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ICE ACCRETIONS ON STRUCTURES

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Abstract. Three basic areas of ice accretion are discussed: (i) the motion of the (supercooled) droplets in air up to collision with the icing object, (ii) the net growth rates, and (iii) the heat balance of an icing cylinder. The weakness of presently used approaches to establish collision efficiency and possible improvements are outlined. Experimental evidence on the icing of cylinders is then reviewed which establishes that the growth rates by icing are far less than expected because up to 80% and more of the initially accreted water can bounce off or be lost by shedding at air temperatures of −20°C. The reduced growth rates have a considerable effect on the heat balance of the icing object, whose temperature and water-phase mixture are controlled by the temperature and phase of the lost water-substance. These latter properties are completely unknown and require further studies. In conclusion the scientific aspects of icing are far from being understood; empirical findings may have to provide guidance for applications.

Résumé. *Accrétions de glace sur des ouvrages.* On discute trois situations de base où peuvent se produire des accrétions de glace: (i) le mouvement de gouttelettes (en surfusion) dans l'air provoquant la collision avec l'objet givrant, (ii) la vitesse nette de croissance de la glace et (iii) le bilan thermique d'un cylindre givrant. On souligne la faiblesse des méthodes actuellement utilisées pour tenter d'établir l'efficacité des collisions et les possibilités de les améliorer. Des résultats expérimentaux sur le givrage de cylindres sont passés en revue. Ils montrent que la vitesse de croissance du givre est bien moindre qu'on ne l'attendait parce que jusqu'à 80% et plus de l'eau qui avait frappé le cylindre peut rejaillir ou être perdue par chute à des températures de l'air de l'ordre de −20°C. La réduction de la vitesse de croissance a un effet considérable sur le bilan thermique de l'objet givrant dont la température et la composition en phases sont contrôlées par la température et l'état physique (eau ou glace) de la matière perdue. Ces dernières propriétés sont complètement inconnues et demandent de nouvelles études. En conclusion, les aspects scientifiques du givrage sont loin d'être compris; des constatations empiriques peuvent avoir à servir de guide pour les applications.

Zusammenfassung. *Zur Vereisung von Objekten.* Drei grundlegende Aspekte atmosphärischer Vereisung werden diskutiert: 1. Die Bewegung der Wolkentröpfchen bis zum Kollisionspunkt auf dem Vereisungsobjekt, 2. die effektiven Wachstrumsraten, und 3. die Wärmebilanz eines vereisenden Zylinders. Die Schwächen der gegenwärtig gebrauchten Methoden zur Bestimmung der Kollisions-wahrscheinlichkeit werden aufgedeckt und Vorschläge werden vorgebracht zu ihrer Verbesserung. Eine Übersicht über neueste Vereisungs-experimente mit Zylindern zeigt, dass die Eiswachstumsraten viel geringer sind als erwartet da sogar bei −20°C bis zu 80% und mehr des ursprünglich angelagerten Wassers durch Abprallen und Abfliessen verlorengeht. Das reduzierte Wachstum hat einen bedeutenden Einfluss auf die Wärmebilanz, da die Temperature des Vereisungsobjektes wesentlich durch Temperatur und Phase des verlorenen H_2O bestimmt wird. Diese Grössen sind jedoch nicht bekannt und verlangen Abklärung in neuen Experimenten. Zusammengefasst muss gesagt werden, dass viele wissenschaftlichen Aspekte der atmosphärischen Vereisung nicht gelöst sind, und dass der Praktiker sich immer noch auf empirische Zusammenhänge verlassen muss.

1. Introduction

Ice accretion on structures is of interest to engineers and scientists particularly as it affects communication equipment, power transmission lines, and helicopter and aircraft operations. Ice deposits are often important in the design of structures because of the additional mechanical loads they may impose through their weight or increased wind loads, as well as the damage falling pieces of ice may cause during the melting phase or due to break-off by aerodynamic forces. If ice accretions are caused by freezing rain, then traffic can easily come to a standstill and plants and trees may be damaged. Operations of ships in cold weather can be affected by heavy icing caused by sea spray, etc., etc. All these situations can affect human safety and are accompanied by economic losses. There is only one beneficial application known to the author: protection of plants by continuous icing during periods of radiation frost. In such situations the constant release of the latent heat of fusion does not allow buds and other sensitive plant parts to cool below about −2°C, i.e. to temperatures at which damage by freezing of the sap would occur.

The studies of the icing of structures has benefitted considerably from research on ice formation in the free atmosphere (snow and hail formation) and it is not surprising that atmospheric physicists have devoted considerable time to this problem (for example, Langmuir, 1944; Melcher, 1951; de Quervain, 1954; Aufdermaur, 1973). Many power companies do or did simulate the natural icing and de-icing of transmission lines under controlled conditions, and may have established operational criteria. The physical phenomena, however, are far from being understood. This paper should demonstrate this by exposing and reviewing the complexity of a simple case like the icing of a cylinder. Three aspects of the icing process will be given special attention: (i) the air flow around the icing object, (ii) the collision, coalescence, bouncing and shedding mechanisms, and (iii) the resulting heat transfer.

In 1972 and 1974 experiments on the growth of rotating and gyrating hailstone models by ice accretion were carried out in the Swiss Hail Tunnel II (List, 1966) through a co-operative effort by the Swiss Eidg. Institut für Schnee- und Lawinenforschung, the University of Alberta in Edmonton, and the University of Toronto. These icing studies required a calibration of the liquid-water content in the air-stream of the tunnel by the icing of slowly rotating cylinders. The cylinder results proved to be very informative and exciting (List and others, 1976; Joe and others, 1976). In particular, it was shown that shedding of accreted water is a major factor in determining growth rates. These results and their consequences will be expanded and further explored here, and will, with a new assessment of the thermodynamics of icing, give an up-to-date version of the state of the art.

2. Droplet collision and collection

The icing of an object in the atmosphere can be separated into different phases. Since we normally are dealing with a situation involving wind or relative air motions—with the possible exception of freezing rain—the flow pattern around the iced object will control the motion of the droplets in the air. (Deposition from the vapour phase is not considered in this paper.) Another aspect relates to the motion of droplets in air. The next phase involves the collision mechanism as such. But growth rates are also determined by water losses from an icing object due to bouncing, partial coalescence, or from shedding of water which has been accumulated earlier.

The *aerodynamic flow* of air around an obstacle is generally not known. Even for cylinders or spheres the approach by Langmuir and Blodgett (1945) of using either viscous or potential flow is still used, not recognizing that neither flow pattern reasonably describes the boundary layer. At higher Reynolds numbers ($(Re) \gg 1$) viscous flow is not appropriate, and potential

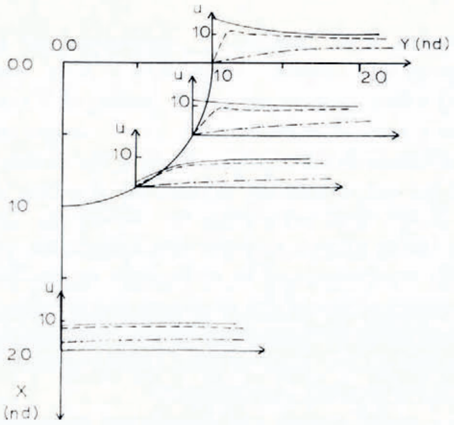

Fig. 1. Velocity profiles of the streamwise component u of the flow around a sphere, normalized to the free stream velocity far away from the sphere; solid lines for potential flow, dashed lines as measured at a Reynolds number of 2.6×10^4, dash–dot lines for viscous flow.

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flow does not adequately describe the flow at the object–air interface. This prompted Joe and others (1976) to measure the real flow around a sphere at a Reynolds number of 2.6×10^4 and to compare it with the two theoretically calculated situations. Figure 1 depicts the results. The differences near the sphere are obvious and the effects of the different treatments are considerable for small droplets with low inertia. The conclusion is that the flow pattern around an icing object should be measured first before calculations on collisions can be made—even if the object has a simple shape. Any theoretical calculation should not be considered adequate without experimental verification.

The *drag of droplets* needs to be known if droplet trajectories are calculated. Joe and others (1976) showed that the motions are extremely sensitive to the assumptions on drag, particularly if the droplets are not moving in the direction of the air flow. It seems advantageous to use the drag correction by Beard and Pruppacher (1969) and build it into the generally accepted formulation of droplet acceleration (see for example Landau and Lifshits, 1959). While the air flow around an icing object is characterized by a larger (Re), the droplet motion relative to the air is at a lower (Re), implying influence of viscous forces. The equation for two-dimensional motion (like that around cylinders) is

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{g} - \frac{\rho_a}{\rho_d}\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{v}-\mathbf{u}) - \frac{9\rho_a}{2\rho_d r}\left(\frac{\nu}{\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{t} \frac{\mathrm{d}(\mathbf{v}-\mathbf{u})}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{(t-\tau)^{\frac{1}{2}}} - \frac{9\rho_a \nu}{2\rho_d r^2}(\mathbf{v}-\mathbf{u})\left[1+\alpha(Re)^{\beta}\frac{(\mathbf{v}-\mathbf{u})}{|\mathbf{v}-\mathbf{u}|}\right], \tag{1}$$

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v}, \tag{2}$$

with $\mathbf{v}$ the droplet velocity relative to a fixed frame of reference attached to the stationary sphere, $\mathbf{g}$ the acceleration due to gravity, ρ_a and ρ_d the air and droplet densities, respectively, $\mathbf{u}$ the air velocity, r the droplet radius, ν the kinematic viscosity of air, t, τ time, (Re) the droplet Reynolds number, $\mathbf{x}$ the droplet position vector in respect to its centre,

$$\text{and} \qquad \alpha = \begin{cases} 0.102 \\ 0.115 \\ 0.189 \end{cases} \qquad \beta = \begin{cases} 0.995 \\ 0.802 \\ 0.632 \end{cases} \qquad \text{for} \qquad \begin{cases} 0.01 < (Re) < 1.5 \\ 1.5 < (Re) < 20 \\ 20 < (Re) < 400 \end{cases}$$

The first term on the right-hand side represents the effect of the weight, the second the added-mass effect or the fraction of the displaced volume of air which has to be accelerated, the third is a history term, and the fourth the Stokes drag as extended by Beard and Pruppacher. The first and fourth terms represent the major contributions to the total acceleration.

The absolute motion of the droplets relative to an icing object can be calculated by embedding them into a known air flow and calculating their relative motion with the help of Equation (1). This procedure is particularly important for the contribution to collision by bounced or shed droplets which collide again.

The *collision efficiency* is a standard concept used to describe the fraction of droplets in a swept-out air volume which is colliding with an object. The "geometrical" definition of the swept-out air volume, based on the cylinder dimensions seems entirely adequate for ice accretions including situations with freezing rain. However, it is normally not spelled out that physical contact is required. Until recently theoretical calculations necessitated a definition of a collision as an approach to within a certain limit (10^{-3} radii of the collector drop, according to Davis and Sartor (1967)). Stuart (unpublished) seems to be the first who was able to produce physical collisions theoretically by using the historical terms in the equation of motion of droplets and without such restrictions. The concept of collision is clouded by situations where a droplet can approach an obstacle and be repelled by a compressed air film without physical contact. This results in bouncing which, when observed in experiments, would normally count as collision. Based on experiments by Whelpdale and List (1971) it is further suggested that collisions with contact always involve partial coalescence, with the bouncing droplet generally smaller than before impact.

It is standard procedure to define the *collection efficiency* as the number fraction of the droplets in the swept out air volume which is accreted onto the collector. This definition needs to be generalized by changing "number fraction" to "fraction of mass". This would include the mass imparted by partially coalescing droplets. However, clarification is required for shedding water which is a key factor in heavy ice accumulation. Not all water losses can be directly associated to individual collisions. A large fraction may originate from bulk water-substance of the collector. This water may have been collected earlier and may in the meantime have lost its original properties and accepted those of the water in the surface skin which often covers all or part of the surface of the icing object.

Hence, a new concept is proposed: the *net collection efficiency*. This new term relates to the permanently accreted water-substance, whereas the "regular" collection efficiency does not distinguish between permanent and temporary accretion; it contains the sum of both. The net collection efficiency is the ratio of the net growth rate, as represented by the difference between accretion of droplets and any loss by shedding and bouncing or similar mechanisms, and the total droplet mass flux in the swept-out air volume. From the point of view of the net mass accretion, this definition is adequate. It will be shown later that the net mass accretion is closely related to the properties (temperature and phases) of the lost water-substance.

Typical *types of water losses* are depicted in Figure 2. Two light sources were used in taking those photographs, illuminating the scene with a 30° angle from behind. The streak separation could then be used to calibrate the drop size (range 10–200 μm) according to a method designed by Cannon (1970). The diameter of the gyrating (Fig. 2a–d), rotating (2e) or fixed (2f) ice spheroids was 2–4 cm.

Figure 2a shows part of the originally impinging droplet re-emerge as a Raleigh jet, which is in the state of break-up. Figure 2b depicts a crown which also results from an impact. Whereas the water in the crown originates from the surface skin and has its temperature, jet water may not have had time to adjust its temperature to that of the collector surface because it mainly consists of water remnants of the colliding droplet. Heat transfer calculations by Gröber and others (1961) show that 50 μm diameter droplets colliding at 10 m s^{-1} and re-emerging after *c.* 2×10^{-5} s may have lost around 80% of the original heat deficit. For 100 μm droplets the figure is *c.* 50–70%. The residence time is increased, hence the effect is not as pronounced as would be expected from thermal considerations alone. Jet ejection events are spectacular but also rare. Critical minimum droplet sizes and impact speeds for their formation are not known.

Sometimes whole pieces of a surface water skin seem to be shaken off and to disintegrate immediately into large clouds of droplets (Fig. 2c). If there are slight protuberances on the surface of a collector, the liquid water will accumulate there in the presence of centripetal and drag forces, as in Figure 2d, and gravity—and shed as soon as the collected drop reaches a large enough size for the combined drag, centripetal and gravity forces to exceed the surface tension. With the surface tension $\sigma = 0.075\,6$ N m^{-1} at 0°C, a water density $\rho = 10^3$ kg m^{-3}, and a cylinder radius $R = 0.012\,5$ m, separation is achieved as a function of the rotation frequency f [Hz] for drop radii r where

$$r > \approx \frac{1}{2\pi f}\left(\frac{6\sigma}{\rho R}\right)^{\frac{1}{2}} = \frac{0.030\,3}{f}. \tag{3}$$

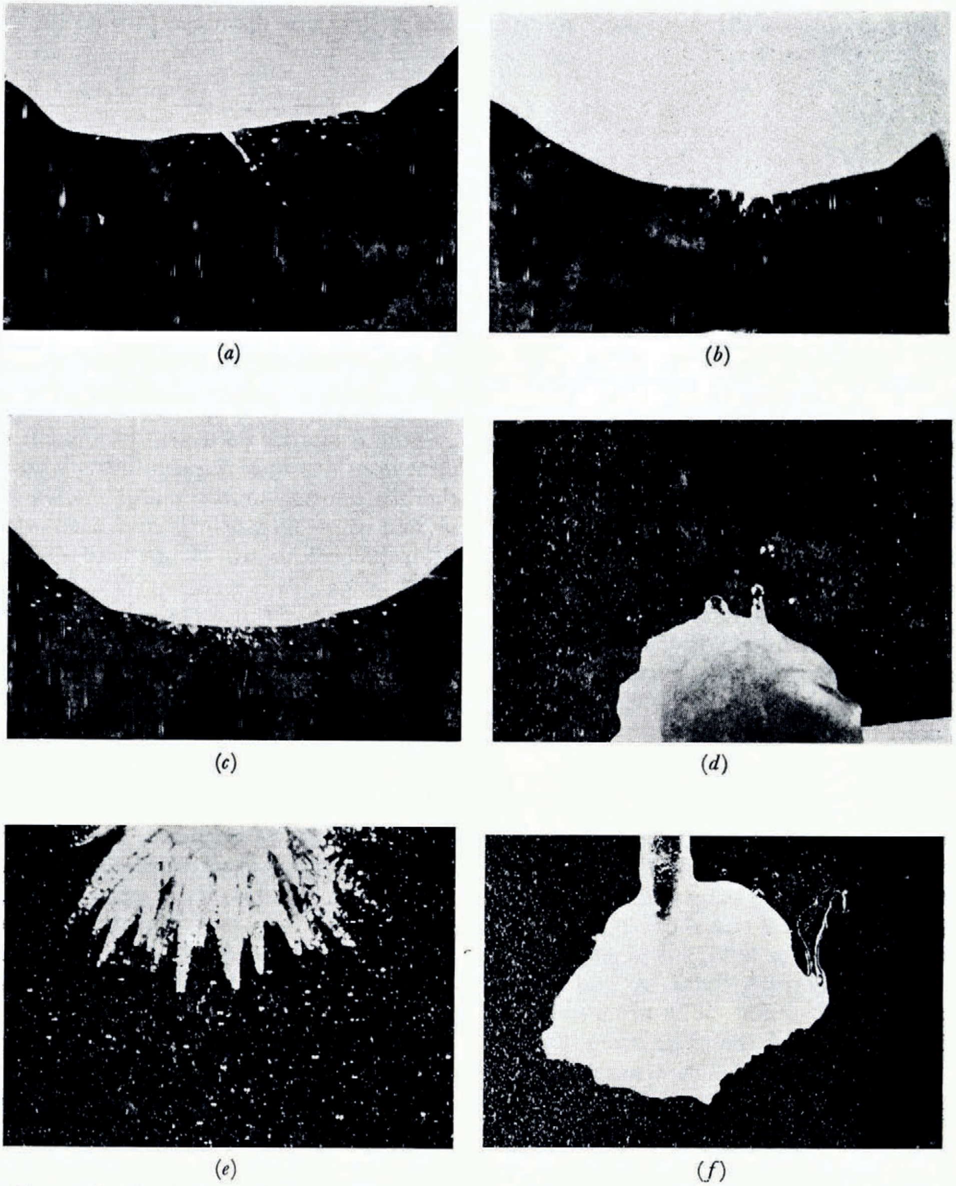

Fig. 2. *Photographs of various water-loss mechanisms which affect the net growth of ice particles (diameter 2 cm; sphere in f, spheroids with axis ratios 0.67 in a–e), as observed in icing experiments in the Swiss Hail Tunnel II. The air flow is from below. The icing conditions for a, b, and c were:* $t_A = -20°C$, $V = 33.4\ m\ s^{-1}$, $W = 2.85 \times 10^{-2}\ kg\ m^{-3}$, *spin frequency* $f_r = -3.17\ Hz$, *nutation-precession frequency* $f_g = 16\ Hz$; *for d:* $t_A = -10°C$, $V = 31\ m\ s^{-1}$, $W = 5.1 \times 10^{-2}\ kg\ m^{-3}$, *otherwise like a; for e:* $t_A = -10°C$, $V = 23.1\ m\ s^{-1}$, $W = 3.87 \times 10^{-2}\ kg\ m^{-3}$, *rotation frequency* $f_r = 6.7\ Hz$, *around a horizontal, fixed axis; for f:* $t_A = -5°C$. $V = 25.8\ m\ s^{-1}$, $W = 2.09 \times 10^{-2}\ kg\ m^{-3}$, *fixed suspension. The pressure for* $-5°C$ *was* 55.5 kPa, *for* $-10°C$ 50.5 kPa, *and* 41.8 kPa *for* $-20°C$. *(a) shows a jet, (b) a crown, (c) a cloud of droplets from a disintegrating water skin, (d) drops detaching from protuberances, (e) drops spinning off from spikes and (f) shedding water.*

With gravity neglected, Equation (3) gives orders of magnitude as listed in Table I. Frequencies for freely falling hailstones of up to 50 Hz are reasonable as Kry and List (1974) point out. Hence, drops shed from rotating hailstones with 3 cm diameters would be about 0.5–2.0 mm in diameter.

Table I. Critical radii r_c of drops which can be shed from a cylinder of a radius $R = 0.0125$ m rotating at a frequency f, and centripetal accelerations a_c acting on drops in multiples of the acceleration due to gravity g

f Hz	$r_c \times 10^2$ m	a_c/g
4	≈ 0.75	≈ 0.8
20	≈ 0.15	≈ 20
40	≈ 0.07	≈ 80

If rotation takes place around a fixed axis then the formation of spikes is to be expected as soon as shedding conditions are approached. While Figure 2e represents a spinning ice particle it can be expected that rotating cylinders exhibit similar patterns. One point needs to be made here: shedding of a drop of a given size does not only require larger separating than holding forces. It is also necessary that there is enough unfrozen and mobile water around to form such a drop. If this is not the case and water freezes or is stabilized in an ice framework, then no shedding will occur unless the drop contains air. Drops with air bubbles have smaller r_c.

If icing objects are fixed in space, the accumulation of water at certain locations and its shedding occur more readily. Figure 2f shows the release of water at the edge of a rigidly suspended ice particle due to aerodynamic forces (the air flow in all situations depicted in Figure 2 is from below). Water can stream from fixed objects just due to the action of gravity if exposed to horizontal wind.

3. Summary of the Swiss icing experiments involving cylinders

The icing experiments carried out in Switzerland involved cylinders (length 0.22 m, diameter 0.022 m), rotating at 2–4 Hz in a horizontal position in an airstream which was directed upward. The icing experiments were carried out at air densities ranging from 0.477–1.005 kg m^{-3} and at a constant temperature of -20°C. Figure 3 gives a typical example of an iced cylinder with a thickness depending on the liquid-water content of the air which was moving by at speeds between 20 and 45 m s^{-1}. The deposit thickness was governed by the net collection efficiency. The mean volume diameters of the drops varied from 41–100 μm, the median volume diameter from 65–144 μm. The accuracy based on statistical and systematic errors in the liquid-water content W of the air is $<25\%$, and $<5\%$ for the air speed V. The relative error for the net collection efficiency is generally $\Delta E/E < 0.5 \Delta W/W$ within the range of speeds of the experiments. For further details see List and others (1976).

A photograph taken along the axis of a rotating icing cylinder is shown in Figure 4 and indicates bouncing, probably of droplets with diameters >80 μm (Joe and others, 1976). The bouncing is not affected by the sense of rotation, since $f \leqslant 4$ Hz implies surface speeds of *c*. 0.3 m s^{-1}—as compared to a wind velocity of 24.2 m s^{-1}. It is not understood yet why bouncing seems to occur predominantly at angles from the flow of $>60°$ The photographic set-up was not sophisticated enough to focus on specific surface elements with bouncing. No shedding is revealed for cylinders; bouncing seems to account for all the losses. This is quite different from the situation for gyrating hailstones where the bulk of the discarded water is shed (Fig. 2d, e and f), the reason being the higher centripetal forces (Joe and others, 1976).

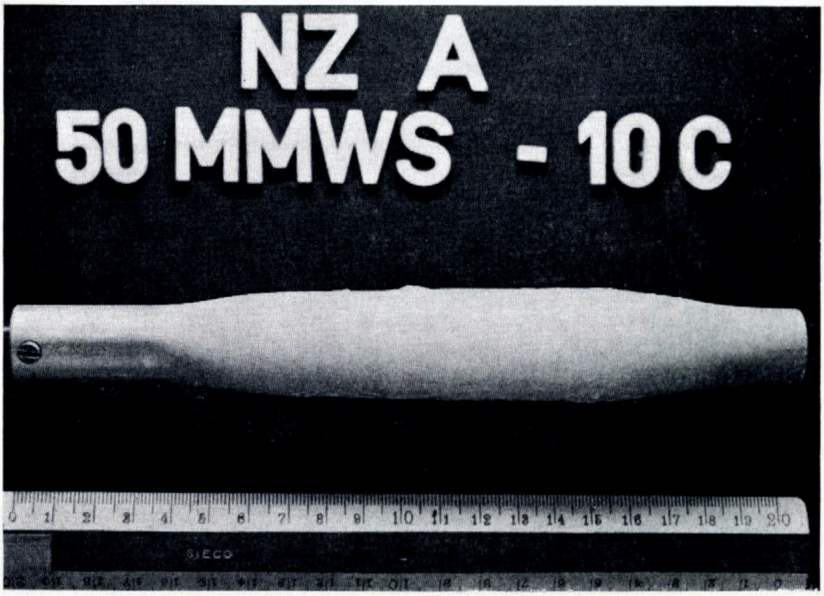

Fig. 3. Cylinder iced for 60 s by nozzle A at a temperature of $-20°C$ *and a pressure of* 50.5 *kPa, equivalent to the* $-10°C$ *level in the atmosphere. The relative air speed was* 38.4 m s^{-1} *with a corresponding velocity pressure of 50 mm of water or 0.49 kPa; scale in centimeters.*

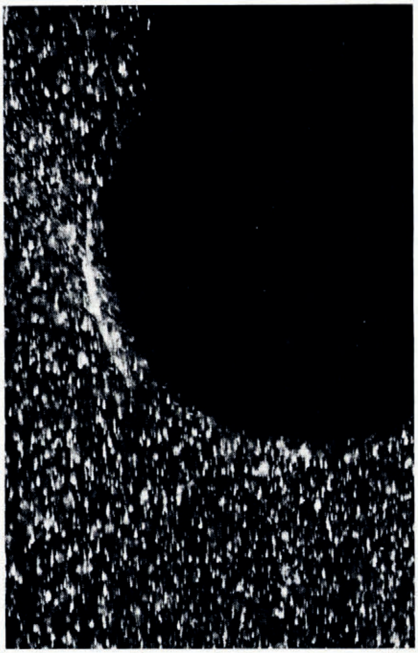

Fig. 4. Photograph taken along the axis of a 2.2 cm diameter cylinder rotating at 4 Hz; water was injected through nozzle B at laboratory pressure (73 kPa), air velocity 24.2 m s^{-1}, *liquid-water content* $\approx 3.2 \times 10^{-2}$ kg m^{-3}, *temperature* $-20°C$.

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According to List and others (1976) the net collection efficiency of cylinders (diameter 2.2 cm) as obtained from the experiments in the Swiss hail tunnel can be described for an air density of $\rho = 1.293$ kg m^{-3} and an air temperature of -20°C by the following equation:

$$E_{\text{net}} = 0.13 + 1.182\,8 \times 10^{-3}(2\,500 - V^2) + \frac{0.87 - 1.182\,8 \times 10^{-4}(2\,500 - V^2)}{1 + 0.425 V^2 W}, \qquad (4)$$

where V is the air speed in m s^{-1} and W the liquid-water content in kg m^{-3}.

The dependence of E_{net} on V^2 is shown for different water contents in Figure 5. Note that there is a limiting E_{net} for $W \rightarrow \infty$ which drops linearly as a function of V^2. E_{net} is shown as a function of W at different speeds V in Figure 6. The net collection efficiencies are generally very low.

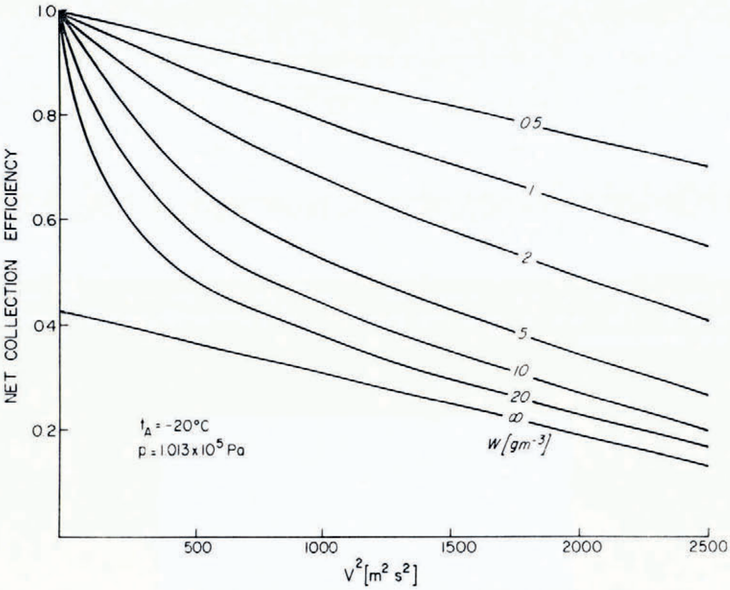

Fig. 5. Net collection efficiency of a cylinder rotating with 2–4 Hz at a temperature of -20°C, as a function of the square of the air velocity; parameter liquid-water content W; according to Equation (4).

While it is not yet known how E_{net} drops with increasing air temperature, it is nevertheless safe to say that E_{net} would drop considerably as the air temperature approaches zero ($E_{\text{net}} = 0$ at an air temperature 0°C).

In the case of freezing rain, no measurements have been made about the retained water, but some of the concepts developed here may well still apply.

4. The heat-balance equation for cylinder icing with shedding

The general heat-balance equation for quasi-steady-state icing of a cylinder was given by List (1962) as:

$$Q^{\star}_{\text{CC}} + Q^{\star}_{\text{ESC}} + Q^{\star}_{\text{CP}} + Q^{\star}_{\text{F}} = 0, \qquad (5)$$

where $Q^{\star}$ represents the heat flux to the cylinder and CC stands for conduction and convection, ESC for evaporation, sublimation or condensation, CP for heat imparted due to the supercooling (relative to the temperature at which freezing takes place) of the accreted cloud particles and F for the heat produced by the release of the latent heat of fusion during freezing.

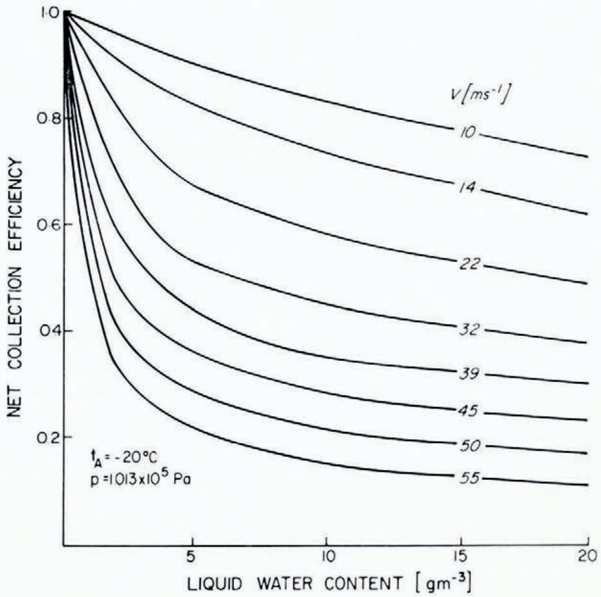

Fig. 6. Net collection efficiency of a cylinder rotating with 2–4 Hz at a temperature of −20°C, as a function of the liquid-water content; parameter air speed V; according to Equation (4).

Gröber and others (1961) give the following relationship for heat transfer of a cylinder in cross-flow for the Reynolds number range under consideration ($2\,000 < (Re) < 60\,000$):

$$(Nu) = 0.24(Re)^{0.6}. \tag{6}$$

(Nu) is the Nusselt number, which is defined as $(Nu) = \alpha D/k$. α is the heat-transfer coefficient, D the diameter and k the thermal conductivity.

This leads to the heat flux per unit of cylinder length

$$Q^{\star}_{\mathrm{CC}} = -0.24\pi(Re)^{0.6}\, k(t_{\mathrm{D}}-t_{\mathrm{A}}), \tag{7}$$

where t_{D} is the temperature of the ice deposit surface and t_{A} that of the air up-stream of the cylinder.

The heat transferred by evaporation can be estimated on the basis of the similarity of the diffusion of heat and the diffusion of H_2O molecules. This is equivalent to saying that the Sherwood number $(Sh) = (Nu)$, which is the case for air at atmospheric pressures. The definition of (Sh) is $(Sh) = \beta D/D_{\mathrm{wa}}$. β is the mass-transfer coefficient and D_{wa} the diffusivity of water vapour in air. Combining with Equation (6) and applying the transfer to the surface of a cylinder segment gives:

$$Q^{\star}_{\mathrm{ESC}} = -0.24\pi(Re)^{0.6}\, C_{1,2} T_{\mathrm{A}}^{-1}(e_{\mathrm{sc}}-e_{\mathrm{v}})\, D_{\mathrm{wa}}. \tag{8}$$

The mass-transfer coefficient β is based on a concentration difference which is replaced in Equation (8) by $C_{1,2}T_{\mathrm{A}}^{-1}(e_{\mathrm{sc}}-e_{\mathrm{v}})$, where $C_1 = 8.663\times10^3$ J K m^{-3} Pa^{-1} for liquid–gas transitions and $C_2 = 9.835\times10^3$ J K m^{-3} Pa^{-1} for solid–gas transitions (see also List and Dussault, 1967); T_{A} is the absolute air temperature, e_{sc} is the saturation vapour pressure over the cylinder, and e_{v} is the saturation vapour pressure at air temperature.

The heat taken away from a cylinder segment due to accretion of supercooled water droplets with temperature t_{A} is supplemented or diminished by the fact that a considerable fraction of the initially accreted water is shed or lost by bouncing (List and others, 1976). This lost water does not contribute to the heat transfer if its temperature is equal to the temperature of the incoming drops (Case I). However, if the shed water were less supercooled or even at 0°C then it would carry heat away and the supercooling would allow further freezing on the cylinder (Case II). There is another possible situation: the lost water-substance may be in a partially or completely (Case III) frozen state. This is equivalent to heat added to the cylinder because heat needs to be removed (towards the cylinder) to cause freezing of the shed water up to the moment of take-off.

With these explanations the heat $Q^{\star}_{\mathrm{CP}}$ imparted to a cylinder segment by icing is

$$Q^{\star}_{\mathrm{CP}} = -\nu(Re)\{E_{\mathrm{net}} W \bar{c}_{\mathrm{W}}(t_{\mathrm{D}}-t_{\mathrm{A}}) + (E-E_{\mathrm{net}})\, W[\bar{c}_{\mathrm{W}}(t_{\mathrm{S}}-t_{\mathrm{A}}) - I_{\mathrm{S}} L_{\mathrm{f}}]\}. \tag{9}$$

where W is the liquid-water content, E the "regular" collection efficiency, E_{net} the net collection efficiency, $\bar{c}_{\mathrm{W}}$ the heat capacity of water averaged over the temperature interval, t_{S} the droplet temperature right after shedding, I_{S} the fraction of shed water-substance which is frozen when leaving the cylinder surface, and L_{f} the latent heat of fusion at the temperature t_{S}.

The amount of heat used to freeze the permanently accreted water is given by

$$Q^{\star}_{\mathrm{F}} = \nu(Re)\, E_{\mathrm{net}} W L_{\mathrm{f}} I, \tag{10}$$

where I is the fraction of the accreted water which is actually frozen.

Substituting Equations (7)–(10) in Equation (5) leads to:

$$L_{\mathrm{f}} I = \frac{0.24\pi (Re)^{-0.4}}{\nu W E_{\mathrm{net}}}\,[k(t_{\mathrm{D}}-t_{\mathrm{A}}) + C_{1,2} T_{\mathrm{A}}^{-1}(e_{\mathrm{sc}}-e_{\mathrm{v}})\, D_{\mathrm{wa}}] + \\ + \bar{c}_{\mathrm{W}}(t_{\mathrm{D}}-t_{\mathrm{A}}) + (E/E_{\mathrm{net}}-1)[\bar{c}_{\mathrm{W}}(t_{\mathrm{S}}-t_{\mathrm{A}}) - I_{\mathrm{S}} L_{\mathrm{f}}]. \tag{11}$$

In order to explore the character of this equation in the rest of this section, the air temperature is assumed to be $t_{\mathrm{A}} = -20$°C (in accordance with experimental data on E_{net} of List and others (1976)). Further, the cylinder temperature is fixed at $t_{\mathrm{D}} = 0$°C.

Using the values for the various parameters given by Baur (1953): $\nu = 1.15\times10^{-5}\ \mathrm{m^2\,s^{-1}}$ (for −20°C), $k = 2.28\times10^{-2}\ \mathrm{J\,m^{-1}\,s^{-1}\,K^{-1}}$, $D_{\mathrm{wa}} = 1.701\times10^{-5}\ \mathrm{m^2\,s^{-1}}$, $L_{\mathrm{f}} = 3.33\times10^{5}\ \mathrm{J\,kg^{-1}}$, $\bar{c}_{\mathrm{W}} = 4.265\times10^{3}\ \mathrm{J\,kg^{-1}\,K^{-1}}$, and given a cylinder diameter of $D = 2.5\times10^{-2}$ m, Equation (11) can be reduced and the ice content I of the deposit can be evaluated for three very specific hypothetical Cases I, II and III. The reduced Equation (11) can be written as follows with V in $\mathrm{m\,s^{-1}}$ and W in $\mathrm{kg\,m^{-3}}$:

$$I = 6.73\times10^{-3}\,\frac{V^{-0.4}}{W E_{\mathrm{net}}} + 0.256 + [(1/E_{\mathrm{net}})-1][\bar{c}_{\mathrm{W}}(t_{\mathrm{S}}-t_{\mathrm{A}})\, L_{\mathrm{f}}^{-1} - I_{\mathrm{S}}]. \tag{12}$$

The third terms T_3 for the three cases are:

$$\left.\begin{aligned} T_{31} &= 0; \\ T_{32} &= 0.256[(1/E_{\mathrm{net}})-1]; \\ T_{33} &= -0.744[(1/E_{\mathrm{net}})-1]. \end{aligned}\right\} \tag{13}$$

Figure 7 depicts the fraction of accreted water I which can freeze as a function of the liquid-water content for Case I. In this situation no thermodynamic interaction takes place during the temporary residence on the cylinder of the shed water. It is interesting to note that the curves for slow speeds or at low liquid-water contents W, i.e. conditions with a high E_{net}, are representative for icing of cylinders without shedding. If the speed if increased, the heat transfers by conduction and convection $Q^{\star}_{\mathrm{CC}}$ and by evaporation $Q^{\star}_{\mathrm{ESC}}$ are also increased. Without shedding this trend would produce lower ice fractions because the accretion of non-freezing water is enhanced. Thus less of the accreted water would be frozen with increasing speed (Fig. 8). Furthermore it is necessary to remember that the accretion without shedding is proportional to the speed. However, the shedding is considerable (see Figs 5 and 6) and the effective W (which causes the net growth) is much less than the indicated true total liquid-water content. For $W = 5\times10^{-3}\ \mathrm{kg\,m^{-3}}$ $E_{\mathrm{net}} = 0.9$ at $V = 10\ \mathrm{m\,s^{-1}}$, but $E_{\mathrm{net}} = 0.27$ at $V = 50\ \mathrm{m\,s^{-1}}$. This explains in general why more of the net accreted water can be frozen at higher speeds with shedding than without (measured) shedding.

The second hypothetical situation assumes that the net accreted water receives the benefit of the initial supercooling of all the incoming water, even that which is later shed, i.e. at the point of departure the latter has a temperature of 0°C. Then the situation changes even more because the shed water carries the maximum amount of heat away from the cylinder (Case II). The icing fraction is increased (Fig. 9), which is particularly visible in the separation of the 10 and 20 m s^{-1} curves which essentially coincided in Figure 7. It is noted that

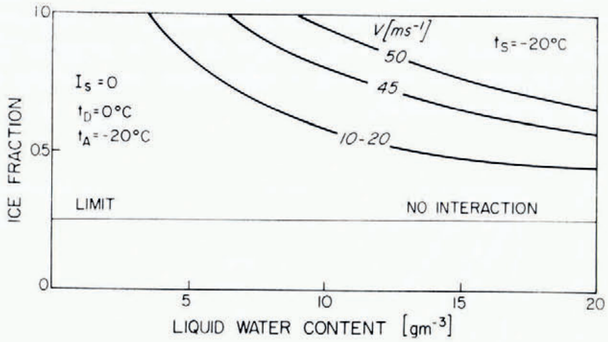

Fig. 7. Fractional ice content in a spongy deposit growing on a slowly rotating cylinder at −20°C, at various speeds, as a function of the liquid-water content. Assumptions: shedding occurs as measured, the shed water has the temperature of the air, $t_S = t_A$, implying no thermal interaction.

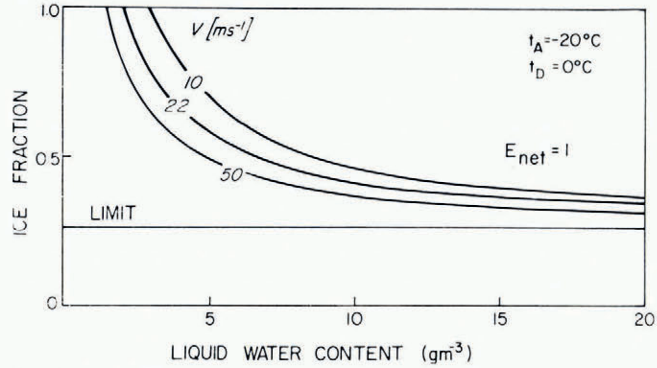

Fig. 8. As in Figure 7, but the assumption is as follows: no shedding is allowed to occur, thus relationships are obtained as in earlier calculations on spongy ice growth.

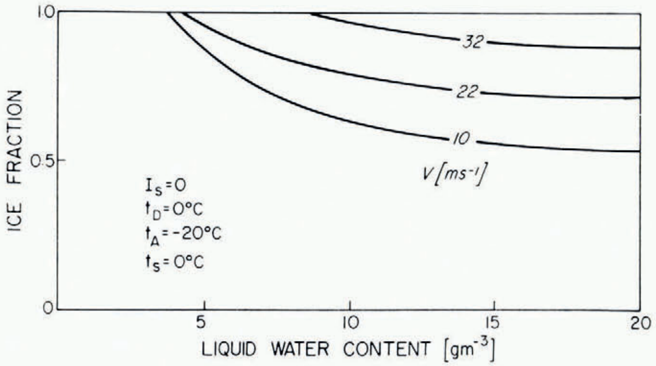

Fig. 9. As in Figure 7, but the assumptions are as follows: shedding as measured, the shed water is supposed to have the temperature of the cylinder surface ($t_S = 0°C$), and no ice is allowed in the shed particles ($I_S = 0$).

at speeds higher than *c.* 35 m s^{-1} all the permanently accreted water will be completely frozen and no liquid water will be contained in interstices. In other words, at high speeds and at a temperature of $-20°$C there is no formation of spongy ice on a cylinder with a diameter of 0.025 m, if the shed water had a temperature $t_s = 0°$C, no matter how high the liquid-water content is.

There is another possibility: the shed water may in part be frozen, or ice particles may be shed or broken from the cylinder surface (Case III). This process would be equivalent to a heating of the cylinder. The extreme case with shedding of completely frozen particles ($I_S = 1$) from a cylinder surface with a temperature of 0°C is depicted in Figure 10. If the shed particles consist of ice only, the warming is substantial, particularly at high speeds, and the formation of spongy ice is more likely.

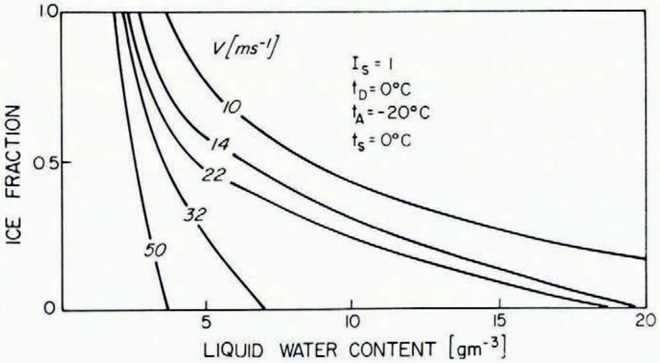

Fig. 10. As in Figure 7, but the assumptions are as follows: shedding as measured, the shed particles consist entirely of ice ($I_S = 1$) and have the temperature of the spongy cylinder surface ($t_S = 0°$C).

A general discussion of the hypothetical cases may be timely now about the bounds $I = 0$, $I = 1$ in Figures 7 to 10. As $I = 0$ implies that all the ice formed is shed, water cannot stay and would be shed continuously because the water skin cannot grow beyond a certain limit. $I = 0$ would even indicate $E_{net} = 0$. The experiments (List and others, 1976) have not indicated such a behaviour of no growth of cylinders at high speeds. Hence, near and at $I = 0$, and with $I_S = 1$, Figure 10 shows only an extreme and improbable situation. One can expect to have ice in detaching drops according to Aufdermaur and Larsen (1976), but it is unlikely that the drops are completely frozen (equivalent to $I_S = 1$) at the time of separation from the cylinder.

The situation in the region $I = 1$ is quite different, because it is physically realistic. $I = 1$ represents the conditions where all the net accreted water is frozen, but the surface temperature is still 0°C. However, a lowering of liquid-water content or speed would keep $I = 1$ and lower the surface temperature.

The three cases of Figures 7, 9 and 10 are combined in Figure 11 for two speeds. It shows for $V = 10$ m s^{-1} that the spread in ice fraction between $I_S = 0$ and $I_S = 1$ ($t_S = 0°$C) at any one liquid-water content is relatively small because E_{net} is just a bit smaller than 1. At 32 m s^{-1} however, E_{net} is markedly lower than 1 (Figs 5 and 6) and the spread in ice deposit conditions is substantial.

These considerations show that the properties of the separating water-substance are key factors in the heat transfer and that they can greatly affect the sponginess or surface temperature of an icing cylinder. Figure 12 relates specifically to this problem and gives the heat carried away from the cylinder by detaching drops for different ice fractions in the shed water,

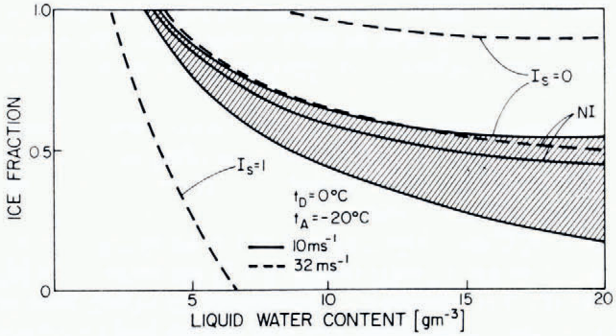

Fig. 11. Comparison of the three special cases given by Equation (12) for a situation involving net collection efficiencies near 1 ($V = 10$ m s^{-1}) and conditions with low E_{net} ($V = 32$ m s^{-1}); $t_A = -20$°C.

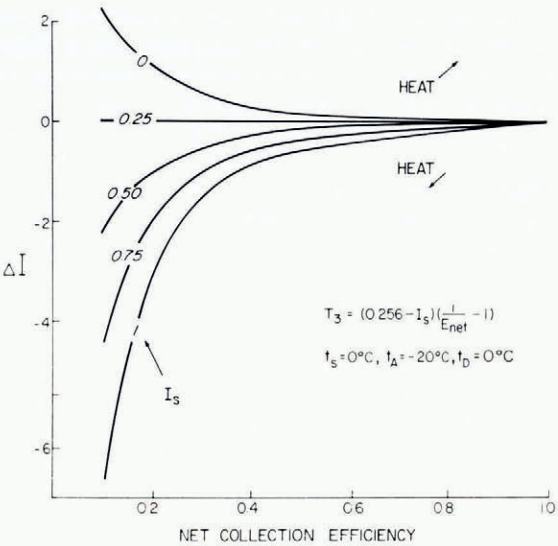

Fig. 12. Contribution ΔI to the heat transfer of a cylinder at $t_A = -20$°C for fractional ice content of lost drops varying from $I_S = 0$ to $I_S = 1$. One unit of ΔI is equal to the heat loss required to continuously transform the cylinder deposit from water to ice.

whereby the unit of heat transfer is given by the energy it takes to transform the net accreted water into ice at 0°C, a heat transfer component which is provided by the shed water. Figure 12 demonstrates that the effect on the permanently accreted water is minimal at $E_{net} \approx 1$. However, at low net collection efficiencies (0.15 has been measured at −20°C), the effect can overpower all the other heat-transfer components. What really happens in Nature or a corresponding laboratory experiment is still unknown but it must be within the bounds discussed above. The hypothetical cases also give an idea about what needs to be measured in new follow-up icing experiments.

5. Summary and conclusions

Experimental results on icing of cylinders at temperatures of −20°C have been extrapolated to sea-level conditions. They show for air temperature of −20°C that shedding of particles is becoming important for the icing rates of cylinders and the related heat transfer, particularly if the relative speeds of the air and cylinder surpass 10 m s^{-1} and the liquid-water

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contents are higher than 2×10^{-3} kg m^{-3}. It is expected that those limits would move to lower values if the air temperature were nearer to the freezing point. Since no measurements have yet been taken under such conditions no quantitative statements can be made in this respect.

Increases in liquid-water content always lead to larger deposit rates ($\approx E_{\text{net}} WV$). The decrease in net collection efficiencies coupled with increasing speed increases the heat transfer such that a larger fraction of the net accreted water can freeze. This trend is the reverse of the situation hitherto assumed with no shedding.

It is shown that, at $-20°$C and at speeds >35 m s^{-1}, no spongy ice ($I = 1$) can be formed on cylinders if the shed water had a temperature of $0°$C, no matter how high the liquid-water content is. This possibility is in complete contradiction to beliefs held up to now and may also have considerable impact on our ideas and concepts on hail formation, where the lack of large fractions of liquid-water in spongy ice shells was hard to understand or led to the assumption that the liquid-water contents of the cloud were always rather low.

The status of the separating water-substance is very important in assessing its effect on the heat transfer. It is obvious that bouncing and shedding may lead to different effects. Bouncing or glancing collisions may not lead to large interactions because of short residence times (order of 5 μs for relative speeds of 10 m s^{-1}). This applies to a lesser degree to Raleigh jets which consist of water just impacted. Crown water, however, is drawn from the original water skin and would separate with a temperature equal to that of the surface: shedding of water is in the same category. Photographic evidence from the cylinder icing experiments indicates that the bulk of the separating water originates from bouncing. A comparison with gyrating hailstones suggests that increases in rotation frequencies from ≈ 2 Hz to ≈ 20 Hz or more would shift the balance more towards shedding of surface skin or water collected on protuberances.

The new information also points to the need for further experiments dealing directly with the characteristics of the separating drops or ice–water mixtures.

The situation in terms of applications to the icing of structures is becoming less promising, because more basic studies are necessary before we can calculate icing rates under different atmospheric conditions. Even for cylinders we have to recognize that collision-efficiency calculations require exact knowledge of the real flow pattern. Approximations by potential or viscous flow are inadequate. Fortunately, excellent drag coefficient measurements are available for droplets and these allow calculations of absolute trajectories with proper "historic" terms if those droplets can be embedded in a known flow around the icing object.

Atmospheric icing does not only occur when supercooled droplets are blown against structures; in some areas of the world freezing rain is even more important. This kind of icing is less understood because no experiments simulating, for example, the icing of power lines from supercooled drops with diameters 1–5 mm have ever been carried out. The problem of generating such drops at terminal fall speed and at temperatures as low as $-10°$C or lower have still not been overcome.

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DISCUSSION

J. W. GLEN: Is the ring of shedding which develops on some of your spheres not a result of streaming flow on the surface of the sphere which undergoes a hydraulic jump at the ring and then is well placed for shedding?

R. LIST: Yes. Rotation rates are of the order 10–30 Hz; but I do not understand how such stream flow is established so rapidly.

M. DE QUERVAIN: It is very important to explore the mechanism of ice accretion, as Dr List has done for many years, but the result is somewhat discouraging in so far that most icing factors (parameters) entering the equations are provided by Nature (temperature, wind, drop size, etc.). If we are confronted with the problem of preventing icing or getting rid of it we have few possibilities to play with. Can you see any except adapting the shape of a body endangered by icing to minimum conditions?

LIST: I do not see any other reasonable measures except the prevention of icing by heating.