We shall write, exactly, as a definition of $\phi(x)$ the identity in x:

$$f(h) \equiv f(x) + (h-x)f'(x) + (h-x)^2K + \phi(x) \qquad (1)$$

Clearly $\phi(h) = 0$; and we can choose K so that $\phi(0) = 0$. Then $\phi(x)$ is a Rolle function for the interval, giving $\phi'(\xi) = 0$, where $0 < \xi < h$.

But by differentiating (1) we have

$$0 \equiv (h-x)f''(x) - 2(h-x)K + \phi'(x)$$

Substituting ξ for x, using $\phi'(\xi) = 0$ and $\xi \neq h$ we have

$$f''(\xi) = 2K$$

Substituting in (1) and writing $x = 0$ we have, since $\phi(0) = 0$, the result

$$f(h) = f(0) + hf'(0) + \tfrac{1}{2}h^2 f''(\xi)$$

A. J. Moakes

CORRESPONDENCE

To the Editor of the *Mathematical Gazette*

Dear Sir,—In your issue of October 1959 Misses Curphey, Kelley and Moffat (CKM) have given a table of the frequency distribution of the digits in the first 10,000 decimal places of π. Whereas their table was intended to replace the inaccurate one of Messrs. Doe, Ogden and Vieri that appeared in the February 1959 issue of the Gazette, this itself is defective because of the inaccuracy, after the 7480th decimal place, of the value of π which seems to have been employed for this enumeration. Same is the case with the observations made by Mr. A. Kodym on some of the numerical properties of these digits and reported in the February 1960 issue.

I have recently investigated in some detail the statistical aspects of the 10,000 digits of π appearing in " Chiffres " **1**, 17–22 (1958). The complete agreement of this value with the correct one, as re-computed by Mr. Felton, has been attested by Prof. J. C. Miller (Math. Rev. **20**, Abs. No. 1436, 1959). The frequency distribution of the digits in $\pi - 3$ is seen to be as follows:

Decimal places \ Digits	0	1	2	3	4	5	6	7	8	9
1–7,000 (CKM)	657	733	692	686	702	730	708	694	680	718
7,001–8,000	97	100	119	95	107	104	108	92	84	94
8,001–9,000	101	103	100	103	101	99	98	97	90	108
9,001–10,000	113	90	110	90	102	113	107	87	94	94
1–10,000	968	1026	1021	974	1012	1046	1021	970	948	1014

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Next, with a view to study the degree of serial correlation among the neighbouring digits, a frequency count of the 10,000 (overlapping) pairs ij of the digits has been done. The observed frequencies f_{ij} of the different types of pairs are given in the following table.

i \ j	0	1	2	3	4	5	6	7	8	9	Total
0	85	103	98	103	98	89	101	93	83	115	968
1	99	99	103	102	121	95	106	90	98	113	1026
2	101	115	110	99	82	118	100	101	100	95	1021
3	102	92	86	94	114	100	90	102	97	98	975
4	95	100	100	89	102	110	103	108	101	104	1012
5	92	117	110	96	108	96	115	107	96	109	1046
6	107	95	117	97	101	124	91	101	90	98	1021
7	89	105	99	91	92	101	95	97	103	98	970
8	86	97	99	93	96	106	114	83	80	93	947
9	112	103	99	110	98	107	106	88	100	91	1014
Total	968	1026	1021	974	1012	1046	1021	970	948	1014	10000

As it should be, $\sum_{l} f_{lm} = \sum_{n} f_{mn}$ for all m except when it is either 3 or 8; the difference ± 1 in these two cases is due to the " end " effects of the open array. The smallest frequency in this table is 80, which is for the pair (88), and the largest one is 124, for the pair (65). Further, we note that out of the one hundred entries as many as seventy show a deviation of less than 10 in magnitude from the expected value of 100, seven deviate exactly by that amount and the remaining twenty-three depart by greater amounts.

Detailed statistics of these digits, as obtained by subjecting them to a larger number of standard tests of randomness, is under study and will be reported elsewhere.

Yours Faithfully, R. K. Pathria

FOR SALE

Mathematical Gazette, Nos. 337 to 349; Collection of mathematics text-books by Routh, Edwards, Hall and Knight (with key), Hobson, Durell, Forder, etc. Replies to L. Malone, 17 West Oakhill Park, Liverpool 13.

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