ON THE ACCURACY OF INFRARED PHOTOMETRY

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ABSTRACT. Photometric errors due to scintillation are considered in detail. Given noise characteristics, the standard deviations, as deduced from observations, are computed for the quantities measured at successive steps of the photometric procedure. This allows us to understand better the errors computed on-line in observatories, and to understand better the overall error. The latter can be minimized by suitable changes in the time sequence of the measurement.

1. INTRODUCTION

Many reports concerning IR photometry are not explicit as to the accuracy of their data, the definition of error bars and/or the possible systematic errors. This probably reflects a real problem in IR photometry, which results from a combination of various sources of noise, whose statistical properties differ from each other and may vary between observations. Thus, although it is highly desirable to obtain average atmospheric characteristics from extended monitoring at each and every observatory, this must be complemented by efforts to characterize fluctuations at each observing session. Resident astronomers have realized this and provided on-line data pretreatment that may be very efficient, if adequately used. We propose here to analyze the meaning of on-line results, to show how to deduce from them an estimate of the accuracy and to discuss how to improve this accuracy. Only extinction fluctuations are considered here because they supersede sky emission fluctuations when the observed object is not too faint. The case where emission noise is dominant was treated elsewhere (Papoular 1983). It is assumed that extinction is independent of direction and that image wandering is negligible.

2. STATISTICAL BACKGROUND

First recall the basic statistical definitions and relations that will be needed. Let X be the random variable to be measured and $W_x(f)$, its <u>noise power spectrum</u>. Then, its <u>variance</u> is $\sigma_x^2 = \int_0^\infty W_x(f)\,df$ and an estimate of the error on X is the <u>standard deviation</u>, σ_x. If $W_x(f)$ is not known, σ_x has to be evaluated by repeating the measurement a large number of times. Usually, however, a small number, N, of measurements are

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performed, from which the sample variance S_X is computed using $S_X^2 = (1/N-1)\sum^N (X-\bar{X})^2$, where $\bar{X} = \sum^N X/N$, and S_X is considered as an estimate of σ_X. But we showed (Papoular and Pegourie 1983) that this is often overly optimistic because the low frequency fluctuations are not properly taken into account. Using the results of Barnes et al. (1971), it can be shown that the most probable value of S_X^2 is not σ_X^2 but

$$s_X^2 = \frac{N}{N-1} \int_0^\infty df \cdot W_X(f) \cdot \left[1 - \frac{\sin^2(\pi f N T)}{N^2 \sin^2(\pi f T)} \right] \quad , \qquad (1)$$

where T is the interval between successive measurements. The difference between s_X^2 and σ_X^2 is not negligible unless all the frequencies in $W_X(f)$ are such that $f > 1/\pi NT$. The monitoring of atmospheric emission and extinction fluctuations in a number of observatories (Allen and Barton 1981, Papoular 1983) yielded a conspicuous "$1/f^\alpha$-noise" spectrum with $W(f)$ still increasing for $f < 0.001$ Hz. As a result, the measuring time NT should, in principle, be very long in photometry ($\sim$ 1/2 hour).

The best estimate of X is taken to be its average, $\bar{X}$. The error in X is often taken to be $\sigma_X/\sqrt{N}$. Again, this is shown in Papoular and Pegourie (1983) to be too optimistic because it neglects correlations between measurements. In fact, the averaging procedure does not uniformly reduce the noise spectrum $W_X(f)$, but preferentially its higher components (Barnes, et al. 1971):

$$W_{\bar{X}}(f) = W_X(f) \cdot \frac{\sin^2(\pi f N T)}{N^2 \sin^2(\pi f T)} \qquad (2)$$

On the other had, integration over a time NT alters the noise according to

$$W_{\bar{X}}(f) = W_X(f) \cdot \frac{\sin^2(\pi f N T)}{(\pi f N T)^2} \qquad (3)$$

Thus, integration acts as a real low-pass filter, with a bandwidth $\sim$ 1/NT, while averaging acts like a multiple-band pass filter.

By definition, $\sigma_{\bar{X}}$ is the integral, over frequency space, of expression (2) or (3), the computation of which requires a knowledge of the noise spectrum $W_X(f)$. It is usually implicitly assumed that this spectrum is white, in which case $S_X/\sqrt{N}$ is a good estimate of $\sigma_{\bar{X}}$. As stated above, this is not acceptable in astronomical photometry. Moreover, the exponent α (in the $1/f^\alpha$ spectrum) changes with time and photometric band.

Photometric procedures also include back-ground subtraction and comparison with standard stars. In these cases, sums and differences of random variables arise, with a constant time difference, T, between the measurements of two variables. If both variables (X,Y) can be assumed to have the same noise spectrum W_X, then the following relations hold (Papoular 1983, Appendix C):

$$W_{X+Y}(f) = 4 \cdot W_X(f) \cdot \cos^2(\pi f T) \qquad (4a)$$

$$W_{X-Y}(f) = 4 \cdot W_X(f) \cdot \sin^2(\pi f T) \qquad (4b)$$

It is possible to compute the variance at any stage of the process, using equations of the form (2) to (4). For purposes of comparison, we may normalize variances by dividing them with the total noise energy, $\sigma_0^2 = \int W_\chi(f)\,df$ over the available spectrum.

3. COMPUTATION OF VARIANCES IN PHOTOMETRY

The sequence of measurements, as well as the symbols used, are summarized in Figure 1. A, B, a, b represent signals; s^2, estimated sample variances; indices $*$ and Σ are for program star and standard star respectively; τ, θ and T are time intervals. Each line of the figure represents an element of the sequence with its name and outcome. Let I be a quantity proportional to the spectral brightness of the point object observed, and $\chi(t)$ be the atmospheric extinction at the same wavelength and direction, and at the time of the observation. Then,

$$a(t) = I \cdot \exp[-\chi(t)] \quad \text{or} \quad b(t) = -I \cdot \exp[-\chi(t)] \tag{5}$$

Here we overlook the sky emission, for reasons stated in the introduction. χ is a random variable with a non-zero average: $\chi(t) = \chi_0 + \tilde{\chi}(t)$. It usually is so small that $\exp[-\chi(t)] \simeq 1-\chi(t)$ and the i th average of a over a time τ is

$$a_i = I \cdot [1 - \chi_0 - \tilde{\chi}_i] \quad , \tag{6}$$

where the average of $\tilde{\chi}_i$ (over a long time) is zero. Let $W_\chi(f)$ be the (unknown!) noise power density of $\tilde{\chi}(t)$. Because of the linearity of this relation, the noise spectra of the photometric signals will be proportional to W_χ, with the coefficient I^2. Table I represents, in a self-explanatory way, the signals at the various levels of the sequence of Figure 1, together with their respective averages, noise spectra and estimated variances, according to the rules of section 2. Descending this sequence, we find a number of quantities that usually are computed on-line during observations: A,B (A-B)/2 (or A-B), I and $\bar{I}$. Also computed are sample variances, such as

$$\begin{aligned}
S_a^2 &= \sum_1^n (a_j - \bar{a})^2/(n-1) \\
S^2_{(A-B)/2} &= \sum_1^N \left[\frac{(A-B)_i}{2} - I\right]^2/(N-1) \\
S_I^2 &= \sum_1^M (I - \bar{I})^2/(M-1)
\end{aligned} \tag{7}$$

By way of an example, at ESO (La Silla), the on-line computer gives E1, E2 and STD, where E1 $\simeq S_a\sqrt{2/n}$; E2 $= 2\,S_{(A-B)/2}$; STD $= 2\,S_I$.

Note that their IT and n correspond to our $n\tau$ and N. Also %E1 $= 100 \cdot \frac{E1}{I} \cdot \frac{1}{(N-1)^{1/2}}$,

$$\%\,E2 = 100 \cdot \frac{E2}{I}\,\frac{1}{(N-1)^{1/2}} \quad , \qquad \%\,STD = 100 \cdot \frac{STD}{I} \cdot \frac{1}{(M-1)^{1/2}} \quad .$$

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The quantity of interest for one star (at a given wavelength) is $\bar{I}$. Since there is usually no time available to determine its variance experimentally, the latter is often estimated by $S_{\bar{I}}^{*} = S_{I}^{2}/M$ or $S_{I}^{2}/(M-1)$. As stated in section 2, this usually is not valid.

The final step usually involves a comparison with a standard star of known spectral brightness $I_{\Sigma,\lambda}$. At a given wavelength, we have (skipping subscript λ):

$$I_{*\Sigma} = \left(\bar{I}_{*}/\bar{I}_{\Sigma}\right) \cdot I_{\Sigma} = I_{*} \exp\left(\tilde{\chi}_{\Sigma} - \tilde{\chi}_{*}\right) \qquad (8)$$

where $\bar{I}_{*}$ and $\bar{I}_{\Sigma}$ are the results of the above sequence of measurements for star and standard Σ, respectively. I_{*} is the true spectral brightness of the star and $\tilde{\chi}_{\Sigma}$, $\tilde{\chi}_{*}$ the average atmospheric extinctions during the respective sequences. Then, the relative error in $I_{*\Sigma}$ is

$$\epsilon \equiv \left(I_{*\Sigma}/I_{*}\right) - 1 \simeq \tilde{\chi}_{*} - \tilde{\chi}_{\Sigma} \qquad (9)$$

which is also to be considered as a random variable, with zero mean value. Assuming the directions of $*$ and Σ are close to one another, the standard deviation (or overall error) σ_{ϵ} can be determined by noting that $\tilde{\chi}_{*}$ and $\tilde{\chi}_{\Sigma}$ are the same functions of time, taken at intervals $T = LMN\theta$. Then eq. (4b) is applicable and gives the last line in Table I; σ_{ϵ} can then be determined by integrating $W_{I_{*\Sigma}}$ over all frequencies.

4 DISCUSSION

Let us assume a simple model for the noise spectrum:

$$\begin{aligned} W_{\chi}(f) &= W_{0}\,(f_{0}/f)^{\alpha} && \text{for} \quad f_{0} \leq f \leq f_{M} \\ W_{\chi}(f) &= W_{0} && \text{for} \quad f \leq f_{0} \end{aligned} \qquad (10)$$

Then it is easy to compute (numerically) all standard deviations in units of σ_{0} as a function of α, for different values of f_{0} and of the parameters, τ, n, N.... It is found, e.g., that $<E1/E2> \simeq 1$ only got $\alpha \simeq 0$, but decreases notably as α increases. Experimentally, it is found that $<E1/E2> \sim 1$ in the middle of photometric nights and falls to ~ 0.1 in the presence of cirrus clouds. Similarly, Figure 2 shows the same trend for $<(\%\ E2/(\%STD)>$. Observations yield values of the order of 0.5 in normal weather at La Silla, which corresponds to $\alpha \sim 1$ (i.e. 1/f noise) around $f \sim 0.01$ Hz. Photometry of bright stars at Mauna Kea (Barnes et al. 1971) indicated dominant noise components down to at least 0.002 Hz. This, together with Milone and Robb (1983) and Allen and Barton (1981) compels us to investigate thoroughly the behavior of errors in the presence of 1/f noise. As an example, Figure 3 shows clearly the advantage of performing the comparison with a standard as early as possible in the sequence of Figure 1, for instance for each wave band instead of once, after completing the whole sequence for the program star. The higher α, the larger the advantage, either in accuracy or in total measuring time, or both (for $\alpha = 0$, the accuracy depends only on the total time spent for each wavelength, whatever the order of operations in the sequence). The total time in each case is equal to $nNM\tau$ as indicated in the Table under Figure 3. This was clearly demonstrated with RADS (Milone and Robb 1983), which performs the comparison immediately after step 1 in Figure 1. Quantitatively, this effect can be traced to the factor $\sin^2(\pi ft)$ in eq. (4b).

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TABLE I

X	$\bar{X}$	$W_x(f) \qquad (\sigma_x^2 = \int W_x\,df)$	$s_x^2 = (N/N-1)\int df.W_x\left[1 - \frac{\sin^2(\pi f N T)}{N^2 \sin^2(\pi f T)}\right]$
a	$\bar{a} = \int_0^\tau dt.a(t)/\tau$	$W_a = I^2 W_x \frac{\sin^2(\pi f \tau)}{(\pi f \tau)^2}$	$\frac{n}{n-1}\int df.W_a\left[1 - \frac{\sin^2(\pi f n \tau)}{n^2 \sin^2(\pi f \tau)}\right]$
$\bar{a} \equiv A_i$	$\sum_1^n a_j/n$	$W_{\bar{a}} = W_a \frac{\sin^2(\pi f n \tau)}{n^2 \sin^2(\pi f \tau)}$	$\frac{N}{N-1}\int df.W_{\bar{a}}\left[1 - \frac{\sin^2(\pi f N \theta)}{N^2 \sin^2(\pi f \theta)}\right]$
$\left(\frac{A-B}{2}\right)_i$	$I = \sum_1^N \frac{(A-B)_i}{2N}$	$W = W_{\bar{a}} \cos^2(\pi f \theta/2)$	$\frac{N}{N-1}\int df.W_{\bar{a}} \cos^2(\frac{\pi f \theta}{2})[\quad]$
I	$\bar{I} = \sum_1^M I_k/M$	$W_I = W_{\bar{a}} \cos^2(\pi f \frac{\theta}{2}) \frac{\sin^2(\pi f N \theta)}{N^2 \sin^2(\pi f \theta)}$	$\frac{M}{M-1}\int df. \quad .\left[1 - \frac{\sin^2(\pi f M L N \theta)}{M^2 \sin^2(\pi f L N \theta)}\right]$
$\bar{I}$		$W_{\bar{I}} = W_I \frac{\sin^2(\pi f M L N \theta)}{M^2 \sin^2(\pi f L N \theta)}$	
$\epsilon \equiv \bar{I}_*/\bar{I}_\Sigma - 1$		$W_\epsilon = 4 W_{\bar{I}} \sin^2(\pi f M L N \theta)$	$\sigma_\epsilon^2 = \int W_\epsilon.df$

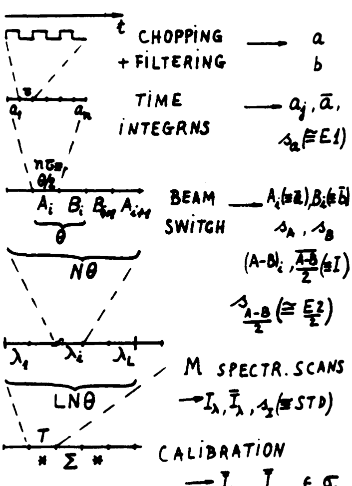

Fig. 1.

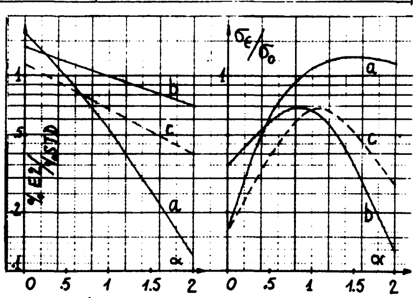

FIG.2 FIG.3

curve	n	N	L	M	f_0	f_M	τ
a	5	4	13	3	10^{-1}	1	1
b	2	2	1	2	"	"	"
c	5	4	1	2	"	"	"