and 0·0203 light-sec (or 6100 km) for the radius of Venus. The data are seen to deviate systematically from the

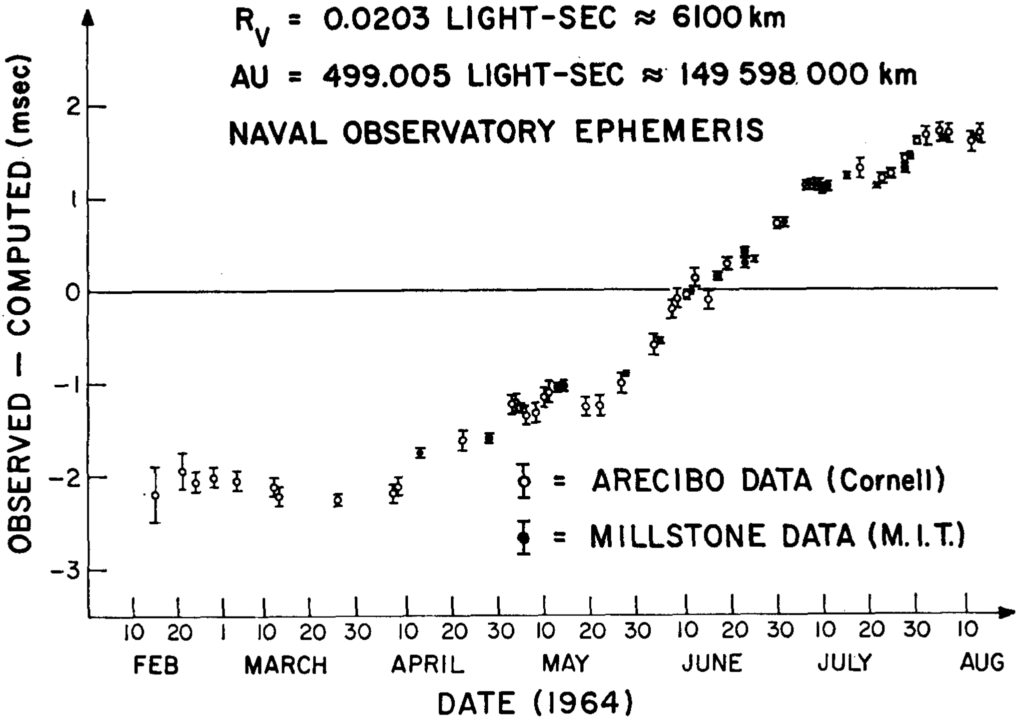

Fig. 2. Earth-Venus time-delay residuals (Naval Observatory ephemeris).

predicted values. An elaborate digital computer program is being developed at Lincoln Laboratory to process all the radar data in a weighted-least-mean-square sense, allowing as many free parameters as are relevant (**22**). Since this program has not yet been completed, a preliminary hand calculation was carried out allowing variations in the A.U. in the radius of Venus, in the relative longitude of Earth and Venus, and in the eccentricities of both orbits. The results are truly only tentative but indicate agreement with the assumed value of the A.U. to within 0·0004 light-sec or, equivalently, within about ±100 km. Similarly, the 6100 km radius appears to be accurate to within about ±50 km. The relative longitude of Earth and Venus requires a change of almost 0″·45, in good agreement with the earlier deductions. Corrections to the orbit eccentricities appear to be small—less than one unit in the sixth decimal place. It should be stressed that these reductions are only preliminary and that they by no means exploit the full accuracy inherent in the measurements.

One interesting feature of the residuals shown in Fig. 2 is the appearance of short-period oscillations of about a 30-day period. The time-delay measurements made at 440 MHz at

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Lincoln Laboratory in 1961 also seemed to indicate a similar periodic effect. Although the accuracy of those data was really insufficient to conclude reliably that such periodic effects were really present, many explanations soon poured forth. One, involving a correction to the lunar mass, required about a 2 per cent change in it and was rejected as exceedingly unlikely. In another, a seemingly significant correlation between the residuals and the solar decimetric flux was turned into an explanation in terms of an *ad hoc* model of the Venusian ionosphere that undergoes complicated changes correlated with the Sun's rotation period. This explanation, too, as we shall see, is untenable. Perhaps the boldest attempt was the development of a theory in which the speed of the radar wave was influenced by its passing near massive bodies (in this case the Moon) by far more than is predicted by general relativity.

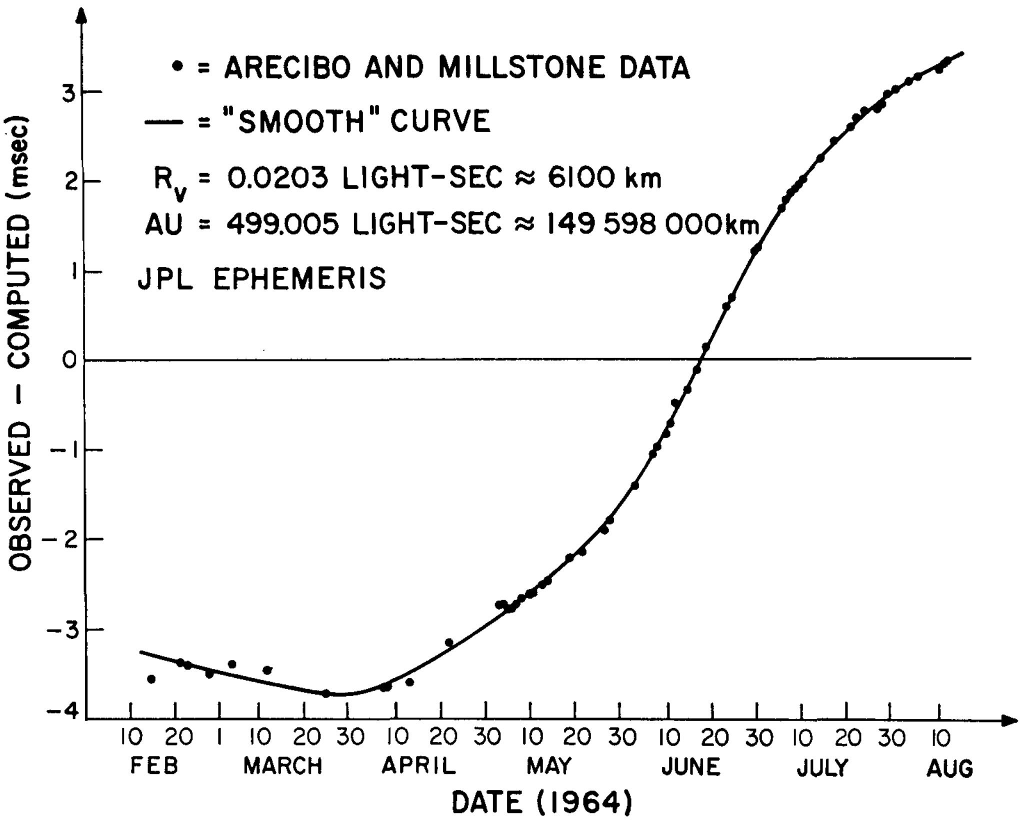

Fig. 3. Earth-Venus time-delay residuals (JPL ephemeris).

As we have already noted, the 1964 time-delay measurements also show this almost-periodic effect in the residuals (e.g. there seem to be local peaks in the residuals near the middle of April, May, June and July). The measurements are now sufficiently accurate that these short-period variations must be considered seriously. A plasma or ionospheric effect is even harder to accept with the 1964 results since, for example, plasma effects can be expected to depend on the square of the frequency, but the Millstone measurements at 1295 MHz are quite consistent with Arecibo's, although made at a frequency approximately three times as great.

The most prosaic explanation is that the short-period residuals merely reflect the fact that the basic ephemeris was prepared from a general perturbation theory which, of course, involved truncated Fourier series. Thus one would expect a smoothly varying function, like the interplanetary time delay, to be represented in a truncated Fourier series expansion by the same smoothly varying function but with high-frequency, low-amplitude oscillations superposed, these latter being the effects of truncation. To test this explanation, the measurements were compared with predictions deduced from numerical integrations of the planetary motions as kindly provided by JPL. The results are shown in Fig. 3. The disappearance of the short-period variations is virtually complete. In addition, we see that the measurements are remarkably consistent with one another since the residuals all lie extremely close to a 'smooth' curve, except for a few of the earliest measurements which had rather large uncertainties in virtue of the much greater Earth-Venus distance and consequent decrease in echo strength at that time.

(The maximum residuals in Fig. 3 are almost twice as large as in the preceding figure because the JPL values for the positions of Earth and Venus did not contain all of the Duncombe corrections.)

Time-delay measurements have also been made this year for radar signals reflected from Mercury. (Because of the large bandwidth and weakness of the echoes, experiments involving Mars and Jupiter have not yet provided useful information from the point of view of celestial mechanics.) The Mercury measurements encompassed approximately two-thirds of its orbit, as can be seen in Fig. 4. The ephemeris used here was based on the American Ephemeris and

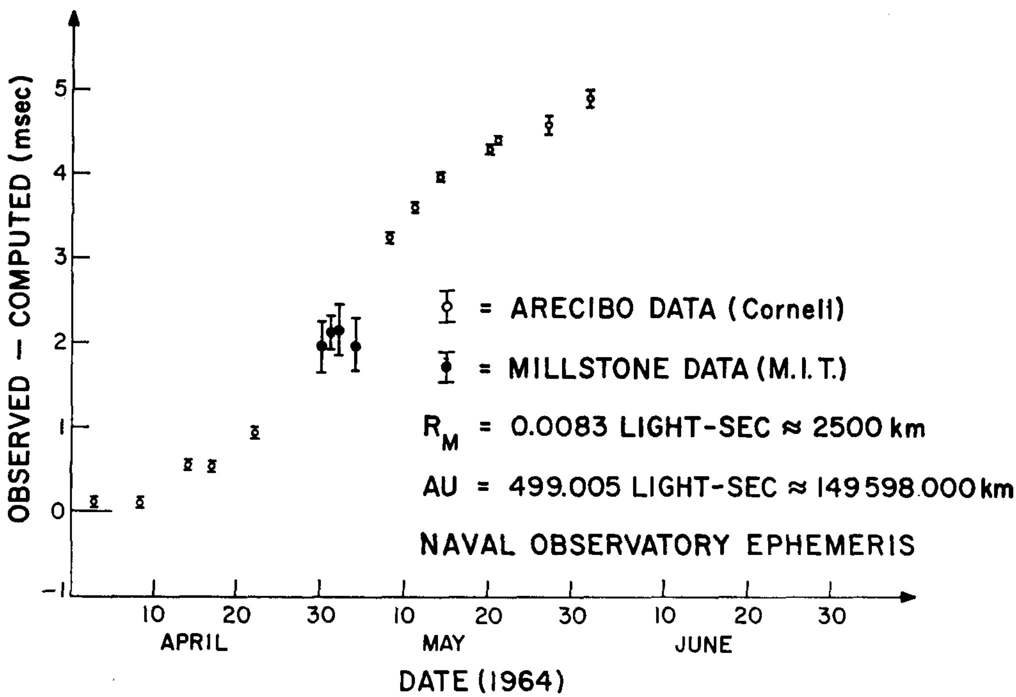

Fig. 4. Earth-Mercury time-delay residuals.

Nautical Almanac values for the heliocentric positions of Mercury, and on the geocentric positions of the Sun as given by Vol. 14 of the *Astr. Pap. Amer. Eph.*, suitably modified by Duncombe's corrections. Mercury's radius was taken to be 0·39 times that of the Earth, i.e. slightly less than 2500 km, and the value of 499·005 light-sec was used for the A.U. Again, the measurements shown were taken at both Arecibo and Millstone, with those at Arecibo being significantly more accurate. Internally, the data are quite consistent, except perhaps for the 4 May measurement at Millstone which appears to be anomalously low. (Previously, in May 1963, JPL also made several accurate measurements of time delay and concluded (**10**) from these that the value of the A.U., as determined by JPL in 1961, was accurate to within 100 km. However, it is very difficult to determine the A.U. reliably from data over such a limited arc since there are so many different possible sources that can contribute to the resultant residuals.) For the 1964 data preliminary hand calculations have been performed in which the sensitivity of all time delays to errors in the A.U., in the radius of Mercury, in the eccentricity of Earth and of Mercury, and in the relative longitude of Earth and Mercury were considered in detail. The general 'S-shaped' appearance is almost surely caused by Mercury's being 'ahead' in its orbit (relative to Earth) by an average of about 1″ of heliocentric arc. Thus, before inferior conjunction, Mercury is closer to Earth than expected on the basis of the ephemeris used, and after conjunction farther away so that the residuals (observed-computed) increased systematically throughout the interval of observation. At conjunction, which occurred on 30 April, the computed value of time delay is insensitive to changes in relative planetary longitudes, and the fact that the observed delays were about 2 m sec (i.e. about 600 km) longer than expected indicates the presence of additional ephemeridic errors. A likely candidate might appear to be the A.U.; however, closer inspection shows that since the sensitivity of time delay is proportional to the delay, accounting for the residual near conjunction by increasing the A.U. leads to intolerably large residuals near the ends of the observing interval. Of course, the corresponding residuals for the Earth–Venus time-delay measurements would also be intolerably large. (We note, in addition, that the 1963 Mercury measurements made by JPL seem to be consistent with those in Fig. 4; only the interpretation is different.)

Although the preliminary hand-computation attempt to understand these 1964 residuals in detail was not completely successful, it does appear that the eccentricity of Mercury's orbit requires a reduction by about 1 unit in the sixth decimal place and that the radius of the disk of Mercury requires a reduction on the order of 100 km from the 2500 km value that was assumed in the ephemeris. (The values of the A.U. and the elements of Earth's orbit were as determined from the preliminary analysis of the Earth–Venus time-delay measurements.) It is, of course, obvious that a reduction in the radius of Mercury leads to an increase in the computed time delays; less obviously, but also understandable from simple arguments, a reduction in the eccentricity of Mercury's orbit increases the computed time delays during this observing interval. A more precise and systematic analysis of these data and, more importantly, future radar measurements will be required to confirm these tentative conclusions. Such measurements are, in fact, currently in progress at Arecibo.

As a concomitant to enabling the radius and some of the orbital elements of Mercury to be improved substantially, the radar time-delay measurements can provide a check on the excess advance of the perihelion of Mercury that is attributed to the effects of general relativity. If the individual radar observations are made with an accuracy of about 30 μ sec and are repeated at about weekly intervals, then it can be shown that the variation in Mercury's argument of perihelion can be determined to within about 1″ of arc per century after 3 years of observations. By contrast, angle data gathered by optical means over two centuries were estimated (**23**) to yield a corresponding probable error of 0″.41 which is about 1 per cent of the value attributed to the effects of general relativity. Thus, within a decade the radar determination should be competitive, even with no improvement over current measurement accuracies.

In addition to the usual general relativistic correction to the advance of a planet's perihelion, there is another contribution stemming from the rotation of the Sun. However, for a solar rotation period of 28 days, the effect on the perihelion motion, even for Mercury, has a centennial magnitude of only about 0".01 and will clearly not be detectable in the foreseeable future. (The corresponding effect on the orientation of the plane of Mercury's orbit is even smaller—about half as large.)

The possibility that at least part of the observed excess in the centennial motion of Mercury's perihelion is caused by the effects of a solar equatorial bulge must be considered seriously. Such a bulge would also cause an advance of the perihelion with a dependence on distance from the Sun that differs only slightly from the corresponding dependence of the relativistic effect. The possible contributions of a solar bulge are usually minimized by the argument that it would cause a correspondingly large motion of the ascending node of Mercury's orbit (**24**). My calculations, however, indicate that, with respect to the ecliptic, a solar equatorial bulge would cause a change in node and in inclination of Mercury's orbit which are each less than 10 per cent of the corresponding secular change in the argument of perihelion. Hence such a bulge could contribute several seconds of arc to the centennial perihelion precession of Mercury without the concomitant effect on node and inclination being reliably detected at present.

Radar observations of the planets can be used not only to determine the motions of their centers of mass but also to investigate the motions about the centers of mass, i.e. the rotation periods. Such a possibility is, of course, especially intriguing in regard to Venus whose surface is perpetually enclosed in a shroud of clouds.

The method of determining a planet's rotational motion is based on the following fact: Each region on the planet's surface reflects back some of the incident radio energy but shifted in frequency by an amount proportional to the component of its velocity along the line-of-sight to the radar. By measuring the spectral width of echoes received from annular regions on the planet, which are each characterized by a given time delay with respect to the sub-Earth point, it is possible to determine with high accuracy an 'instantaneous' angular velocity of rotation. This velocity can be broken down into two parts: (*a*) the intrinsic, inertial angular velocity of the planet, and (*b*) the apparent angular velocity of rotation caused by the observer's motion with respect to the center of the planet (i.e. by the observer's passing by the planet). More precisely, at each instant, the bandwidth is directly proportional to the component, in the plane perpendicular to the radar line of sight, of the sum of these two angular velocities. Hence, from an extended series of bandwidth measurements it is possible, in principle, to determine a planet's rotation period as well as the direction of its rotation axis.

The first attempts to estimate Venus's rotation period from radar observations were made in 1961. At that time it was concluded both by JPL and by Lincoln Laboratory that the rotation period of Venus was very slow—on the order of its orbital period. But the sense of the rotation was not determined. From spectral measurements taken at the time of the 1962 inferior conjunction, JPL and the U.S.S.R. workers both concluded that the rotation of Venus was retrograde. The estimate of rotation period made in the U.S.S.R. was 300 days (**25**) and at JPL two estimates were made—one 248 days, the other 266 days, with errors of about 30 days (**26, 27**). The 1964 measurements at JPL have not yet been completely reduced but are consistent with the 1962 measurements and are expected to yield a more precise value for the rotation period. Measurements made at Jodrell Bank in 1964 also confirm the retrograde aspect of the rotation, and yield a rotation period between 100 and 300 days (**28**). Similarly, the 1964 Lincoln Laboratory spectral measurements are consistent with a retrograde motion of Venus, but are still in the process of being reduced.

Probably the most extensive series of spectral measurements are being made this year at Arecibo. In Fig. 5, the measurements made through 1 August are shown. Each of the bandwidths was determined from individual spectra for different annuli with the results extrapolated to the corresponding limb-to-limb values and suitably averaged.

One might ask why a minimum in the values appears near inferior conjunction. Very loosely speaking, the apparent angular velocity due to the observer's motion is inversely proportional to the distance between Earth and Venus. Hence, this apparent angular velocity is largest at inferior conjunction when the distance is smallest. Since the inertial rotational angular velocity of Venus is retrograde, the apparent angular velocity, which is always smaller in magnitude, has its greatest effect in reducing the vector sum of the two angular velocities, and hence in reducing the bandwidths near conjunction.

We obtained the results shown in Fig. 5 from a weighted-least-mean-square fit to the data.

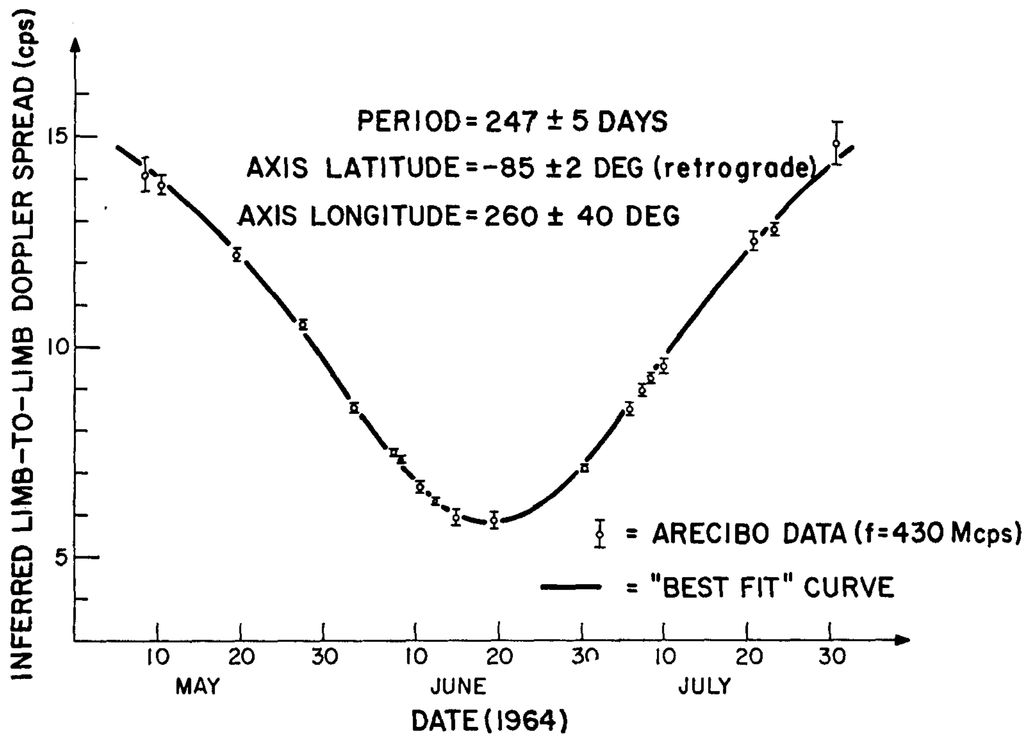

Fig. 5. Rotation of Venus.

The theoretical model used assumed the radius R_V of Venus to be a few kilometers less than 6100, and all motions were considered, including the movement of the radar site with respect to the center of the Earth. (The error in R_V does not affect the result significantly since it depends only on the square root of R_V. Of course, R_V can also be estimated from the bandwidth data; a preliminary attempt, however, indicated that the accuracy achievable would not be substantially better than 200 km.) The rotation period determined by the best fit is 247 ± 5 days, with the celestial latitude of the rotation axis being −85 ± 2 degrees, and the celestial longitude being 260 ± 40 degrees. The rotation axis is almost perpendicular to the ecliptic and, in fact, the corresponding inclination to Venus's orbital plane is about 84 degrees.

The formal probable error in this determination of rotation period, caused solely by the random errors involved in estimating the bandwidths, as shown by the extended data points in the figure, is only about one day. The corresponding formal probable errors for the axis direction are also considerably smaller than the values shown in the figure. The enlarged errors quoted represent an attempt at conservatism, pending the completion of an exhaustive search for possibly significant systematic errors that may have been overlooked in the preliminary analysis.

The final determination of the rotation period of Venus from the 1964 radar measurements will probably be sufficiently accurate to warrant establishing co-ordinates on the surface of Venus so that, for example, radar measurements of surface characteristics made at different conjunctions can be compared in a meaningful manner.

In summary, one can say that radar observations of the planets have already yielded several fruitful results. Most notable and firmly established are: an improvement of almost three orders of magnitude in the accuracy of expressing the Astronomical Unit in terrestrial units; the determination of a $0\overset{''}{.}5$ of arc correction to the relative longitude of Earth and Venus; and the accurate determination of the rotation period and rotation axis of Venus.

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