

# Stellar-mass black holes in star clusters: implications for gravitational-wave radiation

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**Abstract.** We study the dynamics of stellar-mass black holes (BHs) in star clusters, with particular attention to the formation of BH–BH binaries, which are interesting as sources of gravitational waves (GWs). We examine the properties of these BH–BH binaries through direct  $N$ -body simulations of Plummer clusters of  $N \leq 10^5$  low-mass stars with an initial population of stellar-mass BHs, using the NBODY6 code. We find that the stellar-mass BHs segregate rapidly into the cluster core and form a dense subcluster of BHs in which BH–BH binaries form through three-body encounters. While most BH binaries are ejected from the cluster by recoils due to superelastic encounters with the single BHs, we find that for clusters with  $N \gtrsim 5 \times 10^4$ , typically a few of them harden sufficiently so that they can merge via GW emission within the cluster. Also, for each of such clusters there are a few escaping BH binaries that merge within a Hubble time, with most merger times being within a few Gyr. These results imply that the intermediate-age massive clusters constitute the most important class of star cluster candidates that can produce dynamical BH–BH mergers at the present epoch. The BH–BH merger rates obtained from our computations imply a significant detection rate ( $\sim 30 \text{ yr}^{-1}$ ) for the proposed *Advanced LIGO* GW detector.

**Keywords.** gravitational waves, black hole physics, stellar dynamics, scattering, methods:  $N$ -body simulations, methods: numerical, galaxies: star clusters, binaries: close

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## 1. Introduction

Star clusters, e.g., globular clusters (henceforth GCs), young and intermediate-age massive clusters (henceforth IMCs) harbor a large overdensity of compact stellar remnants compared to the field by virtue of their mass segregation. These compact stars, which are neutron stars (henceforth NSs) and black holes (henceforth BHs), are produced by stellar evolution of the most massive stars of the cluster within the first  $\sim 50$  Myr after cluster formation. Since they are heavier than the remaining low-mass stars in the cluster, they segregate quickly (within one or a few half-mass relaxation times) to the cluster core, forming a dense subcluster of compact stars. Such a compact stellar subcluster is of wide interest as it is efficient in producing compact-star binaries dynamically, which are of interest for a wide range of physical phenomena, e.g., X-ray emission (X-ray binaries), gravitational waves (BH–BH and NS–NS binaries) and short gamma-ray bursts (NS–NS and NS–BH binaries). In the present work, we investigate the dynamics of stellar-mass BHs in star clusters, which are formed through supernova explosions of  $\gtrsim 18M_{\odot}$  stars, typically within the mass range  $8M_{\odot} \lesssim M_{\text{BH}} \lesssim 12M_{\odot}$  for solar-metallicity progenitors (Casares 2007). We are particularly interested in the dynamically formed BH–BH binaries, which are strong sources of GWs as they spiral in via GW radiation.

As studied earlier by several authors (e.g., Merritt *et al.* 2006; Mackey *et al.* 2007), BHs, formed through stellar evolution, segregate into the cluster core within  $\sim 0.3$  pc

and form a dynamically isolated subcluster of BHs, where the density of BHs is large enough that BH–BH binary formation through three-body encounters (Heggie & Hut 2003) becomes important. These dynamically formed BH binaries then ‘harden’ through repeated superelastic encounters with the surrounding BHs (Heggie’s law; Heggie 1975). The binding energy of the BH binaries released is carried away by the single BHs and BH binaries involved in the encounters. This causes the BHs and BH binaries to get ejected from the BH core to larger radii in the cluster, and they deposit energy to the cluster stars while sinking back to the core through dynamical friction, thereby heating the cluster. As the BH binaries harden, the encounter-driven recoil becomes stronger and finally the recoil is large enough that the encountering single BH and/or the BH binary escapes from the cluster. This also results in cluster heating because of the associated mass loss from the cluster core that decreases its potential energy correspondingly. These heating mechanisms result in cluster expansion.

In the present work, we make a detailed study of the dynamics of BH–BH binaries formed in a BH subcluster. In particular, we investigate whether hard enough BH binaries that can merge via gravitational radiation in a Hubble time within the cluster or after getting ejected, can be formed in such a subcluster. To that end, we perform a series of  $N$ -body integrations of concentrated star clusters (half-mass radius  $r_h \leq 1$  pc) consisting of  $\leq 10^5$  low-mass stars, to which a certain number of stellar-mass BHs is added, representing a star cluster with an evolved stellar population.

## 2. Computations

To study the dynamics of BHs in star clusters, we perform direct  $N$ -body simulations with star clusters in which a certain number of BHs are added initially, using the NBODY6 code on graphical-processing-unit (GPU) platforms (see Aarseth 2009). The initial cluster follows a Plummer model with half-mass radius  $r_h \leq 1$  pc, consisting of  $N \leq 10^5$  low-mass main-sequence stars in the mass-range  $0.5M_\odot \leq m \leq 1.0M_\odot$ . A specified number of BHs are added to each cluster with the same initial distribution as the stars. The initial number of BHs are chosen according to the BH retention fraction, where we explore both full retention and the case where half of the BHs are ejected from the cluster by natal kicks. For simplicity, we consider only equal-mass BHs in the present work, with the representative value of  $M_{\text{BH}} = 10M_\odot$ . With such a cluster, we mimic the epoch at which the massive stars have already evolved to produce their remnant BHs.

To evolve the BH–BH binaries due to GW emission, the well-known Peters’ formula (Peters 1964) is used in NBODY6, according to which the merger time  $t_{\text{mrg}}$  of an equal-mass BH–BH binary due to GW emission is given by

$$t_{\text{mrg}} = 150 \text{ Myr} \left( \frac{M_\odot}{M_{\text{BH}}} \right)^3 \left( \frac{a}{R_\odot} \right)^4 (1 - e^2)^{7/2}, \quad (2.1)$$

where  $a$  is the semimajor axis of the binary and  $e$  is its eccentricity. Numerical simulations of BH–BH mergers indicate that for unequal-mass BHs or even for equal-mass BHs with unequal spins, the merged BH product acquires a velocity kick of typically  $100 \text{ km s}^{-1}$  or more, due to asymmetry in momentum outflow from the system, associated with the GW emission. Therefore, in our computations, we provide an arbitrarily large velocity kick of  $150 \text{ km s}^{-1}$  immediately after a BH–BH merger, to make sure that the merged BH escapes. Also, we evolve the clusters isolated, i.e., in absence of a galactic tidal field as the formation of the BH core through mass segregation and its dynamics remain largely unaffected by the presence of a tidal field, which mainly affects stars near the

**Table 1.** Summary of the computations performed for isolated clusters and those with reflective boundary (see Section 2). The meaning of different columns is as follows: (1): Identity of the particular model; similar values with different names (ending with A, B, etc.) imply runs repeated with different random seeds. (2): Total number of stars,  $N$ . (3): Number of simulations,  $N_{\text{sim}}$ , with the particular cluster. (4): Initial half-mass radius of the cluster,  $r_h(0)$  (isolated cluster), or radius of reflective sphere,  $R_s$ . (5): Initial number of BHs,  $N_{\text{BH}}(0)$ . (6): Total number of BH–BH binary mergers within the cluster,  $N_{\text{merg}}$ . (7): Times,  $t_{\text{merg}}$ , corresponding to the mergers. (8): Number of escaped BH pairs,  $N_{\text{esc}}$ ; the three values of  $N_{\text{esc}}$  are for  $t_{\text{merg}} < 3$  Gyr, 1 Gyr and 100 Myr, respectively.

Model name	$N$	$N_{\text{sim}}$	$r_h(0)$ or $R_s$ (pc)	$N_{\text{BH}}(0)$	$N_{\text{merg}}$	$t_{\text{merg}}$ (Myr)	$N_{\text{esc}}$
Isolated clusters							
C10K20	10000	10	1.0	20	0	— —	— —
C25K50	25000	10	1.0	50	0	— —	3 1 1
C50K80	45000	1	1.0	80	1	698.3	3 1 0
C50K80.1	45000	1	0.5	80	2	217.1, 236.6	3 2 1
C50K40.1	45000	1	0.5	40	0	— —	1 1 1
C50K200	50000	1	1.0	200	2	100.8, 467.8	0 0 0
C65K110	65000	1	1.0	110	1	314.6	4 2 1
C65K110.1	65000	1	0.5	110	0	— —	4 3 1
C65K55.1	65000	1	0.5	55	1	160.5	1 0 0
C100K80	100000	1	1.0	80	2	219.4, 603.2	5 2 1
C100K200	100000	1	1.0	200	0	— —	5 4 4
Reflective boundary							
R3K180	3000	1	0.4	180	1	1723.9	5 3 1
R4K180A	4000	1	0.4	180	1	3008.8	2 2 1
R4K180B	4000	1	0.4	180	2	100.2, 1966.5	2 1 0
R3K100	3000	1	0.4	100	2	3052.8, 3645.9	1 1 0
R4K100A	4000	1	0.4	100	2	104.4, 814.2	3 3 1
R4K100B	4000	1	0.4	100	1	1135.3	3 3 3

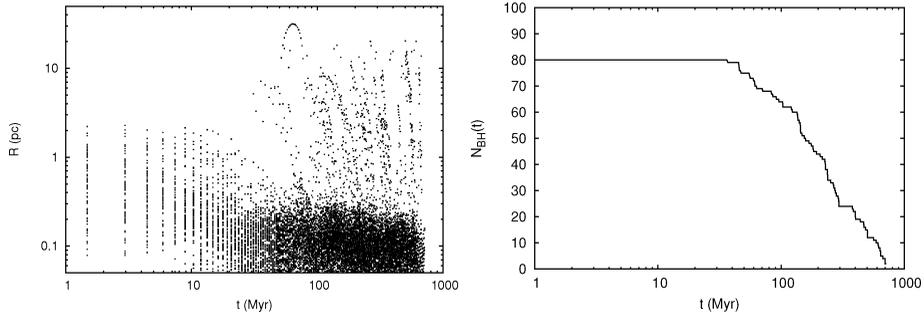
tidal boundary. Further, for simplicity, we do not take into account primordial binaries in this initial study.

### 2.1. Simulation of a GC core: reflective boundary

We also perform integrations with a smaller number of stars and BHs that are confined within a reflecting spherical boundary (Banerjee *et al.* 2009). With such a dynamical system, one can mimic the core of a massive cluster, where the BHs are concentrated after mass segregation. The advantage of this approach is that one can compute the evolution of a massive cluster with many fewer stars, allowing much faster computation. We integrate  $N = 3000 - 4000$  stars, confined within 0.4 pc, which provides a stellar density of  $\sim 10^4 M_{\odot} \text{pc}^{-3}$ , appropriate for the core density of a massive cluster. However, stars and BHs faster than a pre-assigned speed,  $v_{\text{esc}} \approx 24 \text{ km s}^{-1}$ , representing the escape speed of the parent cluster, are allowed to escape through the reflective boundary.

## 3. Results

Table 1 summarizes the results of our computations. For demonstration purposes, we use one of the models, C50K80 (see Table 1). Other models generally possess similar properties. Figure 1 (left panel) demonstrates the mass segregation of the BHs in the cluster, which takes about 50 Myr. As the BHs segregate within about 0.3 pc of the



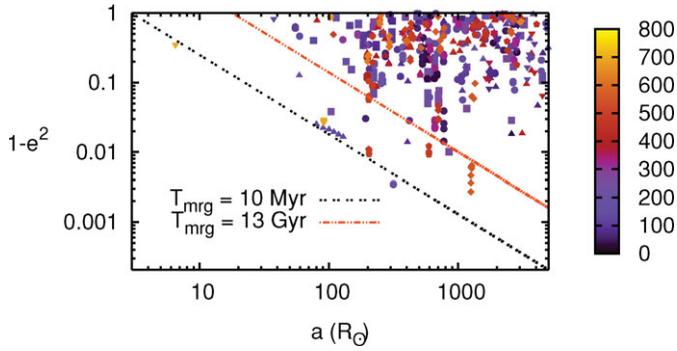
**Figure 1.** Mass segregation of BHs shown by the radial position  $R$  versus time  $t$  (left panel) for model C50K80. The BHs segregate in  $\sim 50$  Myr, during which  $N_{\text{BH}}$  (the number of BHs bound to the cluster) remains unchanged (right panel). As the BH subcluster becomes dense enough that BH–BH binaries begin to form through three-body encounters, single BHs and BH binaries are ejected from the cluster, resulting in a decrease of  $N_{\text{BH}}$ .

cluster core, the BH density of this subcluster becomes high enough to form BH–BH binaries through three-body encounters. Once BH–BH binaries start forming, single BHs and BH binaries begin to escape from the BH core due to encounter recoils (see Section 1). In Figure 1 (left panel), one can clearly distinguish the two phases of the BH subsystem, the initial segregation phase and BH-core formation, the radial positions of the BHs being scattered outwards in the latter phase due to the recoils. The resulting decrease of  $N_{\text{BH}}$  during this phase is also shown in Figure 1 (right panel).

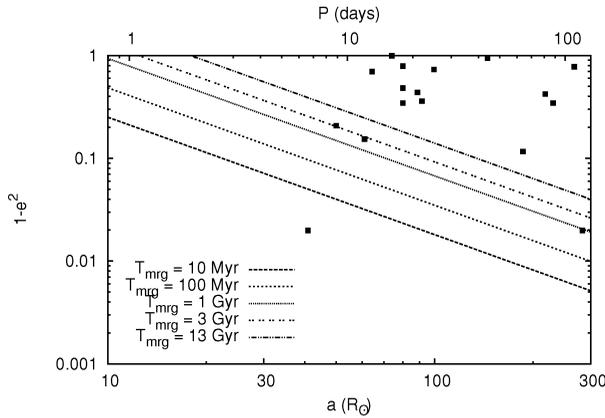
### 3.1. Mergers and escapers

To study the possibility of BH–BH mergers, we consider the positions of the BH–BH binaries within the cluster in an  $a$  versus  $(1 - e^2)$  plane, as shown in Figure 2, where each pair is represented by a different symbol and color coded with the evolution time in Myr. For each BH pair,  $a$  and  $e$  fluctuate over the plane, the changes occurring over a collision timescale, of order Myr. Since the orbital periods of the binaries corresponding to these points are much shorter (from  $\sim$  days to years), these points generally represent binaries which are stable over many orbits. Overplotted in Figure 2 are lines of constant GW merger time  $t_{\text{mrg}}$ , as given by Equation (2.1). While most points correspond to very large merger times, a few of them lie close to the  $t_{\text{mrg}} = 10$  Myr line. This indicates that these binaries are indeed hardened up to small enough  $a$  and/or acquire sufficient eccentricity that if they are left unperturbed, they can merge via GW emission within several Myr. However, these merging BH pairs can still be perturbed by further encounters on timescales of  $\sim$  Myr, which can often prevent them from merging. In the particular example shown in Figure 2, only one of them could merge within the cluster. This feature is found to be generally true for all models reported here (see Table 1), i.e., each produces BH pairs that are capable of merging within several Myr. On the other hand, among the escaped BH binaries, all of them with GW merger times of less than a Hubble time are of interest, since they remain unperturbed afterwards. The positions of the escaped BH binaries from the model C65K110 are shown in Figure 3 as an interesting illustration.

Figure 4 (left panel) shows the distribution of merger times,  $t_{\text{mrg}}$ , of BH binaries that occurred within the clusters, which indicates that  $N_{\text{mrg}}$  decreases with  $t_{\text{mrg}}$ . This can be expected, as at later times  $N_{\text{BH}}$  in a cluster decreases, decreasing the hardening rate of BH binaries correspondingly. Figure 4 (right panel) shows the distribution of the merger times for the escapers, which also shows a decrease with time, as can be expected from Equation (2.1). The above merger-time distributions indicate that most mergers occur within the first few Gyr of cluster evolution.



**Figure 2.** Positions of all BH binaries within the cluster on the  $1 - e^2$  versus  $a$  plane, where different symbols are used to distinguish between different BH pairs (for model C50K80). The color coding of the points, as indicated by the color scale, represents the time (in Myr) at which they appear at a particular location in the above plane. Lines of constant GW-merger times  $t_{\text{mrg}} = \text{constant}$  are overlotted.

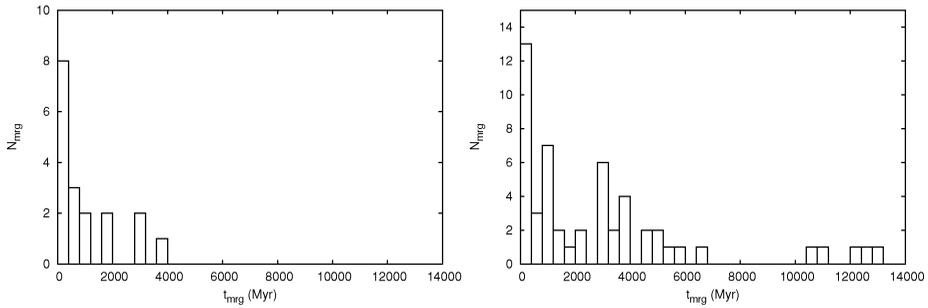


**Figure 3.** Positions of all escaped BH binaries in the  $1 - e^2$  versus  $a$  plane for model C65K110. Lines of constant merger times are overlotted, as in Figure 2.

These results imply that an important class of candidates for dynamically forming BH binaries that merge at the present epoch are the star clusters with initial mass  $M_{\text{cl}} \gtrsim 3 \times 10^4 M_{\odot}$ , which are few Gyr old. Such clusters represent IMCs with initial masses close to the upper limit of the initial cluster mass function in spiral and starburst galaxies (Weidner *et al.* 2004; Gieles *et al.* 2006; Larsen 2009). Old GCs, which can be about 10 times more massive, are typically much older ( $\sim 10$  Gyr), so that they cannot contribute significantly to the present-day merger rate, as most of their mergers would have occurred much earlier. On the other hand, young massive clusters with ages younger than 50 Myr are generally too young to produce BH–BH mergers, as the segregation of BHs and the formation of the BH core itself takes longer. Hence, IMCs seem to be the most likely star clusters for dynamically producing present-day BH–BH mergers.

### 3.2. Detection rate of BH–BH mergers

Results from Table 1 give an average merger rate of  $\approx 0.4 \text{ cluster}^{-1} \text{ Gyr}^{-1}$  for BHs within the clusters and  $\approx 0.9 \text{ cluster}^{-1} \text{ Gyr}^{-1}$  for the escapers, totalling  $\mathcal{R}_{\text{mrg}} \approx 1.3 \text{ cluster}^{-1} \text{ Gyr}^{-1}$ . To estimate the detection rate of BH–BH mergers from IMCs, we adopt the space density derived by Portegies Zwart & McMillan (2000) for young populous clusters,  $\rho_{\text{cl}} \approx 3.5 h^3 \text{ Mpc}^{-3}$ , where  $h = 0.73$  is the Hubble parameter. The above merger



**Figure 4.** (*left*) Distribution of the merger times,  $t_{\text{mrg}}$ , for BH binary mergers within the cluster for the models in Table 1. (*right*) Distribution of  $t_{\text{mrg}}$  for escaped BH binaries for the models in Table 1.

rate implies (Banerjee *et al.* 2009) that IMCs would yield a BH–BH-merger detection rate of  $\mathcal{R}_{\text{AdLIGO}} \approx 30 \text{ yr}^{-1}$  for the future *Advanced LIGO (AdLIGO)* detector (see, e.g., Belczynski *et al.* 2007). The present-day *LIGO* detection rate is, however, negligible,  $\mathcal{R}_{\text{LIGO}} \approx 4.6 \times 10^{-3} \text{ yr}^{-1}$ . Interestingly, the above dynamical BH–BH-merger detection rate can be more than 10 times higher than that from individual primordial binaries (Belczynski *et al.* 2007). Our results imply that dynamically formed BH–BH binaries constitute the dominant contribution to BH–BH merger detection. Thus, the dynamical BH–BH inspirals from star clusters seem to be a promising channel for GW detection by *AdLIGO*, although their estimated detection rate with the present *LIGO* detector is negligible, conforming with the hitherto nondetection of GWs.

### Acknowledgements

We thank the Symposium organizers for selecting this work for presentation. SB acknowledges support from the Alexander von Humboldt Foundation. HB acknowledges support from the German Science Foundation through a Heisenberg fellowship.

### References

- Aarseth, S. J. 2009, <http://www.sverre.com/>
- Banerjee, S., Baumgardt, H., & Kroupa, P. 2009, *MNRAS*, in press (arXiv:0910.3954)
- Belczynski, K., Taam, R. E., Kalogera, V., Rasio, F. A., & Bulik, T. 2007, *ApJ*, 662, 504
- Casares, J. 2007, in: V. Karas & G. Matt. (eds.), *Black Holes from Stars to Galaxies: Across the Range of Masses*, IAU Symp, 238, p. 3
- Gieles, M., Larsen, S. S., Scheepmaker, R. A., Bastian, N., Haas, M. R., & Lamers, H. J. G. L. M. 2006, *A&A*, 446, L9
- Heggie, D. C. 1975, *MNRAS*, 173, 729
- Heggie, D. C. & Hut, P. 2003, *The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics* (Cambridge: Cambridge University Press)
- Larsen, S. S. 2009, *A&A*, 494, 539
- Mackey, A. D., Wilkinson, M. I., Davies, M. B., & Gilmore, G. F. 2007, *MNRAS*, 379, L40
- Merritt, D., Piatek, S., Portegies Zwart, S., & Hemsendorf, M. 2004, *ApJ*, 608, L25
- Peters, P. C. 1964, *Phys. Rev. B*, 136, 1224
- Portegies Zwart, S. F. & McMillan, S. L. W., 2000, *ApJ* (Letters), 528, L17
- Weidner, C., Kroupa, P., & Larsen, S. S. 2004, *MNRAS*, 350, 1503