and the cone which touches the planes BOC, COA, AOB and the coordinate planes is

$$\sqrt{l_1 l_2 l_3 x} + \sqrt{m_1 m_2 m_3 y} + \sqrt{n_1 n_2 n_3 z} = 0.$$

Substituting for $l_1 l_2 l_3$, $m_1 m_2 m_3$, and $n_1 n_2 n_3$, we obtain the equations of these cones.

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A Method of obtaining Examples on the Multiplication of Determinants.

In the ordinary text-books on Algebra there is a lack of suitable examples on Multiplication of Determinants. Most of the examples that are given are particular cases of the theorem

$$D \Delta = D^n,$$

in which

$$D = \begin{vmatrix} a_1 & a_2 & \dots & a_n \\ b_1 & b_2 & \dots & b_n \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}, \qquad \Delta = \begin{vmatrix} A_1 & A_2 & \dots & A_n \\ B_1 & B_2 & \dots & B_n \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix},$$

where A_1, A_2, ..., B_1, ..., are the co-factors of a_1, a_2, ..., b_1..., in D.

If the determinant D is chosen at random, in most cases the second determinant Δ will be too complicated. It is easy, however, to choose D so that factors can be taken out of Δ; and thus a sufficiently simple second determinant is obtained.

For example, let

$$D = \begin{vmatrix} b & a & a \\ a & b & a \\ a & a & b \end{vmatrix} = (2a+b)(a-b)^2.$$

Then

$$\Delta = \begin{vmatrix} b^2-a^2 & a^2-ab & a^2-ab \\ a^2-ab & b^2-a^2 & a^2-ab \\ a^2-ab & a^2-ab & b^2-a^2 \end{vmatrix}.$$

Let the factor $b-a$ be taken out of each row of Δ. Then, multiplying the determinant so obtained by D, we have

$$\begin{vmatrix} b & a & a \\ a & b & a \\ a & a & b \end{vmatrix} \begin{vmatrix} a+b & -a & -a \\ -a & a+b & -a \\ -a & -a & a+b \end{vmatrix} = \begin{vmatrix} b^2+ba-2a^2 & 0 & 0 \\ 0 & b^2+ba-2a^2 & 0 \\ 0 & 0 & b^2+ba-2a^2 \end{vmatrix}$$

$$= (b-a)^3 (b+2a)^3.$$

Similarly, starting from the first of the two following determinants, we obtain

$$\begin{vmatrix} a & b & b & b \\ a & b & a & a \\ b & b & a & b \\ a & a & a & b \end{vmatrix} \begin{vmatrix} a & -a & b & -a \\ -b & b & -b & a \\ b & -a & a & -a \\ -b & a & -b & -b \end{vmatrix} = (a-b)^8.$$

In this example the two determinants multiplied are really identical.

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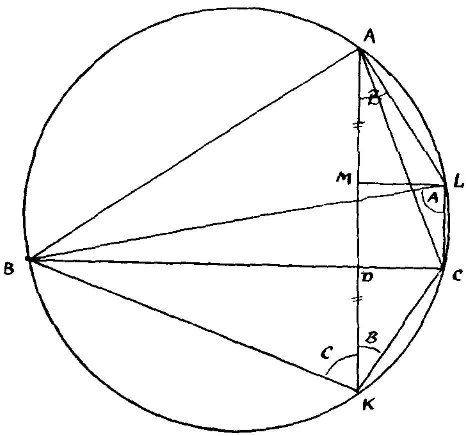
Geometrical Proof of tan A + tan B + tan C = tan A tan B tan C.



Let ABC be the $\triangle$, $AD \perp$ to BC produced to meet circumcircle in K, BL a diameter of circumcircle, $LM \perp$ to AK. Let BK, KC, CL and LA be joined.

$$\tan A = \frac{a}{CL}, \quad \tan B = \frac{DC}{DK}, \quad \tan C = \frac{DB}{DK};$$

$$\therefore \tan A + \tan B + \tan C = \frac{a}{CL} + \frac{a}{DK} = \frac{a(DK+CL)}{CL \, . \, DK}$$

$$\tan A \, . \, \tan B \, . \, \tan C = \frac{a}{CL} \times \frac{DC}{DK} \times \frac{DB}{DK} = \frac{a \, . \, AD}{CL \, . \, DK},$$

since $\qquad AD \, . \, DK = BD \, . \, DC.$