ORIGIN AND SCALE OF COORDINATE SYSTEMS IN SATELLITE GEODESY

J. B. Zieliński
Department of Planetary Geodesy, Space Research Centre,
Polish Academy of Sciences, Warsaw, Poland

ABSTRACT

The center of mass of the Earth is commonly taken as origin for the coordinate systems used in satellite geodesy. In this paper the notion of the "geocenter" is discussed from the point of view of mechanics and geophysics. It is shown that processes in and above the crust have practically no impact on the position of the geocenter. It is possible however that motions of the inner core may cause variations of the geocenter of the order of 1 m. Nevertheless the geocenter is the best point for the origin of a coordinate system. Mather's method of monitoring geocenter motion is discussed, and some other possibilities are mentioned. Concerning the scale problem, the role of the constant GM and time measurements in satellite net determinations are briefly discussed.

INTRODUCTION

In a number of coordinate systems, applied in practice as well as proposed, the center of the Earth's mass, "geocenter", is designated as the origin. This is the case for systems realized by means of satellite observations as well as for the global geodetic system and astronomical systems connected with directions to very distant space objects. The notion of the "geocenter" was widely discussed during the Torun Colloquium by Bursa (1974), Moritz (1974), Groten (1974) and mentioned by many others. Some authors expressed the fear that in a non-rigid Earth the geocenter is not stable, but can change its position with respect to the Earth's surface. This undermines the significance of the geocenter as the best origin of the reference frame for geodynamics. Therefore it is worthwhile to reconsider the properties of this partiular point.

DEFINITION, MECHANICAL PROPERTIES

Let the physical body be set up within some arbitrary but fixed coordinate system. Each material point is described by the position

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vector $\underline{x}(x_1, i = 1, 2, 3)$ and a mass dM. The coordinates of the center of mass are

$$x_{0i} = \frac{\int_E x_i \, dM}{M} , \qquad (i = 1, 2, 3) \qquad (1)$$

where M is the total mass of the body, and E is the space of the body.

Integration is extended over the entire volume of the body and the boundary of integration must be specified. In the case of the Earth and the geocenter there exists a certain ambiguity because the limit of the atmosphere is not clearly determined. The author suggests that some conventional altitude be specified, below which the atmosphere will be considered as belonging to the Earth mass system. This altitude height coincide for instance with the juridical limit of the Earth space which probably be established as the 100 km elevation above sea level. However, as we shall soon see that the height of this limit is not very important.

In a freely-rotating rigid body the axis of rotation passes through the center of mass. In the case of the Earth, the observed instantaneous axis of rotation, which from now on we shall call the "spin axis", is connected with the solid Earth, while the definition of the geocenter comprises also the atmosphere. So, neither the spin axis nor the geographical axis, as defined by Munk-MacDonald (1960), need pass exactly through the geocenter. This fact should be taken into account when discussing the rigorous definition of the reference system.

Precise definition of the Earth's rotation axis is not easy because different fractions of the Earth have some differences in rotation. Therefore, a so-called "Tisserand axis" is introduced, on a minimum condition (Moritz 1979a):

$$\int_E (\underline{v} - \underline{\omega} \times \underline{x})^2 \, dM = \min \qquad (2)$$

where $\underline{v}$ is the velocity vector of the particle dM, and $\underline{\omega}$ is the angular velocity vector of the total mass M. The integral is minimized by proper choice of the vector $\underline{\omega}$ which in turn defines the axis of rotation. This axis is not detectable in reality, but has the advantage that it is connected by definition with the geocenter.

Also connected with the geocenter are the axes and moments of inertia. The inertial tensor [I] has components

$$I_{ij} \int_E \left(\sum_{k=1}^{3} x_k^2 \, \delta_{ij} - x_i x_j \right) dM \qquad (3)$$

where i,j = 1, 2, 3 and δ_{ij} is the Kronecker delta. If the coordinate system is centered at the geocenter and the axes are directed in such a way that $I_{ij} = 0$ when $j \neq i$, then the diagonal components of [I] are called principal moments of inertia: $I_{11} = A$, $I_{22} = B$, $I_{33} = C$. They form a triaxial ellipsoid of inertia which is always centered at the geocenter.

If, on the contrary, the x_i-system is <u>not</u> centered at the geocenter, but the axes are parallel to the former ones, then

$$I_{ii} + M\Delta x_i^{\,2} = A\delta_{i1} + B\delta_{i2} + C\delta_{i3} \qquad (4)$$

according to the Huyghens-Steiner theorem (Suslov, 1946). This property enables us to determine the influence of the center of mass shift on the rotation rate of the body using the angular momentum conservation law:

$$\underline{H} = [I] \cdot \underline{\omega} = \text{const.} \qquad (5)$$

The notation of "geocenter" plays an equally important role in the theory of the Earth's figure. If we expand the gravity potential of the Earth in terms of spherical harmonics, the first-degree terms are simple functions of the coordinates of the geocenter (Heiskanen and Moritz, 1967):

$$U_{11} S_{i1} + U_{12} \delta_{i2} + U_{10} \delta_{i3} = \frac{GM}{R^2} X_{01} \qquad (6)$$

where G is the constant of gravitation and R is the Earth's mean radius. Hence, these terms vanish if the origin of the system is located at the geocenter. Solving the boundary value problem by the Stokes formula we implicitly locate the origin of the reference frame at the geocenter because of the intrinsic property of the boundary value condition

$$\Delta g = - \frac{\partial T}{\partial r} - \frac{2T}{r} = \frac{1}{R} \sum_{n=0}^{\infty} (n - 1)\, T_n(\theta, \lambda) \qquad (7)$$

This expression does not contain the first-degree terms. So, if we do not make other assumptions, a gravimetrically determined geoid has the center of mass identical with the center of the reference ellipsoid (Heiskanen and Moritz, 1967).

The meaning of the center of mass is most evident in artificial satellite motion theory. The differential equation of motion of the material point in the force field is usually divided into three parts, e.g.

$$\underline{\ddot{r}} = \underline{\nabla} V + \underline{\nabla} T + \underline{\nabla} F \tag{8}$$

The first term on the right hand side describes the influence of the so-called "central force" which is the gravitational attraction of a point mass. T is a perturbing potential reflecting the fact that the central body is not a point but has a certain structure and finite dimensions. Other forces have also some potential, either harmonic or not, denoted by F. Only the motion in the central force field is described by the closed analytical theory. Kepler's laws require the orbit to be a conic section and the central mass to lie in the plane of the conic at one of the foci.

In the real situation of an artificial Earth satellite, the orbit is never an ellipse, but if we know the perturbations we can reconstitute at any instant the so-called osculating orbit which is a Keplerian one whose plane passes through the geocenter. Having reconstructed this orbit we have a direct relation between the satellite position in space and the geocenter, in terms of direction as well as distance.

The problem is, how exact is our calculation of the perturbations? We have neither a complete model of all acting forces nor a perfect theory of perturbation. The process of improvement of these factors still continues, and the present accuracy is best reflected by the satellite position errors obtained by ephemeris computation, which is nowadays at the centimeter level.

STABILITY OF THE GEOCENTER, GEOPHYSICAL CONDITIONS

When talking about motion we must always define the frame with respect to which the motion takes place. If the discussion concerns the possible motion of the geocenter in relation to the non-rigid Earth the situation becomes really complicated. However, as we have access only to points situated on the surface of the Earth we are interested in relations between the geocenter and these points. Of course we have to accept that each of these points (stations) moves relative to the others, but at a given epoch each point has its fixed coordinates x_i in an arbitary system:

$$x_i^t \in E \quad , \qquad (i = 1, 2, 3)$$

where E is the Earth space, and t is the epoch.

Coordinates of the geocenter at epoch $t = t_0$ are found from (1)

$$x_{0i}(t) = \frac{\int_E x_i{}^{t_0} dM}{M} \tag{9}$$

Now, suppose that at another epoch $t = t_1$ a majority of points have preserved their positions, so that the system x_i is preserved, but in the limited subspace E' there is a mass displacement

$$x_i{}^{t_1} \neq x_i{}^{t_0}$$

This displacement will be reflected by a corresponding shift in the position of the geocenter in the system x_i:

$$x_{0i}^{t_1} - x_{0i}^{t_0} = \frac{\int_{E'} (x_i{}^{t_1} - x_i{}^{t_0})\ dM}{M} \tag{10}$$

This expression makes possible calculation of the change in the position of the geocenter resulting from mass deplacement in the limited volume, in relation to other points which are at rest. This is an intuitively supposed "motion" of the geocenter.

Let us make some very simple calculations to estimate the influence of some geodynamical phenomena.

The disappearance of the Antarctic polar ice cap would bring a change of about 30 m in the geocenter position. There is no need to consider such cataclysms in geodynamical investigations, but it gives some feeling about the sensitivity of the geocenter position to the mass changes occurring on the surface of the Earth.

In reality episodic changes occur in connection with earthquakes. Very large earthquake fields can extend over some 1000000 km^2 with displacements of the order of several meters. Suppose there is an uplift of 10 m over this area, and the depth of the displacement is 10 km. Such a tremendous earthquake would move

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the geocenter by only 0.5 mm. These calculations show that isolated episodic events on the surface of the Earth as well as in the crust have practically no impact on the position of the geocenter with respect to the rest of the globe.

The same is true of the seasonal changes in the atmosphere, analyzed by Stolz (1976). Using data on seasonal variations of the air pressure as well as on ground water storage be estimated the possible range of the geocenter position oscillations as 2.8 mm within a six-month period. The model was certainly simplified, but the order of magnitude will be the same using more sophisticated expressions. Anderson et al. (1975) found that the center of mass of the solid Earth and oceans differs in position from that of the geocenter by less than 5 cm. It seems that this estimate is one order of magnitude too large and that the variations indicated by Stolz make up the most we can expect as the influence of the atmosphere.

Special attention should be paid to the influence of Earth tides. If the Earth were a uniformly elastic body, tidal forces would produce only symmetrical deformations. It is possible, however, that inhomogeneity of the mantle may produce unequal response to the tidal forces causing small oscillations of the geocenter. It seems unlikely that amplitude of these oscillations could exceed a few mm, but we have not done any model calculations.

We do not know very much about mass displacements inside the globe, i.e. in the mantle and/or in the core. Today mantle convection is a commonly accepted hypothesis. In this case the quantity of mass involved in the motion is large in comparison to the total Earth mass, but the rate of the motion is slow. The phenomenon is of a global scale and secular character, deformations extend over the entire globe and the system to which we could refer geocenter motion is lost. The same is true for tectonic plate motion associated with convective streams in the mantle. Another possible phenomenon – rotation of the mantle with respect to the core – has an analogous pattern.

Yet another theoretical possibility of the geocenter motion is mass displacement in the core. According to the currently accepted Earth model, convection appears also in the outer liquid core (Stacey 1977, p. 197-204). On the other hand, Barta (1974) suggested asymmetry of the core structure, based on interpretation of gravity anomalies. Teisseyre (1979) supposes that this asymmetry can be supported by convection (Fig. 1).

Stability can easily be disturbed by processes occurring in the Earth's outer layers – geochemical phase transformations. These transformations occur constantly, causing changes in pressure, viscosity and temperature. They constitute a mechanism

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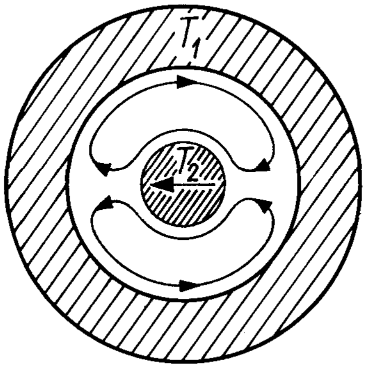

Figure 1. Core assymmetry caused by core convection.

pushing the inner core in different directions (Teisseyre, private communication). In this situation some motions of the inner core cannot be excluded. Unfortunately, we do not have mathematical model permitting quantitative estimates. However, even this hypothetical phenomenon is not capable of changing the geocenter position very much. The density difference between the inner and outer core is of the order of 1g/cm^3, hence the mass surplus influencing the geocenter is 7.5×10^{24} g. This is less than 10^{-3} of the total Earth mass and in consequences a 10 m displacement of the inner core (which may be supposed admissible) will give less than 1 cm change in the geocenter position. Such variations if detected would offer interesting information about the Earth's interior.

DETERMINATIONS OF THE GEOCENTER POSITION

By determining the positions of surface stations in the geocentric coordinate system we automatically determine the position of the geocenter in relation to these stations. This has been done in the Goddard Earth Model, the Smithsonian Standard Earth (SSE), the GRIM and others. It is not the purpose of this paper to compare different Earth models. Present estimates of the mean accuracy of the geocentric position in different models vary from approximately 5 m (Groten 1978). The author is inclined to believe that we are now close to the lower number. In recent years laser observations of passive satellites (Lageos and Starlette), as well as an immense quantity of Doppler observations of Transit satellites, contributed a great deal to the improvement of solutions. Comparison of different types of data made it possible to eliminate certain systematic errors or misinterpretation of results.

What is more dangerous is the correlation which appears between different unknowns in the huge systems of equations in-

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dispensable for combination solution. Therefore it is necessary to envisage some independent, more direct method for finding the position of the geocenter. As has been shown in §3, the spin axis can pass off the geocenter by some few cm at most, hence the determination of the offset position of the axis is practically equivalent to the determination of two coordinates of the geocenter. The first application of this method used Doppler observations of deep space probes for independent checking of the SSE (Gaposchkin 1973).

In the case of distant spacecraft the observed range rate can be expressed, after introducing necessary reductions, as

$$s = r + \omega r_s \cos\delta \sin t \tag{11}$$

where r is the geocentric radius vector, δ,t are the declination and hour angle of the spacecraft, and r_S is the spin-axis distance of the observer. The last parameter can be calculated, assuming that the ephemeris of the probe is known. The accuracy estimated in SSE III was about ± 2m for r_S, but taking into account the steady development in instrumentation and special planning of missions, an improvement by a factor of 10 can be expected. It appears that this method can be efficiently used with the help of GPS satellites.

Another method has been proposed by Domaradzki and Zielinski (1979). It consists in measuring the angles and distances of the object while the Earth is rotating at a certain angle (Fig. 2).

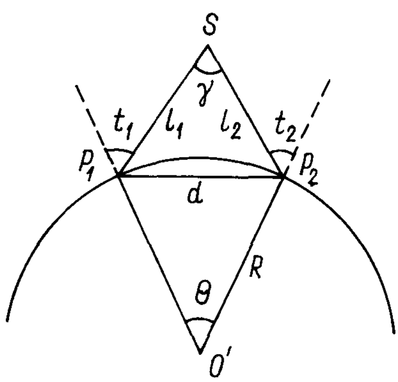

Figure 2. Determination of geocenter using angle and range measurements.

This method was first applied at four SAO stations and in three cases the solution was found. The degree of accuracy of the calculated spin-axis distance was about 15 m, too much to investigate the geocenter motion, but this error resulted from lack of precision of the photo camera direction data. The same method can be applied using GPS satellites or the Moon, measuring distance by laser beams and angles by interferometry, with an accuracy expected to rise to the decimeter level.

A totally different method based on the properties of the gravity field has been proposed by Mather et al. (1977). The idea is to use the fact that the first-degree harmonic terms in development of the Earth gravitational potential vanish only if the origin of the reference system coincides with the center of mass. By differentiating (6) and substituting spherical coordinates one gets an expression

$$\delta g = c(\Delta x_1 \cos\psi \cos\lambda + \Delta x_2 \cos\psi \sin\lambda + \Delta x_3 \sin\psi) \qquad (12)$$

where $c = -3.08$ μgal/cm, δg is the measured absolute gravity difference between two epochs, and Δx_i is the displacement of the geocenter. In principle three equations will suffice for determination of the three unknowns Δx_i, but Mather et al. suggested a minimum of at least 25 stations. This suggestion cannot be accepted without certain reservations. As we have seen, any changes of the geocenter position are caused by the redistribution of mass, but this redistribution also produces changes in gravity anomalies i.e. in terms of every degree in the development of the geopotential, not only in the first. When measuring gravity, absolute or relative, we are always faced with having to solve the well-known boundary value problem. However, by accepting the hypothesis that only the inner core is responsible for geocentric motion, Mather's method can work. Only a limited number of terms of low degree will be significantly affected by motion of the inner core, so even with a limited number of stations a solution to the problem can be found.

SCALE

One can consider the scale as a metric property of the space stretched upon the triad of unit vectors defining the three-dimensional coordinate systems (Grafarend et al. 1979). Nevertheless one can also approach the problem more pragmatically, considering the scale as one of seven parameters of transformation between two reference frames. In this sense we understand a scale difference to be the ratio of numbers describing the same length in both frames.

In the past we have observed some scale difference between particular solutions. They were especially apparent in comparisons between combination solutions and pure Doppler solutions. However, a thorough analysis of the software revealed the sources of the scale discrepancies (Langley et al. 1979, Hothem 1979). They proved to be connected with some of the constants used,

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and with the reduction method, rather than with the physics of observations.

In this connection let us note one of the peculiarities of the coordinate system realized by satellite observations. The size of a satellite orbit is essentially determined by the third of Kepler's laws:

$$n^2 a^3 = GM \tag{13}$$

where n is the mean motion of the satellite, and a is the semi-major axis of the orbit. This is a trivial statement for anyone working on orbit determination, but it is a very important constraint ensuring a uniform scale for a global net. Thanks to (13) the distance measurement errors can be effectively adjusted. The final accuracy depends on the precision of time measurement (Zielinski 1968). At present there are two types of approach: one is to accept some conventional value for GM which establishes the scale for the solution, the other is to improve this value in the course of the solution. In the latter case the primary scale source is the length measurement standard defined by the velocity of light, but after adjustment the corrected GM value is compatible with an obtained scale. In any case GM plays a crucial role in the determination of the linear scale of the coordinate system.

In physical geodesy we have again the possibility of determing the size of the reference ellipsoid and of the geoid if GM is known (Heiskanen and Moritz 1967):

$$\Delta a = N_0 = \frac{GM}{R_0 \gamma} - \frac{\Delta W}{\gamma} \tag{14}$$

where $\Delta a = N_0$ is the correction to the semimajor axis of the reference ellipsoid, γ is the mean value of gravity, and Δw is the correction to the potential on the geoid surface. Knowing the exact value of GM we can find Δa and improve the parameters of the geodetic datum.

I would like to stress, however, that both procedures are valid only if our calculations are based on the system connected with the center of mass of the Earth. Satellite orbit theory, as well as the theory of the geoid, can be used in a convenient way if the system origin is equivalent to the geocenter. Only then can we obtain uniform scaling for our reference system.

Numerical values of the geocentric gravitational constant and the Earth equatorial radius were discussed at the General Assembly of IUGG at Canberra by the Special Study Group on

Fundamental Geodetic Constants. The values adopted were (Moritz 1979b):

$$GM = 3986005 \pm 0.5 \times 10^8 \; m^3 s^{-2}$$

$$a = 6378137 \pm 2 \; m$$

These are certainly the best current estimates based on the latest results. Nevertheless it would seem that the compatibility of these two figures should be made a subject of further study.

CONCLUSIONS

From the above deliberations we can derive the following conclusions:

1. The motion of the geocenter can be discussed only if a number of points conserving this frame exist on the Earth's surface. We must confine our considerations to cases in which mass displacements are limited in space. If deformations affect the entire Earth, the frame is destroyed and the idea of "motion of the geocenter" becomes meaningless.

2. This implies that we can consider motion of the geocenter only on a short-term scale. But, as was shown above, any short-term phenomena occurring near the surface, e.g. earthquakes, ocean tides, atmospheric changes etc., exert a negligible influence on the position of the geocenter.

3. Processes within the core, and possible motion of the inner core, are the only admissible sources of detectable motion of the geocenter, of short-period and attaining a possible level of a few centimeters. The detection of such motion would be extremely interesting from the point of view of geophysics.

4. The geocenter is a well defined and extremely stable point, accessible to measurement. The location of the origin of coordinate systems at this point is thus fully justified.

5. To ensure a uniform length scale, the scale of the global coordinate system must be connected with the geocenter.

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