

COORDINATE SYSTEM IN GENERAL RELATIVITY

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Abstract. The proper reference frame comoving with a system of mass-points is defined as a general relativistic extension of the relative coordinate system in the Newtonian mechanics. The coordinate transformation connecting this and the background coordinate systems is presented explicitly in the post-Newtonian formalism. The conversion formulas of some physical quantities caused by this coordinate transformation are discussed. The concept of the rotating coordinate system is reexamined within the relativistic framework. A modification of the introduced proper reference frame named the Natural Coordinate System (NCS) is proposed as the basic coordinate system in the astrometry. By means of the concept of the natural coordinate system, the relation between the solar system barycentric coordinate system and the terrestrial coordinate system is given explicitly. To illustrate the concept of NCS, we quote in the following the definition of the non-rotating NCS comoving with the Earth, i.e. the Terrestrial Coordinate System (TCS) (Fukushima et al., 1986a, 1986b):

- 1) Consider a fictitious spacetime with the metric obtained by subtracting the direct terms due to the Earth from the true metric in the solar system Barycentric Coordinate System (BCS).
- 2) The time coordinate axis of the TCS is defined as the worldline of the geocenter, i.e. the timelike geodesic of the geocenter in the above fictitious spacetime.
- 3) The unit of time in the TCS, terrestrial second s_T , is defined as the unit of time in the BCS, barycentric second, multiplied by a certain factor so that there exist periodic differences only between the time coordinate of any event in the TCS, i.e. TDT, and the corresponding time coordinate in the BCS, i.e. TDB.
- 4) The space coordinate axes of the TCS are defined as three geometrically straight lines satisfying that they and the time coordinate axis of the TCS are orthogonal to each other at the geocenter in the above fictitious spacetime, and that the coordinate triad constructed by them is symmetric.
- 5) The unit of length in the TCS, terrestrial meter m_T , is defined as the length so that $c = 299792458 m_T/s_T$.

The introduced concept of the NCS has the following merits:

- 1) The non-rotating NCS has no secular rotation referred to the background spacetime. This makes it easy to realize the NCS by use of distant-objects-fixed coordinate systems such as the VLBI coordinate system or usual star catalogs. Also this suits well with the convention that the amount of geodesic precession is included to that of the general precession.
- 2) The coordinate transformation defining the NCS is explicitly obtained. Then one can build a hierarchy of the coordinate systems which makes the relations among them clear.
- 3) The conversion formulas of all physical quantities including the light direction, coordinate velocity, coordinate acceleration, metric tensor, angular momentum, electromagnetic field are explicitly derived.
- 4) The NCS is a natural extension of the relative coordinate system in the Newtonian Mechanics. For example, the force in the equation of motion in the TCS is the sum of the direct attraction of the Earth and the tidal forces due to the other celestial bodies in the first approximation.
- 5) The introduction of the NCS brings no formal changes in the aberration formula.

References

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