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AN INTRODUCTION TO PROBABILISTIC NUMBER THEORY

Despite its seemingly deterministic nature, the study of whole numbers, especially prime numbers, has many interactions with probability theory, the theory of random processes and events. This surprising connection was first discovered around 1920, but in recent years, the links have become much deeper and better understood.

Aimed at beginning graduate students, this textbook is the first to explain some of the most modern parts of the story. Such topics include the Chebychev bias, universality of the Riemann zeta function, exponential sums, and the bewitching shapes known as Kloosterman paths. Emphasis is given throughout to probabilistic ideas in the arguments, not just the final statements, and the focus is on key examples over technicalities. The book develops probabilistic number theory from scratch, with short appendices summarizing the most important background results from number theory, analysis, and probability, making it a readable and incisive introduction to this beautiful area of mathematics.

Emmanuel Kowalski is Professor in the Mathematics Department of the Swiss Federal Institute of Technology, Zurich. He is the author of five previous books, including the widely cited *Analytic Number Theory* (2004) with H. Iwaniec, which is considered to be the standard graduate textbook for analytic number theory.

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An Introduction to Probabilistic Number Theory

EMMANUEL KOWALSKI
Swiss Federal Institute of Technology, Zürich



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