Appendix D: Cross sections and probability

Consider a flux I_0 of collimated, monoenergetic particles impinging upon a target. The number of atoms per unit volume in the target material is

$$n_a = \frac{N_A \text{ (atoms/mol)} \times \rho \text{ (g/cm}^3\text{)}}{A \text{ (g/mol)}} \tag{D.1}$$

where ρ is the target density, A is its atomic weight, and N_A is Avogadro's number.

In an infinitesimal thickness dx of the target there will be $n_a\,dx$ atoms/cm^2 in the path of the beam. As the beam traverses the target, interactions take place, and the beam intensity is reduced. Let dI refer to the change in flux. This quantity will be proportional to both the incident flux of beam particles and the number of target atoms/cm^2 in the beam's path.

$$dI = -\sigma I n_a\,dx$$

The constant of proportionality σ is referred to as the total cross section and has the units of area. A convenient unit for nuclear work is the barn, where

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

Let us assume that the material is homogeneous and that the target is thin enough so that the particle's velocity is not significantly reduced. Then σ is not a function of x, and if we integrate over the target thickness, we find that

$$I(x) = I_0 \exp(-\sigma n_a x) \tag{D.2}$$

Thus, the intensity of particles satisfying the initial conditions drops off exponentially as the beam traverses the target.

If we define Pr to be the probability that a particle interacts in the target, then the probability that the particle does not interact after crossing a thickness L is

$$1 - \Pr = I(L)/I_0 = \exp(-\sigma n_a L) \tag{D.3}$$

The quantity

$$\lambda_I = (\sigma n_a)^{-1} \tag{D.4}$$

is called the interaction length. When $x = \lambda_I$, the beam intensity in Eq. D.2 drops to I_0/e, so λ_I represents the mean free path between interactions. When the target thickness $L \ll \lambda_I$, we can expand the exponential in Eq. D.3 to get

$$\Pr = \sigma n_a L = L/\lambda_I \tag{D.5}$$

The inverse of the interaction length

$$\mu = 1/\lambda_I = \sigma n_a \tag{D.6}$$

is called the attenuation coefficient.

Intuitively, we may consider each atom to present a circular target of area σ to the beam particle. If we assume that the beam particles are randomly distributed over a 1-cm^2 area and that an interaction takes place whenever a beam particle hits one of the circular areas, then $n_a \sigma\, dx$ represents the fraction of the total area in which an interaction will take place, or equivalently the probability of an intersection.

Now consider a scattering experiment using a short target of length $L \ll \lambda_I$ where the scattered particles are only detected in a small solid angle $\Delta\Omega$ around the direction (θ, ϕ). The detected intensity is then given by

$$I(\theta, \phi) = I_0 n_a L \frac{d\sigma}{d\Omega}(\theta, \phi)\, \Delta\Omega \tag{D.7}$$

The constant of proportionality $d\sigma/d\Omega$ is called the differential cross section. The form of the functions σ and $d\sigma/d\Omega$ depends on the dynamics of the scattering process. The probability of an interaction is

$$\Pr = n_a L \frac{d\sigma}{d\Omega}\, \Delta\Omega \tag{D.8}$$