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Is there a Low Surface Brightness Universe ?

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Abstract.
I have been asked to review some of the arguments for and against the existence of a significant population of Hidden Galaxies, and to concentrate on the distant past and the possible future of this subject, leaving more recent observations to others. In particular I ask how should we correct for selection effects and how are we to resolve contradictory evidence?

1. INTRODUCTION

Our subject is built upon the following conjecture: *"Most of the galaxies, even in our neighbourhood, remain to be discovered because galaxies are extremely hard to detect through our atmosphere and against our sky."*

Some of the conjectured missing galaxies will be so dim (i.e. be of such low surface brightness) as to be lost completely below our parochial sky; of others, which I call "Crouching Giants", we shall see only their bright cores - causing us to misconstrue them as dwarfs. Many genuine nearby dwarfs and high SB galaxies will be hidden among a sea of apparently more numerous background giants while others, "Masquerades" as I call them, will appear so bright and compact as to be indistinguishable, at first glance, from stars.

Examples of all these types of "hidden galaxy" certainly exist; the controversy enters in assigning a significance to them relative to the well-known and long studied population of "Normal galaxies". Some dismiss them as rare oddities while others believe them to contain the majority of galactic light - or mass. Lacking certain knowledge it is a good question to ask which of the two extreme points of view will make for the better working hypothesis. I contend that, for the galaxy explorer at least, it is more fruitful to assume that large hidden populations remain to be found and to seek new observing techniques to detect and count them. At the same time we all recognise that certain (very difficult) observations, such as detection of the Extra-Galactic Background Light, must set upper bounds to our wildest imaginings. For now these upper bounds are rather uncertain (but improving) so that it is still not unreasonable to contend that hidden galaxies might contain several times more light in total than the known population. If eventually they prove to emit less light they may nevertheless contain the preponderance of mass, in the form of dark matter, while we already know for sure that their total cross-section - important for the absorption of background radiation (e.g. Quasar absorption line systems), should comfortably exceed that of conventional galaxies (e.g. Linder, 1998).

"The Conjecture", as I shall call it, is heuristically a strong one, in the sense that it challenges us all to refute it using every observational and theoretical tool to hand. Indeed, it has been decisively refuted several times already - only to rise again and again from its grave. At this meeting we shall certainly hear at least one more new and apparently decisive refutation. Are we not therefore, as honest scientists, bound to abandon it for good? Popperian logic urges "yes", while historical precedent answers "no". How are we to reconcile these contradictory reactions to the same pieces of evidence when only one of them, surely, can be sound? The answer, as I shall argue in Section 7, is to be found in the Bayesian approach to evidence.

My own interest in the subject was aroused in 1969 while making my first observations of galaxies, using the brand new 90-inch at the University of Arizona. Spectroscopic observations had to be guided by watching the object on the slit with the naked eye and I spent many a cold night so staring at NGC ellipticals. It was natural to wonder if there was anyone out there staring back at the Milky Way. I estimated that from the core of an elliptical, where the local sky would necessarily be very bright, our own galaxy, and indeed most of the galaxies in the Universe, would be quite invisible. My sympathy for elliptical based astronomers was cut short when I realised that we might be blinded by the same effect ourselves. At about the same time Ken Freeman (1970) noted, without explanation, that the best studied disk galaxies all have the same surface brightness (SB) while, before us both, Arp (1965) had pointed out that any object, to appear diffuse, must be confined to a certain (rather broad) band in the apparent size/apparent magnitude plane. But I have to thank some typically sceptical remarks from Ron Ekers, during a seminar of mine at Groningen, for forcing me five years later to put my own ideas (Disney, 1976) into a quantitatively defensible form.

2. VISIBILITY THEORY - AND ITS PITFALLS

Our objective must be to enumerate the <u>true</u> number-density of galaxies $N_T(L_T, \Sigma)\, dL_T.d\Sigma$ in the total-luminosity (L_T)/ Surface-brightness (Σ) plane - which I call the Bivariate Brightness Distribution (BBD). To correct the <u>observed</u> number $N_0(L_T, \Sigma)$ for selection effects, one needs to know the "Visibility" $V(L_T, \Sigma)$ - which denotes the relative volumes within which different galaxies can be detected using the observational procedure in question. Clearly $N_T \propto N_0/V$.

It is easy to show that V is a <u>separable</u> function of L_T and Σ (Disney & Phillipps, 1983) i.e.

$$V(L_T, \Sigma) = L_T^{3/2}\Lambda(\Sigma) \tag{1}$$

so that one can FIX L_T and examine V as a function $\Lambda(\Sigma)$ of the SB alone. My first idea (1976) was that $\Lambda(\Sigma) \propto \Theta_{ap}^3$ - where Θ_{ap} is the apparent or isophotal size of a galaxy seen against our sky. Sure enough, and most excitingly, $\Lambda(\Sigma)$ turned out to be a steeply humped function centred, in the case of both ellipticals and spirals, at the very values of SB which turn up most commonly

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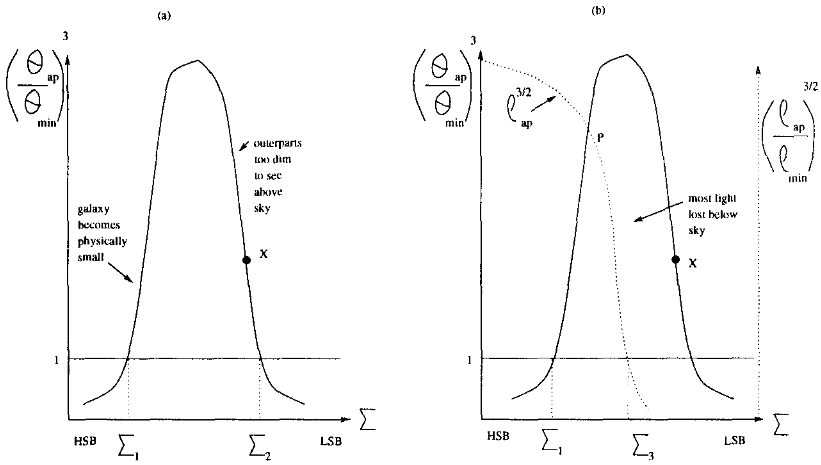

Figure 1. The Visibility Volume $\Lambda(\Sigma)$ of a galaxy as a function of SB Σ in schematic form. Assume the galaxy is of FIXED total luminosity L_T and lies at a fixed distance d_0. Its apparent (isophotal) angular size Θ_{ap} is a humped function of Σ and so is $d_{max}/d_0 = \Theta_{ap}/\Theta_{min}$, where d_{max} is the maximum distance to which the galaxy could be seen as larger than some minimum catalogue limit Θ_{min}. Figure (a) shows $\Lambda(\Sigma) \propto (d_{max}/d_0)^3 \sim (\Theta_{ap}/\Theta_{min})^3$. Any such galaxy with $\Sigma_1 > \Sigma > \Sigma_2$, including the dim object X would be visible out to d_0 or further. But what if there is also an apparent (isophotal) luminosity limit l_{min} (implicit or explicit)? Now $d_{max}/d_0 = (l_{ap}/l_{min})^{1/2}$ and $(l_{ap}/l_{min})^{3/2}$ is the dotted line in (b). The modified Visibility volume is now $\Lambda'(\Sigma)$, always the lesser of the two lines. $\Lambda'(\Sigma)$ is very sharply peaked at P (where $\Sigma_{CAT} \equiv l_{min}/\Theta^2_{min}$) and only galaxies in the much narrower range $\Sigma_1 > \Sigma > \Sigma_3$ would be visible at d_0, and X, although large enough, is selected against because it is apparently far too faint.

in catalogues (Fig 1). There is a peak because HSBGs *are* physically too small, while LSBGs, *appear* too small, with only their centres visible. Such coincidences impressed me then, and many authors continue to use the same SB Visibility function $\Lambda(\Sigma)$ today (e.g. Impey & Bothun 1997, in their excellent review).

But $\Lambda(\Sigma)$ is far from the whole story. To begin with, it yields a peak which is too broad to fit the very narrow SB distributions observed. The full-blown Visibility Function $\Lambda'(\Sigma)$ (Figure 1b) which includes limits on both apparent size (Θ_{lim}) AND apparent luminosity (l_{lim}), is both more accurate, and a great deal more interesting than the earlier $\Lambda(\Sigma)$ (Disney & Phillipps, 1983). Since $\Lambda'(\Sigma)$ is the lower envelope of two plunging and intersecting curves (Figure 1b) it is not continuous, and its peak is very sharp. More importantly it fits the data better and explains why dim galaxies like X are so hard to see (see caption). Samples need to be tested (V/V_{max}) for completeness in both angular size and apparent magnitude. (Note that CCD samples have median SBs no dimmer than plates because both Θ_{lim} and l_{lim} are smaller so $\Sigma_{CAT} \sim$ same.)

As one who has got it wrong myself, I can ruefully claim that Galaxy Visibility is a difficult subject, if only because it is three-dimensional. And it is difficult (for experienced astronomers in particular), to believe its dramatic consequences. Thus Allen and Shu (1979) projected onto the wrong (i.e. degenerate) plane and got almost everything wrong subsequently. Thus van der Kruit (1987) used the correct theory but normalised his two Visibility curves arbitrarily (which you mustn't) and failed to check (which you must) that his sample was complete in both Θ_{lim} and l_{lim} (which it wasn't). Thus McGaugh (1996) has introduced scalelength h in place of L_T then claimed that $\Lambda(\Sigma)$ is not peaked. He's wrong first because he doesn't hold L_T fixed while varying Σ, and second because $V(h, \Sigma) = L_T^{3/2}\Lambda(\Sigma) = (h^2\Sigma)^{3/2}\Lambda(\Sigma) = h^3 F(\Sigma)$, and although $F(\Sigma)$ may not be peaked, $\Lambda(\Sigma)$ still is. If very intelligent astronomers can make such mistakes, so can we all. I believe the majority of workers using Visibility today are still using it wrongly - in particular, they are seriously under-counting LSBGs like X by sticking to the simplistic $\Lambda(\Sigma)$, instead of using $\Lambda'(\Sigma)$.

3. LUMINOSITY-VISIBILITY AND LUMINOSITY FUNCTIONS

I have been asked to say a few words about this much simpler - but still contentious subject. Why do nearly all LF's (not only for optical galaxies) have a faint-end slope close to or just below $\alpha = 1.5$? The answer, I believe, is "because in almost all cases they are based on flux-limited surveys with not enough distances known". In other words the 1.5 is meaningless and proves only that space is roughly Euclidean out to the median distance of the survey. The proof is simple for if we assume that α is only a slowly varying function of L then the <u>local</u> slope of the Schechter function is:

$$\frac{dlogN(L)}{dlogL} \sim -\alpha - \frac{L}{L_\star} \qquad (2)$$

Because of the $L_T^{3/2}$ in (1) the *observed* number (per dL) in a flux-limited sample will peak sharply at $L_{max} = L_\star(3/2\text{-}\alpha)$ where the local slope <u>must</u> therefore be 1.5! If you force-fit a Schechter function to such flux-limited data then, in order to accommodate the exponential choke at the high luminosity end, you will find an α of slightly less than 1.5 for the low luminosity end.

For the above reason, it is not surprising to me that observations in clusters (e.g. Driver at al. 1994, Trentham 1997, Phillipps et al. 1998) find steeper LF's than ones found in the field. Cluster LF's may have their own problems, but they do not require the vast numbers of redshifts (usually lacking) that are needed to unambiguously establish the flat LF's often claimed for the field (e.g. Disney 1997, Marzke et al 1994).

4. ARE THERE HIGH SB GALAXIES ?

Allen and Shu (1979) put forward often quoted, but actually circular arguments against the existence of many HSBGs. At the same time our own analysis of the selection against HSBGs was totally inadequate (Disney 1976, Disney &

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Phillipps 1983) because most HSBGs will be missed, not because they will masquerade as stars as we assumed, but because they will masquerade as background giant galaxies - a far more serious selection effect (Disney, 1998a).

HSBGs will only be uncovered in numbers by large spectroscopic or multiband photometric surveys, or with a large F-O-V space-camera. I am impressed with the early 2DF fibre work by Drinkwater & Gregg 1998) which finds unsuspected HSBGs at the few per cent level ($\sim$ 5 per square degree brighter than B_J=19.7) and this is likely to be an underestimate because galaxies containing absorbing dust cannot attain very high SBs in *the optical.* If you contract a disc galaxy in the radial direction, the effectiveness of its dust as an absorber increases as it is pushed nearer to the accompanying stars - and there will be an upper limit to that SB not much greater than the Freeman value (Jura 1980; Disney & Phillipps 1987; Disney, Davies & Phillipps 1989). However, the question of how much absorption there is in discs is a tricky one, and the early work on inclinations and IRAS observations is now regarded as inconclusive (see Davies & Burstein, 1995, for the debate) but should shortly be illuminated by the first submillimeter survey of bright galaxies being carried out by SCUBA (Dunne et al. 1998). The high sub-mm background derived from COBE suggests that half the starlight from galaxies is absorbed by dust (Dwek et al. 1998, Trewhella et al. 1997).

5. THE STRONGEST ARGUMENTS IN FAVOUR OF THE CONJECTURE

are currently, in my opinion:

(a) The large number of fortuitous numerical coincidences (about 8) otherwise required to explain the observations, if the Conjecture is false (Disney and Phillipps, 1987).

(b) A complete lack of correlation between Blue SB and Blue colour (B-V) - which makes no sense at all unless it is due to selection. Increased star formation should push both parameters up together (Disney & Phillipps 1985, Bothun, Impey & McGaugh 1997, O'Neil 1997).

(c) The close correspondence between calculated Visibility and median observed distance in a large sample of spirals (Davies et al. 1994).

(d) The large number of QSOALs containing metals. What else could they sensibly be - if not LSBGs and dwarfs (Phillipps et al. 1991, Linder, 1998) ?

(e) The lack of correlation observed between the apparent SBs of galaxies, and their galactic latitudes, even in latitudes where the foreground absorption is known to be significant (Davies et al. 1993)

(f) Equal numbers of galaxies found per 1 magnitude bin in SB between $\mu_0(B)$ = 21 and 26.5 (Turner et al. 1993, McGaugh 1996).

6. THE STRONGEST CHALLENGES TO THE CONJECTURE

at present are:

(a) The failure to find optically invisible galaxies in deepish 21-cm pencil beams (Zwaan et al. 1997).

(b) The claim that all QSOALs can be accounted for by enormous gaseous halos surrounding giant galaxies (Lanzetta et al. 1995)

(c) The claim that fluctuations in the EBL are too low to accommodate many LSBGs (Vogeley, 1998).

7. REFUTATION AND THE BAYESIAN APPROACH TO EVIDENCE

How can honest minds reconcile such contradictory evidence? I believe the answer lies in the Bayesian approach to scientific reasoning which is now undergoing a renaissance. The Bayesian's belief in some hypothesis h, as it is affected by some evidence e, is given by P(h|e) "the probability of h given e" which, according to Bayes' theorem is given by:

$$P(h|e) = \frac{P(e|h).P(h)}{P(e|h).P(h) + P(e|\sim h).P(\sim h)} \tag{3}$$

where $\sim$h $\equiv$ "not h".

Such an approach is explicitly subjective because it relies on the assignation of an explicit prior belief or probability P(h) to the hypothesis under question. As such it was severely criticised by Fisher and other "classical" statisticians who proposed allegedly more objective inference tests in its place, and it went out of fashion. It is now enjoying a renaissance because the classical tests have been found to suffer from previously unacknowledged subjective ailments of their own - see for example Howson & Urbach's (1989) readable polemic on the subject.

Consider what conclusions two astronomers A (whose long previous experience lends him high confidence in the Conjecture) and B (who is pretty sceptical about it) draw from some new evidence, e.g. from Zwaan et al's (1997) recent failure to turn up invisible galaxies in an Arecibo strip survey.

If h = "The Conjecture" then:

A(*Believer*)		B(*Sceptic*)	
Prior: $P_A(h)=0.9$,	Thus $P(\sim h=0.1)$	$P_B(h)=0.2$,	Thus $P_B(\sim h)=0.8$
$P(e\|h)=0.3^*$,	Thus $P(e\| \sim h)=0.7$	$P(e\|h)=0.2^*$,	Thus $P(e\| \sim h)=0.8$
Bayes: $P_A(h\|e)=$	$\frac{0.3 \times 0.9}{0.3 \times 0.9 + 0.7 \times 0.1}$	$P_B(h\|e)=$	$\frac{0.2 \times 0.2}{0.2 \times 0.2 + 0.8 \times 0.8}$
=	0.8 (down from 0.9)	=	0.06 (down from 0.2)

(*There is also a lesser subjective element here, hence the slightly different estimates of P(e|h) by A and B).

Thus according to the Bayesian approach, both can maintain their different opinions with integrity, though A's confidence in the Conjecture is slightly eroded whilst B's scepticism is reinforced. And surely that is how it should be. Through their respective priors, each can factor into the equation all the other evidence which they believe bears on the matter. The fiction of a single refutation being enough to change an honest mind, particularly in an observational science, is seen for what it is.

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8. BLIND 21-CM HI SURVEYS

The obvious way to get a completely dark sky has always been to use "redshift" to discriminate local brightness from distant dimness - and where better to do that than 21-cm? Unfortunately receiver noise - and other forms of system noise "illuminate" the 21-cm sky in their own fashion, so that any radio telescope, irrespective of size, has a column density limit (Disney & Banks 1997)

$$N_{HI}(cm^{-2}) \approx 10^{18} T_S \sqrt{\Delta V(kms^{-1})/t_{obs}(sec)} \quad (4)$$

where T_S = system temperature, ΔV is the HI line-width, and t_{obs} the integration time. Generally speaking, one might expect LSBGs to have low N_{HI}'s as well (see de Blok et al. 1996), so that earlier 21-cm blind surveys set few interesting limits on the population of LSBGs - although claims were sometimes made to the contrary (e.g. Shostak 1977).

But technology is changing fast; lower noise amplifiers; faster and cheaper correlators; multi-beam foci on single dishes and wider bandwidth receiver systems on interferometers, mean that over the next few years extreme LSBGs (i.e. with $\mu_0 > 27$ Bμ) will become rather easy to pick up at 21-cm if they exist and if they have "normal" M_{HI}/L_B's in the range 0.3 to 3. The one fly in the ointment is the possibility that such low column density systems might be ionised by the intergalactic radiation field - as some have argued (Maloney, 1993, Dove & Schull, 1994).

What we need to do is look. A start has been made with HIPASS (HI Parkes All Sky Survey, see Staveley-Smith et al. 1996) which is covering the entire southern sky out to 12700 kms^{-1} with sufficient sensitivity to pick up some 10^4 galaxies independently of their optical properties, while Jodrell Bank is taking on the Northern sky. We are at the same time carrying out deeper surveys of selected regions so that we should soon know, one way or another, if large numbers of HI-rich LSBGs exist. The shallow HIPASS survey will only pick up optically invisible galaxies (on the DSS) if they have extreme M_{HI}/L_B's of > 25 in solar units but deeper surveys will be more interesting. In a 5600 seconds/point search of a $4° \times 8°$ patch, we find (Disney et al. 1998) 106 sources, of which 5 have no optical counterparts on the DSS.

9. OUTSIDE THE ATMOSPHERE

The sky will be darker and different, as was pointed out by O'Connell (1987). For instance, in the optical, Bryn Jones and I (1997) have looked at the SBs of galaxies in the HDF - to find the extraordinary result that they are much the same as those seen from the ground (see also Schade, 1995). Yet because SB $\propto (1+z)^{-4}$, they ought to be down by 3-5 magnitudes - depending on the uncertain k-corrections. I can think of only three alternate explanations for this extraordinary result - and it is extraordinary though barely remarked upon, all of which are implausible: selection bringing to light a rich population of intrinsically HSBGs not seen so far from the ground; an evolutionary conspiracy in which SB conveniently rises into the past so as to just cancel the cosmological effects; or the Universe isn't expanding, i.e. the redshift has failed the classical Tolman (1930) test.

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To either side of the optical window the sky ought to be invitingly dark, but instruments planned to specifically exploit this opportunity to look for low SB structures seem sadly to be lacking from NASA's repertoire. For instance in the NUV, O'Connell (1987) showed that the contrast between a galaxy SED and the sky should be 2-3 magnitudes larger than it is from the ground-based optical, while the NIR sky seen from the HST with NICMOS is 200 times darker than it is from Earth. Surely there are some exciting opportunities here; maybe we can design the last camera on HST - the WFC-3, or the first on NGST to exploit these windows. Interested parties please e-mail me.

10. IS THE CONJECTURE REALLY TRUE?

Suppose that for every galaxy in the UGC catalogue ($\sim 10^4$ of them) there exists another Shadow Galaxy of the same luminosity and at the same distance - but with 10 times its radius. Such "Shadows" would have extremely low SBs (μ_0(B)'s > 27Bμ) and HI column densities ($N_{HI} \sim 10^{18}$ cm^{-2}). In order to be sure of finding only 25 members of such a clustered population, I estimate you would need to take 10^4 very deep CCD frames, or 10^4 even deeper 21-cm integrations. Since we haven't made such an effort, or anything approaching it, we cannot be sure that the Shadow Universe is not there. And if it is there, it is most likely to turn up serendipitously - as it did with the Crouching Giant Malin 1 (Bothun et al. 1987). But the most promising place to look is among QSOALs, where I believe it is turning up already. The Shadow population would have a total cross-section 2 orders of magnitude greater than the familiar one and would neatly account for many of the otherwise puzzling properties of QSOALs.

Having confessed to a Bayesian leaning I include in Table 1, for what it's worth, my own estimate of the odds against the truth of the Conjecture, as they have been changed by various pieces of evidence over the years.

11. THE FUTURE OF OUR SUBJECT

There is nothing wrong with ambition in science. We may all ask (see Hardy, 1984):

"Here, on the level sand, Between the sea and land, What shall I build or write, Against the fall of night?"

but ambition is sometimes carried to such lengths by aggressive individuals as to poison a whole subject area, drive out the competition and thus slow progress to a crawl. To maintain our own momentum, by preserving the enjoyment in our fascinating area, I hope we will therefore:

(a) Recognise there is such a lot more to do that we are unlikely to be "scooped". According to Harwit(1981) we have made no more than 10% of the key discoveries in astronomy.

(b) Remember that "Astronomy is not Physics" - indeed it is so incomplete that one refutation can never be enough. According to the Bayesian view there is usually room for more than one interpretation of the same evidence.

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Epoch	Odds Against	Evidence
1975	$10^6 : 1$	Original prior belief
1976	50 : 1	First selection effects paper (Disney 1976)
1977	$10^3 : 1$	Failure to find "Icebergs" in off-beams (Shostak 1977)
1983	10 : 1	Visibility calculations done properly (Disney & Phillipps 1983)
1985	5 : 1	No correlation between SB & colour (Disney & Phillipps 1985)
1987	3 : 1	Many LSBGs found in Fornax (Phillipps et al. 1987)
1987	2 : 1	Malin 1 found (Bothun et al. 1987)
1990	1 : 1	Shostak's failure explained (unpub. see Disney & Banks 1997)
1993	1 : 2	No correlation found between SB and b_{II} (Davies et al. 1993)
1994	1 : 4	Distribution of ESO SBs fits Visibility (Davies et al. 1994)
1995	1 : 2	Huge halos invoked to explain QSOALs (Lanzetta et al. 1995)
1995	1 : 3	LSBGs in equatorial survey (Sprayberry et al. 1993)
1997	1 : 4	Blind HI survey of CenA group (Banks et al. 1998)
1997	1 : 5	LSBGs found in deep CCD survey (Dalcanton et al. 1997)
1997	1 : 3	No very dim galaxies in Arecibo strip (Zwaan et al. 1997)
1998	1 : 10	HSBGs found in redshift surveys (Drinkwater & Gregg 1998)
1998	1 : 5	Low SB fluctuations in HDF (Vogeley, 1998)

Table 1. Odds Against The Conjecture

(c) Never write anonymous referees reports either on papers or proposals. The referees job is surely to encourage and improve good science, not discourage or disprove bad, for in an open society bad science will sink to its own level quickly enough.

(d) Celebrate and reference each others work, and expect the same in return. In that connection, I want to sincerely thank Greg Bothun, Julianne Dalcanton and Chris Impey for being overgenerous to me.

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