THE ORGANIZATION OF SPACE: FRAMES, SYSTEMS AND STANDARDS

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ABSTRACT. The term "frame of reference" or "reference frame" has long been established in physics. The recent emergence of the use of this very term in astronomy to denote "a catalogue of the adopted coordinates of reference points that serves to define, or realize, a particular coordinate frame" is misleading because "a catalogue of adopted coordinates" must necessarily consists of estimates which cannot uniquely define a coordinate system.

Physical events happen in space-time and their complete description requires the assignment of spatial coordinates and epochs. This self-evident fact has led physicists and astronomers to consider certain relevant concepts and to agree on names for them. It can be argued that astronomy and physics are two branches of the same science; their concerns intersect often, perhaps more often than those of any other two sciences, and one would therefore expect that astronomers and physicists call the same entities by the same names. It will lead to confusion if a particular name, preempted in one of the disciplines for one concept is used in the other for a different concept, especially since contemporary physicists frequently work on astronomical problems and vice versa. Such a practice discourages graduate students from choosing astronomy and especially astrometry as a field of specialization.

At the beginning of this century the advent of the theory of relativity opened a discussion of space-time and the various systems used to assign coordinates to points in space-time. The term "inertial frame of reference" emerged for those spaces in which Newton's first postulate and third law of motion are valid. This is the set of spaces (with certain properties) in which certain events take place and this does not imply that these spaces must be organized by any one particular coordinate system. It is, in fact, well known that there is no particular inertial system of coordinates which is privileged, but that all coordinate systems whose axes do not rotate with respect to any of the others but whose origins move uniformly with respect to each other can be used to organize the same inertial frame of reference. The understanding of the community of physical scientists is well expressed by C. A. Murray (1989): "...a 'reference frame' is a physical entity, independent of its numerical realization, just as a vector or tensor is a physical entity independent of any triad (or tetrad) by which it is described. Thus we have reference frames defined by the solar system, the stars in our galaxy and distant matter such as quasars. Nearer home, we have the terrestrial reference frame and also local inertial frames of different observers. These are in accordance with the usual terminology of physics in which 'frames' do not necessarily imply specific coordinates..." This passage states how the term "frame" or "frame of reference" has been used in physics for almost a century, and this is how it should be

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interpreted in astronomy, especially since many who are now astronomers received their formal training as physicists. Unfortunately, a movement has recently emerged to use this term in a different sense (cf. Kovalevsky and Mueller 1989).

In order to operate within a particular reference frame, it must be *organized*, which means that a system of coordinates must have been defined which is in a known (not necessarily fixed) relationship with the frame and allows one to assign coordinates, that is numbers which characterize the locations of specific points within the reference frame. In order to minimize the necessity for frequent revisions, this assignment should preferably be conceptual (Eichhorn 1983), that is, based on some physical (or geometric) concept which dictates and determines unambiguously the choice of the coordinate axes. These axes determine what has been called a "coordinate system" or a "system". The coordinate systems in kinematic astronomy are examples: The H (horizon) system at a particular point of the Earth's surface has its z-axis pointing in the direction opposite to that of the vector of gravitation at this point, and the y-axis points in the direction of the vector $\omega \times \mathbf{k}$, where $\mathbf{k}$ is a vector along the z-axis and ω is a vector in the direction of the Earth's axis.[2] The Q_t-system, the true equatorial system as we understand it at this time, is defined purely conventionally: Let n_E be a vector normal to the ecliptic - a prime example for a conventionally defined entity - pointing toward its northpole, and n_{Q_t} a vector pointing toward the CEP, that is, the conventional celestial ephemeris pole. The x-axis of Q_t is parallel to $n_{Q_t} \times n_E$. Note that the nonrigidity of the Earth would (or could) render the ω which belongs to a particular volume element of the Earth different from one place on the Earth to the next (even though ever so slightly and at this time immeasurably little) if ω were in the direction of the axis of any volume element's instantaneous rotation. This potential dependence of ω on the location of the observer is avoided by the introduction of the CEP, whose direction, by definition, cannot depend on the observer's position on the Earth's surface.

The term "system" has been used in astronomy for rigorously defined constructs which organize (the space-like components of) frames of reference. These definitions may also involve the fixing of the time dependence of the system with respect to the frame; note, for example, that the time dependence of Q_t with respect to an inertial frame is fixed by the theories of precession and of nutation, including the numerical parameters in these theories.

Once such a system has been defined, it becomes possible to estimate the coordinates of points[3] with respect to the system itself and thus with respect to the frame which the system organizes. If this is done for a representative sample of points, such as the stars in the FK5, we obtain what astronomers in the past have - somewhat loosely - also called a system, or perhaps more specifically a "system of standards" or a "reference system". The most accurate and self-explanatory description of such an entity would be "Estimated System" (Owen 1990). Even though the statement is trivial that no system of standards (or estimated system) can consist of the actual coordinates of the points in the system it represents, astronomers are making statements such as "catalogue X is 'on the system' of the FK5". The rather recent suggestion by Kovalevsky and Mueller (1989) to use the term "frame of reference" or "reference frame" for such sets of coordinate estimates has been followed by quite a number of investigators.

This paper suggests that we abandon this practice for two reasons: one merely historical but the other more substantive. We have already stated the historical reason: the term "frame of reference", which emerged in the connotation suggested by Kovalevsky and Mueller only recently in fundamental astronomy, has long been preempted for a different concept in physics, a

[2]After the adoption of the CEP (celestial ephemeris pole), this is no longer conceptually defined, because the CEP itself is conventionally defined. A conceptual definition would require that ω itself be conceptually defined, either as the direction of the instantaneous axis of rotation of the observer's element of the Earth's surface or the (physically uniquely defined) principal axis of inertia.

[3]Without restricting generality, we can restrict our considerations to points rather than to extended bodies which may, after all, themselves be regarded as sets of points.

very closely related discipline; its parallel and simultaneous use for a totally different concept in astronomy cannot but lead to confusion and will discourage some young scientists at the beginning of their career from entering a field where such confusion reigns.

The second, substantive reason is that there cannot be an unambiguous or rigorous relationship between a system of standards and its target system, that is, the coordinate system it is intended to represent. This was pointed out by Eichhorn (1982). The values which constitute a (system of) standard(s) are the results of measurements and therefore affected by unknown errors. These errors are not only accidental (having their origin in the random errors of the relevant measurements) but unavoidably also systematic. The reason for this is that the process of estimating the values which constitute the (system of) standard(s) also involved the estimation of certain parameters, each of which was used in the calculation of at least some groups of standard values. Only parameter estimates and not the parameters themselves are available for the calculation of the values which make up the standard. These estimates will therefore deviate systematically from those quantities which they are intended to estimate as a consequence of the (unknown and unknowable) differences between the actual values of the parameters and their estimates. This unavoidable parameter variance (cf. Eichhorn and Williams 1963) is present even in the absence of noticeable systematic trends in the adjustment residuals (Eichhorn and Cole 1985). It is thus impossible to establish empirically a bias-free set of estimates of the coordinates of any sets of objects with respect to any given system; all we can hope to achieve is to establish a standard which contains estimates whose systematic and random errors are small, at least below whatever upper limit is imposed by the nature of the task toward which they are to be used.

Let it be further emphasized that the use of a standard as reference for the computation of coordinates in the system, from measurements on objects whose coordinates are not part of the standard, cannot yield coordinates of the nonstandard objects whose systematic errors against the target system are identical to those of the standard in the domain covered by standard as well as nonstandard values. The reason (cf. Eichhorn 1982) is again the parameter variance: the differences between the actual values of the parameters involved in the reduction and their estimates will cause systematic differences of the secondary standard values against the primary reference standard. This means, for example, that there is no finite operation by which one could reduce an independent catalogue "to the system of the FK5". The end/product of any operation undertaken toward this purpose will always be a set of position estimates whose systematic errors are different from those of the FK5 (cf. Cole 1988). Even worse: different investigators, making different judgements about reduction models and reduction methods will, from the same set of measurements on the same objects, arrive at different sets of estimates for what are physically the same values, all claiming to have achieved a reduction onto the FK5 system. There will obviously be systematic differences as well between the results of these different reductions. Yet how can sets of estimates, all intended to estimate the identical target quantities, be "on the same system" if there are systematic differences between them?

The incongruities inherent in the statement that a set of estimated positions can define a coordinate system is further illustrated by the following considerations: both polar coordinates of any object and one of the polar coordinates of a second object will fix the coordinate system to which these objects' coordinates are referred. Any combination of three coordinates will fix the system, and one can determine as many different systems found as combinations of coordinates can be put together, because all estimated coordinates are affected by unknown errors, random as well as systematic. One would have to agree on an unambiguous procedure to get "the" system by somehow averaging all these slightly different systems. One can see that the possible ambiguities of the mode of averaging and the questions concerning systematic errors make the *definition* of coordinate system by a set of coordinate estimates with respect to it unpracticable.

In view of the fact that each realization of a system is of necessity of a stochastic nature, we propose any of the terms "standard", "standard system", "realized system" or "estimated system" instead of the misleading term "frame of reference", which connotes solidity and firmness, properties which are conspicuously absent from these standards.

"Quasi-inertial" is another term suggested by Kovalevsky and Mueller which deserves some comment. What they mean by this phrase has been known among relativists as a "local inertial system" and, as in the case of "frame", there is no conspicuous reason why long established terminology ought to be changed. One hears and reads, however, the term "quasi-inertial", now with a different meaning, also applied to entities such as the FK5. This is because the FK5 represents the best effort to date by the community of astrometrists to approximate, empirically, an inertial system in terms of star positions. A much more precise and accurate standard for the realization of an inertial reference frame is available in terms of the ephemerides of selected bodies in the solar system, cf. Williams and Standish (1989). For reasons already mentioned above, the positions in the FK5 (and all its not yet constructed successors and substitutes) show random and systematic differences against a true inertial system. The prefix "quasi-", when used in the literature, always had a clear and well defined meaning (cf. "quasiperiodic function"). We ought to respect this tradition and avoid the term "quasi-inertial" altogether. Instead, we should use the established terms "local inertial system" or "local inertial reference frame" when we mean these entities and characterize the property of, e.g., the FK5, the dynamical standard DE 200 or the contemplated galactic reference standard as "nearly inertial", which seems to convey the nature of these standards more clearly than the ambiguous "quasi-inertial".

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