

TROTTER–KATO PRODUCT FORMULA AND AN APPROXIMATION FORMULA FOR A PROPAGATOR IN SYMMETRIC OPERATOR IDEALS

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The Trotter–Kato product formula is a mathematical clarification of path integration in quantum theory [4]. It gives a precise meaning to Feynman’s path integral representation of the solutions to Schrödinger equations with time-dependent potentials. In this thesis, we consider the Trotter–Kato product formula in an arbitrary symmetrically F -normed ideal closed with respect to the logarithmic submajorisation.

An abstract nonautonomous evolution equation is widely used in various fields of mathematics and quantum mechanics. For example, the Schrödinger equation and linear partial differential equations of parabolic or hyperbolic type [3, 5]. The second problem we consider is the existence of the propagator for such an equation and its approximation formula in an arbitrary symmetric Banach ideal. The approximation formula in the autonomous case corresponds to the Trotter product formula.

Some of this research has been published in [1, 2].

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