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A completely new kind of variable star has recently been discovered (McGraw, et. al. 1979). Designated as PG1159-035 (hereafter PG) this star is distinguished not only by the complete lack of hydrogen in its spectrum but also by an effective temperature that exceeds $8 \times 10^{4} \mathrm{~K}$ (McGraw, private communication). The photometric data show that this star is pulsating with two periods - 539 seconds and 460 seconds and the light curve is very reminiscent of that of a ZZ Ceti variable star (DA white dwarfs pulsating in non-radial modes). However, its spectral characteristics show that it cannot be included in this class since analysis of both optical and IUE spectra show that the major atmospheric constituents are probably helium and carbon and that its surface gravity is considerably lower than $10^{8} \mathrm{~cm} \mathrm{sec}^{-2}$ characteristic of a DA white dwarf. Its energy distribution suggests a small amount of reddening and since it is far out of the plane it must be at a distance at least 1 kpc . This estimate is supported by a null proper motion over a 13 year baseline (Luyten 1979, private communication to J Liebert). These data suggest that its luminosity exceeds $10^{2} \mathrm{~L}_{6}$. In any case, it would need a luminosity as large as $10 \mathrm{~L}_{\odot}$ to fall on or above the white dwarf cooling curve at $\mathrm{T}_{\mathrm{e}}=8 \times 10^{4} \mathrm{~K}$ (Lamb and Van Horn 1975). All of these facts suggest strongly that this star is unique and a new kind of pulsating variable.

This star does not fall near any of the known regions of instability in the $H R$ diagram which suggest that the instability mechanism will not be helium and hydrogen ionization as in the Cepheid variables. Therefore, in order to discover the cause of the instability in PG we have turned to more unusual compositions.

We used PG's approximate position in the HR diagram as a guide and compared it to an evolutionary track computed by Schönberner (1979). This track (see Figure 1) was obtained by evolving a $1.0 \mathrm{M}_{6}$ star through the phase of helium-shell-flashing and gradually removing mass from the surface at a rate consistent with a Reimers (1975) formula.


Figure 1. The mass loss very late evolution of a $1 M_{\Theta}$ star according to Schönberner.

By the time the star has reached its farthest excursion to the blue, its mass has decreased to $\sim 0.6 M_{0}$. Its core is nearly pure carbon and its envelope contains helium and carbon. Most of the hydrogen has been ejected. If we assume that PG has undergone such an evolution, then we can assume a similar mass and composition and construct stellar envelopes to test for instability. Our model is at point $E$ in Figure 1.

A stellar envelope with 400 Lagrangian zones has been constructed with a mass of $0.6 \mathrm{M}_{\odot}, \mathrm{T}_{\mathrm{e}}=80,000 \mathrm{~K}$, and a surface luminosity of $\mathrm{L}_{\mathrm{s}}=$ $5151 \mathrm{~L}_{6}$. This included 95 percent of the mass and 89 percent of the radius. About 30 zones were above the photosphere. The surface composition is half helium and half carbon by mass to a depth of $3 \times 10^{-10}$ of the mass ( $175,000 \mathrm{~K}$ ). Deeper layers are pure carbon. In this carbon core there are two thin convection zones, and insignificant one between 0.24 and $0.41 \times 10^{6} \mathrm{~K}$ and another where convection carries up to 30 percent of the total luminosity at that level between 0.55 and $0.81 \times 10^{6} \mathrm{~K}$. Opacities and equations of state were obtained from the Astrophysical Opacity Library of Huebner, et a1. (1977).

This model has a variable luminosity to schematically allow for the cooling of the carbon core along the track of Figure 1. The formula used is

$$
L=L_{s} \frac{q-q_{0}}{q_{1}-q_{0}} \geqslant L_{0}, \quad q_{0} \leqslant q \leqslant q_{1}, q^{\prime}=1-q
$$

with q the mass fraction $\mathrm{M}(\mathrm{r}) / \mathrm{M}$, $\mathrm{q}_{0}$ the core mass with luminosity $\mathrm{L}_{0}$ and $\mathrm{q}_{1}$ the mass fraction above which the luminosity is the surface luminosity $\mathrm{L}_{\mathrm{S}}$. In this model for $\mathrm{PG} \mathrm{q}_{0}^{\prime}=1.9 \times 10^{-8}, \mathrm{~d}_{1}=3 \times 10^{-10}$, and $L_{0} / L_{s}=0.0126$.

This structure of the internal $T, \rho$, and opacity variation is given in Figure 2 where the abscissa is the logarithm of the surface mass fraction.


Figure 2. Temperature, opacity, and density structure of the model for PG1159-035. Pulsation driving is in the outer $10^{-7}$ of the stellar mass at a iractional radius of 0.95 of the star.

An analysis was made for pulsational instability using the Los Alamos version of Castor's (1971) Iinear nonadiabatic computer program. The two observed periods are matched with the second (2H) and third (3H) overtones with the theoretical period ratio being 0.854 compared to the observed 0.853 . The eigenvalues give kinetic energy growth rates of $1.3 \times 10^{-3}$ and $1.9 \times 10^{-2}$ per period for these two modes. The lower modes seem less unstable by a factor of 10 for 1 H and a factor of 100 for the fundamental mode. The next higher overtone ( 4 H ) has about the same growth rate as 3 H , but 5 H is down by a factor of 10 and 6 H is stable. Our model predicts a possible period at 405 seconds. Another possibility is that the two modes seen are 3 H and 4 H and that the surface luminosity and its internal variation need further adjustment.

Figure 3 gives the radial eigenfunctions for the second and third overtones. They are similar to those shown for even higher overtones in the case of radial modes of cool white dwarfs (Cox, Hodson, and Starrfield 1979; Starrfield, Cox, and Hodson 1979). In this present
case, however, there is a low upper limit to the unstable overtones.


Figure 3. Radial pulsation eigenvectors for the two modes of the model which have the observed periods.

The work done per zone to drive this pulsation is given for the two modes in Figure 4. The peak at zones 251 and 252 (for the second


Figure 4. Work per zone to cause pulsation in the model versus zone number. The third overtone (3H) is reduced by a factor of 10 . The peak carbon ionization driving at zone 251-252 is of a surface mass of $3 \times 10^{-8}$ and a radius fraction of $x=0.95$.
and third overtones) is at a temperature of $0.46 \times 10^{6} \mathrm{~K}$, just where the luminosity reaches its low core value, $\mathrm{L}_{0}$. This is at a surface mass fraction of $3 \times 10^{-8}$ and a radius fraction of 0.95 . The deeper driving and damping spikes are due to small changes in derivatives at, respectively, $0.5,0.6,0.7,0.8,0.9,1.0$, and $1.1 \times 10^{6} \mathrm{~K}$ where there are material property table entries. They give no appreciable contribution to instability. The deeper damping is suppressed in this model because there is very little luminosity in the cooling carbon core.

The cause of this instability can be seen by examining the plot of carbon opacity versus temperature given in Figure 5. There is a "bump" in the opacity between temperatures $10^{5}$ and $10^{6} \mathrm{~K}$ and all densities. Our calculated stellar envelope plotted here passes directly through this region and, therefore, suffers a large change in the opacity and equation of state derivatives at this point. It is commonly assumed that irregularities in the opacity curves indicate a region where driving can occur. In fact, such an irregularity in the opacity is usually associated with the partial ionization region of some element and in this case it is the ionization of the last two electrons (1s2) of carbon. This implies that we have discovered a completely new excitation mechanism for pulsation in stellar envelopes - carbon ionization.

If the carbon is mixed with helium or oxygen the bumps are smoothed and the driving is reduced. We have not considered how much dilution of the carbon is permissible, however.


Figure 5. Opacity for carbon versus temperature for various densities. The model structure is given by the line
which ronges In density from $3 \times 10^{-7}$ to $7 \times 10^{2} \mathrm{~g} / \mathrm{cm}^{3}$

Additional models have been studied along a line of constant period. We seem to find instability for the second and lower overtones at $100,000 \mathrm{~K}$ and for only the fundamental at $110,000 \mathrm{~K}$. Cooler temperatures have not been investigated yet because PG is not cooler than $80,000 \mathrm{~K}$. Further studies of the radial pulsation for PG should await a better calculation of the internal luminosity structure. It may indeed happen that as the effective temperature continues to get hotter as in Figure 1 and the periods get shorter, the $\mathrm{q}_{0}$ gets so deep that radiative damping stabilizes all modes.

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