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THE NUMERICAL SOLUTION OF SECOND KIND FREDHOLM INTEGRAL EQUATIONS

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This thesis examines certain methods for the numerical solution of second kind Fredholm integral equations of the form

(1) $$y(t) = f(t) + \int_D k(t,s)y(s)ds \ , \ t \in \overline{D} \ ,$$

where D is a bounded domain in $\mathbb{R}^d$, $d \geq 1$ (though in Chapters 2 and 3, d will be 1 so that D is a bounded interval in $\mathbb{R}$).

The collocation and iterated collocation methods will be considered in Chapter 2 and the collocation solution will be sought in a space of piecewise polynomials which are either discontinuous or just continuous at the knots. Under suitable smoothness conditions on the kernel k and the solution y and with appropriate choices of collocation points, we shall see that the iterated collocation solution exhibits global superconvergence, that is, it converges faster than the collocation solution itself.

In practice, it is sometimes necessary to use numerical integration to approximate certain integrals required in the collocation and iterated collocation methods. The resultant discrete collocation and discrete iterated collocation methods are analysed in Chapter 3. We see that these two discrete collocation methods have the same order of convergence as their exact counterparts if the kernel is sufficiently smooth and a quadrature rule of sufficient precision is used.

In Chapter 4, the results of Chapter 2 are extended to integral equations in which D is a bounded rectangular region in $\mathbb{R}^2$.

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Other well-known methods for the numerical solution of (1) are the Galerkin and iterated Galerkin methods. Like the collocation methods, these two methods may require the approximation of certain integrals, and hence in Chapter 5, we give an analysis of the discrete Galerkin and discrete iterated Galerkin methods. We shall also consider the discrete Galerkin methods when applied to the regularised equation

$$z = Kf + Kz ,$$

where $z = Ky = \int_D k(t,s)y(s)ds$.

Bateman's method has been known for over 60 years, but it appears that little theoretical work has been done on this method. In Chapter 6, we prove the convergence of Bateman's method for integral equations in which the integral operator K is non-negative definite. The convergence proof, however, does not indicate the rate of convergence, so we consider the situation in which D is a bounded interval in more detail. In this situation we see that the order of convergence of Bateman's method depends strongly on the smoothness of the kernel.

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