The dielectronic satellite lines from the $1s^2 3L$ and $1s 3L_b$ double-excited configurations of lithium-like ions have been studied experimentally and theoretically under dense-plasma conditions. Good agreement between theory and measurements performed in laser-target interactions at 0.53 m wavelength, $10^{14}$ W/cm$^2$ laser irradiance show that collisional equilibrium conditions are reached between $n - 3$ singly and double excited levels at electron densities greater than $10^{22}$ cm$^{-3}$ in Al and Si.