PART I

EVOLUTION OF STARS IN THE POST-MAIN SEQUENCE STAGE

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(Dedicated to Arnulf Schlüter on the occasion of his 60th birthday)

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I would like to start with a short historical introduction. In 1938 thermonuclear reaction rates for hydrogen burning became available. This made it possible to fit a convective core into the point source model of stellar structure integrated by Cowling three years earlier. The free parameter in the fitting process could be fixed with the thermonuclear reaction rates, the first realistic stellar model for a massive main sequence star was constructed! After the war electronic computers became available, and one was able to do more complicated models like those on the lower main sequence, like realistic models for the sun with its helium enriched interior, and one tried already to follow in time the exhaustion of hydrogen in the central regions of stars numerically. There was not too much progress for stars at the upper end of the main sequence. As soon as the stellar model tried to leave the main sequence and to march towards the region of the red supergiants the methods known at that time failed to produce models. For less massive stars the exhaustion of hydrogen could be followed up more easily and, in 1955, the great paper by Hoyle and Schwarzschild came out, which showed how these stars from the main sequence move into the red giant branch and move up parallel to what we now call the Hayashi line (which was not yet known at that time). But when helium started to burn the methods also failed.

The field stagnated for several years until, in 1961, the Henyey method became available. Henyey's famous talk at the IAU in Berkeley on that topic had its 20th anniversary about two weeks ago. We owe almost everything we have learned about stellar models on computers to this method; furthermore, if you look in recent issues of Ap. J. for stellar evolution calculations you will find that it has not yet been replaced by anything substantially better. With this method it was possible to follow the evolution of stars of intermediate mass and also of massive stars through several types of nuclear burning. It was possible to follow these stars through their different stages of Cepheid pulsation and into stages of very complicated evolution, with features about which I will report later. The method also made it possible to follow low mass stars through the helium flash. If, today, we get stuck with a computer simulation of stellar evolution it is normally not the numerical method,

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but rather our lack of understanding the physics which is responsible for the failure. We do not know enough hydrodynamics to deal with convective overshooting, with semiconvection and with effects of stellar rotation. We do not know all the mechanisms which bring material which has just undergone a nuclear process to the surface, although the observations suggest it, and we do not know the mechanisms which produce strange chemical abundances, even for stars on the main sequence. We do not know the mechanisms which blow material from the stellar surface into space, sometimes causing mass loss which cannot be neglected in stellar evolution calculations.

In my report I will confine myself to the evolution of stars of intermediate mass (1.4-10 $M_\odot$) and of low mass ($M < 1.4\ M_\odot$). A good review appeared in vol. 12 of Annual Review of A & A by Icko Iben seven years ago (Iben, 1974).

I. THE EVOLUTION OF STARS OF INTERMEDIATE MASS

It is well known from observation and from model calculations that stars, when exhausting their hydrogen in the central regions, become red giants or supergiants. Some evolutionary tracks are given in Figure 1 computed by our group almost 20 years ago. Although opacities and nuclear reaction rates have improved in the meantime, nothing has changed qualitatively. The loops in the red giant region are extended more or less, de-

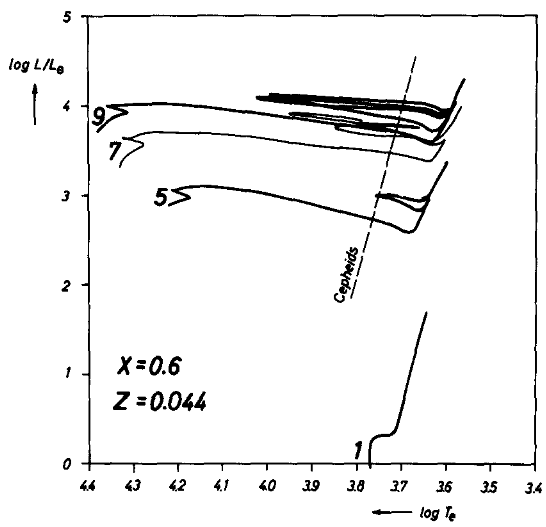

Figure 1. Evolutionary tracks for 1, 5, 7, 9 $M_\odot$ for an extreme population 1 (according to v. Sengbusch, 1967; Hofmeister et al., 1964b,c and Hofmeister, 1967).

pending on different parameters. They are also sensitive to inaccurately computed helium abundances in the central region. Anyone who is careless with his models on the main sequence as regards the central enrichment of helium, may later have to pay for this by receiving completely wrong loops in the red giant region (Lauterborn et al., 1971).

For a long time the boundaries between convective and radiative regions were determined by the Schwarzschild criterion. Also the computations on which the figures 1 and 2 are based use the Schwarzschild criterion and ignore the effect of overshooting. But, following the recent work of Shaviv and Salpeter (1973) one learned that convective overshooting at the surface of the convective core can have dramatic effects. This has been shown first by Maeder (1975) for a star of 2 $M_\odot$. If the mixing length is taken equal to the scale height ($\alpha = 1$), overshooting increases the mass of the central core over which mixing occurs from 13 % to 17 % of the total mass. The results depend on the assumption of different parameters which appear in the application of the mixing length theory to overshooting. The result is more drastic for more massive stars. In his thesis at Hamburg University, Wassermann (1978) showed that for 5 $M_\odot$ the Shaviv-Salpeter type of overshooting increases the central region which after the exhaustion of central hydrogen is polluted by newly formed helium by 18 % to 26 % of the total mass of the star (again for $\alpha = 1$). It is obvious that this effect increases the main sequence lifetime. But the more striking effect, already predicted by Weigert (1975), is that the chemical profile in the central region will have an effect on the occurence or disappearance of loops in the evolutionary stages after the onset of helium burning. This has the consequence that the overshooting has an influence on the mass range of those stars which during their life have the chance to stay long enough in the Cepheid strip. The latest result of the Hamburg school (Matraka et al. 1981) is rather disturbing. For stellar masses below 6 $M_\odot$ as soon as overshooting with $\alpha = 1$ is taken into account the loops avoid the Cepheid strip. In order to make these stars to become Cepheids again, the ratio of mixing length to pressure scale height in the convective envelope has to be reduced below the canonical value of 1.5 down at least to 1. It seems difficult to obtain Cepheids with the *same* value of α for the convective envelope and the convective core. Unfortunately, in the treatment of overshooting in the very interior of stars the uncertainties in the mixing length theory become important, whereas in the old treatment, the mixing length uncertainties were important only in the outer convective zones.

The motion of a star in the HR diagram to the right and then back and forth is steered by events in the very interior, events which can be seen from Fig. 2 for a star of 5 $M_\odot$. There the evolution is described from the zero age main sequence (left) to the formation of a carbon oxygen core in a double shell model where, at the end (right), the hydrogen burning shell is exhausted but, sometime later, when the helium burning shell has eaten further outwards, hydrogen will reignite and both shells will move outwards parallel in the mass scale, as we will see later. Actually the carbon oxygen core is due to ignite

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carbon, but this process is delayed because of cooling by neutrinos.

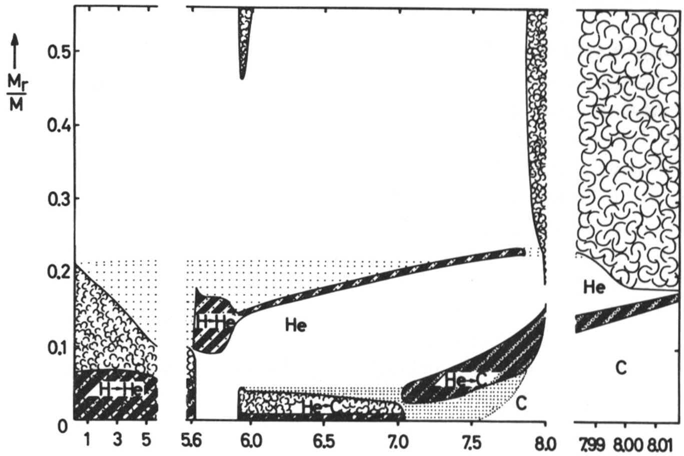

Figure 2. Variation in time of the interior of a star of 5 $M_\odot$ (Kippenhahn et al. 1965). The abscissae give the time after the zero age main sequence stage, the ordinate the mass M_r in units of the total mass. "Cloudy" regions correspond to convective zones. Regions in which the nuclear energy generation rate exceeds 10^3 c.g.s. units are hatched. Dotted regions indicate areas in which the hydrogen decreases inwards or where the helium abundance decreases inwards. Two dredge-up phases are seen: The first when the outerconvective zone reaches down to the $M_r/M \approx 0.47$, the second when it comes down below 0.2 and reaches the helium shell.

There are not too many ways of checking whether the events described in Fig. 2 will take place not only in computers but also in nature. Certainly, the evolutionary tracks, the isochrones which can be derived from them and the comparison with observed HR-diagrams of clusters are a help. Studying the pulsational properties of the models when they cross the instability strip can also indicate whether the computations have given the right picture, but these tests are not very compelling. They were not even able to tell us whether mass loss in the red supergiant region will reduce the mass in the hydrogen rich envelope appreciably during that phase of evolution or not.

Several authors have discussed the effect in which the outer convective zone (which becomes rather thick not only in radius but also in mass when the star comes close to the Hayashi line) mixes material which has been recently processed by nuclear reactions to the surface. This effect is called *dredge-up*. In Fig. 2 one can see two dredge-up phases for the star of 5 $M_\odot$. Although the first dredge-up process seems not to go very far into the interior, since it always stays away from the hydrogen burning shell, it is still very effective. The CNO cycle and its subreactions occur not only in the hatched regions in Fig. 2 but also in their neighbourhood. During the first dredge-up the outer convective zone reaches down to $M_r/M = 0.47$. At this layer transformation of ^{12}C into ^{14}N has already taken place and half of the carbon is already transformed into nitrogen. Therefore, the ratio of the two elements in the outer layers will be changed (Iben, 1964). Simultaneously, ^{7}Li as well as B and Be deplete, since they are destroyed at the hot bottom of the convective envelope (Boesgaard, 1977).

The second dredge-up phase occurs after the exhaustion of helium in the central region. Then, the outer convective zone eats even into the shell which contains pure helium (despite a small amount of heavier elements being already present in the interstellar material out of which the star has formed). Consequently the ^{14}N enrichment becomes even more pronounced and in addition helium will be enriched in the envelope although this is not too drastic an effect.

It was a great surprise when Schwarzschild and Härm (1965), Weigert (1966) and Rose (1966) found that there occur thermal runaways in shells of nuclear burning under nondegenerate conditions. After the phenomenon was first encountered it was easy to understand it. It is explained in the first of the three papers mentioned with an analytical approximation. The two other authors following up the thermal runaway showed that the phenomenon repeats cyclically. In intervals of some thousand years the luminosity of the helium shell temporarily increases by several powers of 10. The phenomenon is called *thermal pulses*, sometimes also *shell flashes*. The phenomenon is shown in Figures 3 and 4 which come from Weigert's classical paper. In Fig. 3 the convective shell which during each pulse develops above the helium burning core is just indicated by spikes, the first and the sixth pulse are plotted with a better time resolution in Fig. 4. From the first to the sixth pulse the phenomenon gets more and more violent. After the discovery of the pulses there was hope that after a few more pulses the convective shell and the convective envelope would merge bringing photons into a region where helium burning takes place and causing nuclear reactions which provide the local stellar material with neutrons and enable a build up of the s-process elements. Now, however, we know that this unification of the two convective regions does not take place. Instead a third dredge-up phase occurs.

With more computing time and with refined numerical methods it was possible to show that after the growing of the subsequent pulses a "steady state" occurs in which the pulse amplitude remains practically constant from pulse to pulse and the differences in subsequent pulses are only due to the fact that between two pulses the two shells have moved outwards together in mass a little bit. The fully developed pulse is described in Fig. 5 which is taken from Paczyński (1977). There

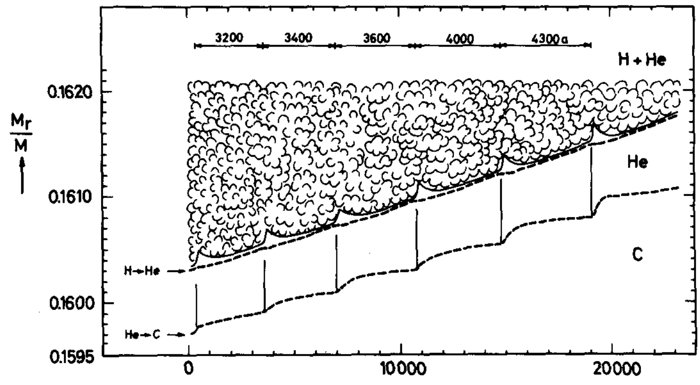

Figure 3. Weigert's 6 thermal pulses for a star of 5 $M_\odot$. This evolutionary phase follows after that depicted on the right-hand side of Figure 2. The outgoing helium burning shell has reignited the hydrogen burning shell which was extinguished for some time and both shells now move outwards parallel in mass scale. The thermal pulses are sharp increases in the energy production of the helium burning shell for a short time during which convective shells develop which, due to the insufficient resolution in time in the graph, are vertical spikes. The abscissa is given in years, starting at an arbitrary zero point. The first and the last thermal pulse with better time resolution are given in Figure 4.

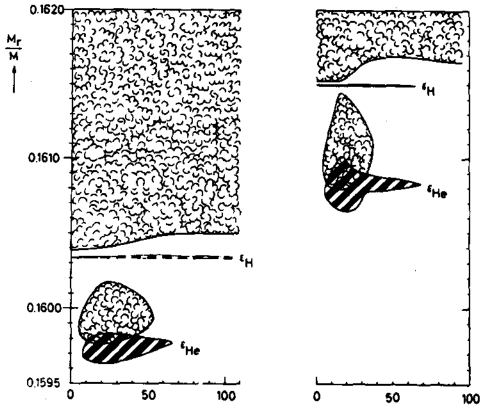

Figure 4. The first and the sixth pulse of Figure 3 with better time resolution. The abscissae give time in years starting at arbitrary zero points. The convective envelope and the convective shells are indicated by cloudy regions. Hatched regions correspond to a nuclear energy generation greater than 10 c.g.s. units.

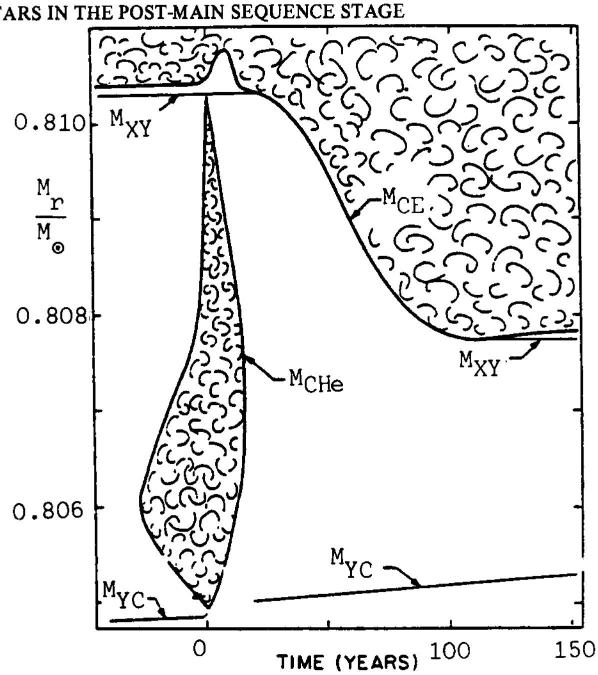

Figure 5. Paczyński's fully developed thermal pulse. Cloudy regions correspond to the outer convective zone and to the convective shell. M_{CE} gives the base of the convective envelope. M_{XY} the transition between hydrogen rich outer part and helium shell. M_{YC} indicates the transition between helium envelope and carbon core while M_{CHe} indicates the boundaries of the helium convective shell. The total mass of the model is 8 $M_\odot$. The dredge-up phase after the pulse is clearly indicated. The bottom of the outer convective zone after the disappearance of the convective shell reaches down into a layer in which carbon formed during the pulse has been brought up by the convective shell.

is one quantitative difference between Weigert's pulses and Paczyński's, and if one looks carefully into Fig. 3 one can see that there is also a tendency in this direction in Weigert's pulses. During the peak of the pulse, a convective shell appears while the convective envelope is retreating in the mass scale. But after the peak of the pulse and after the convective shell has disappeared the outer convective region penetrates inward into a region where the convective shell had released some of the ashes of helium burning. This material is now dredged-up by the outer convective zone. Here only half of the expected mixing takes place: no protons are brought into the helium burning region but matter from the helium burning region is brought to the surface. This dredge-up process had first been seen by Gingold (1975) for low mass stars which also show the phenomenon as we shall see later. The main effect of the dredge-up after each pulse is the enrichment of the surface composition by ^{12}C. Paczyński (1977) gives the following explanation for the post-pulse penetration of the outer convective zone. During the pulse the luminosity of the helium shell has a peak. Consequently, a heat wave propagates outwards, causing a peak in the total luminosity of the star. If now (for given M_{core}) M is sufficiently high, then the depth of the outer convective zone increases with luminosity, and therefore, during the peak of the total luminosity, the convective zone eats inwards. The effect seems to occur only if the core mass is sufficiently high, no dredge-up has been found for core masses below 0.6 $M_\odot$.

But if one wants to fix the bottom of the outer convetive zone one encounters a problem. The opacity at the interface is determined by electron scattering, and consequently the temperature gradient will be discontinuous at the interface where it should have its adiabatic value. Therefore the Schwarzschild criterion can only be fulfilled either coming from the outside to the interface *or* from the inside, but not, as should be the case, from both sides. This difficulty is well-known from the problems of semiconvection. A intermediate region will build up in which the Ledoux criterion (which takes into account the gradient in molecular rate) will be marginally fulfilled. This is not enough. While such a region is being built, helium has to be lifted into outer layers and up to the surface, whereas hydrogen is pushed deeper inwards. This needs energy, and so during the process the luminosity, which is an important parameter in the stability criteria, and which determines the depth of the outer convective zone will be diminished, since part of the energy is used for mixing.Although Paczyński guesses that these effects reduce the dredge-up considerably, nevertheless a certain amount of dredge-up is to be expected.

At the moment, it seems that the thermal pulses give the only reasonable explanation for the formation of s-process elements. Fujimoto et al. (1976); Sugimoto and Nomoto (1975), and Iben (1975a,b) suggested that the process $^{22}Ne(\alpha,n)^{25}Mg$ can take place at the peak temperature of the flash, which is about 3.1×10^8 ^oK. At this temperature the lifetime of ^{22}Ne is of the order of 10 years and this process provides the neutrons necessary to build up the series of s-process elements by neutron capture of ^{56}Fe and the subsequent nuclei. The dredge-up process described above will bring the newly formed elements to the surface.

As mentioned already, the post-pulse dredge-up phases will bring carbon to the surface. This might be an explanation for the occurrence of carbon stars, but it seems that quantitatively there are difficulties, as has recently been pointed out by Iben (1981). Since the post-pulse dredge-up occurs only for a stars with core masses sufficiently large, calculations indicate that there should be no enrichment in carbon for stars below the bolometic magnitude -5^m. But fainter carbon stars are observed in the Magellanic clouds. The carbon enrichment seems not to be present in stars brighter than -6^m, which cannot be explained by theory unless one assumes that the brighter stars with carbon enriched outer layers hide behind an opaque envelope.

Recently the question came up of whether the pulse phenomenon does really occur in stars. Prialnik et al. (1981) suggested that diffusion might become important in the helium burning shell of stars on the asymptotic branch. Diffusion is normally neglected in stellar evolution theory (although it might be of some importance in regions near the surface for stars which have no convection and no meridional circulation there). But in the stage during which the thermal pulses occur the transition zone between the helium shell and the hydrogen rich envelope contains very little mass and simple estimates indicate that diffusion will become important there, preventing the transition zone to become too narrow. It is known already from the paper by Schwarzschild and Härm (1965) that a necessary condition for the occurrence of the pulses is that the shell be geometrically thin. Prialnik et al. (1981) give estimates which indicate that, because of diffusion, the shell might never become thin enough to undergo a thermal runaway. I am sure that people soon will repeat their thermal pulse calculations including diffusion and learn whether or not the pulses disappear. It would be a pity if they vanish, a nice theory for the formation of s-process elements would disappear with them and the occurrence of carbon in the surface layers of carbon stars would need a new explanation.

I will stop here the section dealing with stars of intermediate mass.

II. THE EVOLUTION OF LOW MASS STARS

Low mass stars, which, on the main sequence, have no appreciable central convective cores, develop degenerate helium cores and move slowly to the red giant branch, as was first pointed out by Hoyle and Schwarzschild (1955). More recent calculations are given in Fig. 6. The events in the interior are shown in Fig. 7. One can clearly see the first dredge-up phase which corresponds to the first dredge-up for stars of intermediate mass. Helium is mixed to the surface but there is no change in the ^{12}C and ^{14}N abundances as for more massive stars. In the HR-diagram the star moves up the red giant branch (see Fig. 9) until finally helium is ignited. Because of neutrino cooling the maximum temperature is not in the centre but in a sphere which in Thomas' model was at $M_r/M = 0.3$. However, Thomas at that time used neutrino rates which have since been improved. (Another computation he carried out later with better neutrino rates showed that a star of 1.3 $M_\odot$ ignites helium at $M_r/M = 0.1$, (Thomas 1970, see also Demarque, Mengel, 1971)). Figure 8 describes the events at and after the onset of helium burning.

After the ignition of helium in the degenerate region a thermal runaway starts and a convective shell is formed. Due to degeneracy the energy released in the helium burning shell is used to a large extent to increase the temperature and makes the phenomenon even more violent - until the temperature has increased so much that the degeneracy is reduced and the relevant shell can expand to enable nuclear burning to

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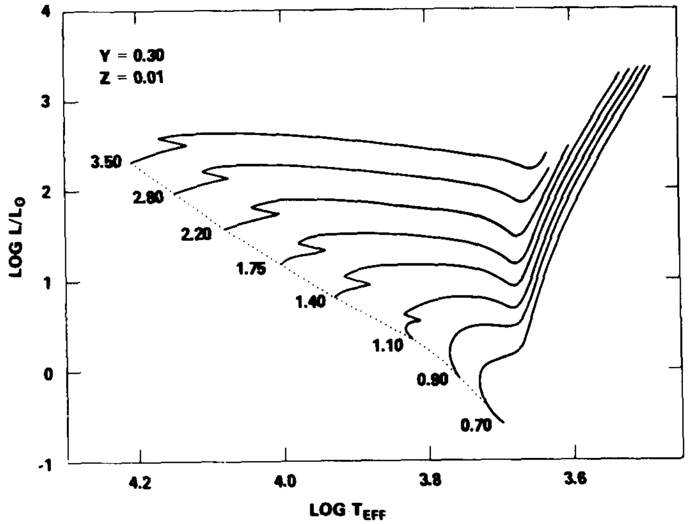

Figure 6. Evolutionary tracks for low mass stars according to Sweigart, 1978.

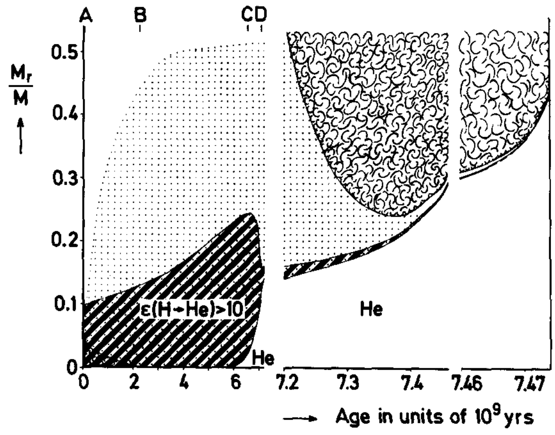

Figure 7. The changes in the interior of a star of 1.3 $M_\odot$ after exhaustion of hydrogen in the centre (after Thomas, 1967). The symbols have the same meaning as in Fig. 2. But hatched regions indicate the areas in which the nuclear energy generation exceeds 10 c.g.s. units. The first dredge-up occurs when the convective envelope reaches down below $M_r/M \approx 0.25$.

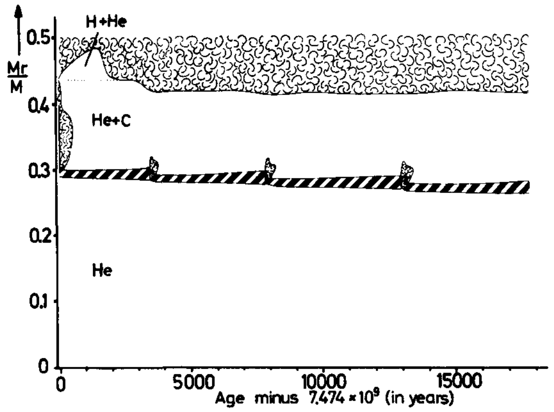

Figure 8. The events in the interior after Thomas' off-centre helium flash (Thomas, 1967). The hatched regions indicate areas in which the nuclear energy generation exceeds 10^3 c.g.s units.

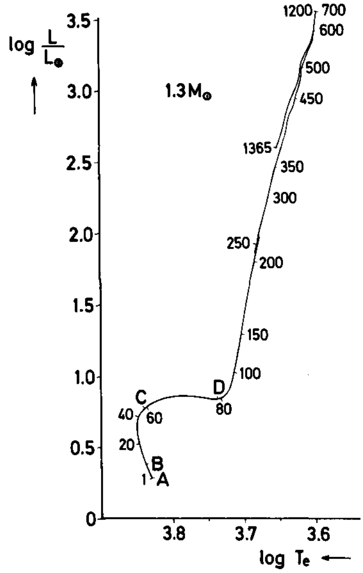

Figure 9. Thomas' evolutionary track (Thomas, 1967). The letters A - D correspond to the phases indicated in Fig. 7. The numbers correspond to model numbers. The short phase during which the luminosity decreases around model number 250 corresponds to the first dredge-up. After the flash the luminosity decreases (number 1200 - 1365). There is no indication that the star tries to settle down on the horizontal branch.

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take place in a secularly stable way. The convective shell does not merge with the outer hydrogen-rich envelope, but a post-flash dredge-up phase occurs in which carbon is mixed to the surface. Thomas expects an enrichment of $X_C = 0.008$ whereas, more recently, Pacyński and Tremaine (1977), by varying the depth of the layer in which the flash occurs, came to the conclusion that X_C could reach 0.012. This could explain the carbon enrichment in stars in globular clusters. The luminosities of these stars reach down to the horizontal branch luminosity and they therefore cannot be explained by dredge-up caused by thermal pulses on the asymptotic branch.

After the flash nuclear burning takes place in a non-degenerate region more or less quietly interrupted by some smaller thermal pulses, as can be seen in Fig. 8. In the star therefore, a carbon-rich shell is formed, surrounding a helium core. There is material of higher mean molecular weight above a region of lower molecular weight and, consequently, a secular instability is expected, causing thermohaline mixing. It has been estimated by Kippenhahn et al. (1979) that this mixing is ineffective and will not change the chemical composition in a time shorter than the nuclear timescale.

In the HR-diagram the star moves down towards the horizontal branch luminosity but it always stays close to the ascending branch and no horizontal branch can be formed with these models. It is known that only if an appreciable amount of mass is taken away from the envelope, can agreement with the observed horizontal branch be obtained. This can be seen in Fig. 10. The models on the dashed line have all the same core mass, but they have undergone a different amount of mass loss. In a cluster with an age of 12.5×10^9 years, but without mass loss, there would be no stars to the left of the point indicated by an arrow. The more mass loss the star has undergone, the more its position is shifted towards the blue. The horizontal branch is direct evidence of mass loss! The branch crosses the instability strip where one has the RR Lyrae stars. In the post-horizontal branch evolution the star moves to the right and to the left and later again to the right. The instability strip is crossed in several directions. Van Albada and Baker (1972) have shown that these different directions of crossing can be used to explain the Oosterhoff groups of RR Lyrae stars. If in post-horizontal branch evolution the strip is crossed again, one expects there the W Vir stars of shorter periods (Kraft, 1972). When the stars move to their asymptotic branch nuclear burning takes place in two shells. This constitutes an ideal situation for thermal pulses! The behavior of the stars in the HR-diagram at this stage of evolution is not very clear. It depends on several different parameters. The stars during the thermal pulses may leave their ascending branch and move to the left for a short time (Schwarzschild, Härm, 1970; Schwarzschild, 1970). If they then on these loops cross the instability strip they start to pulsate and this might be the state in which they are seen as the W Vir stars of longer period (Kraft, 1972).

During the thermal pulses dredge-up occurs and one could ask whether observable changes in chemical composition occur. Several attempts have

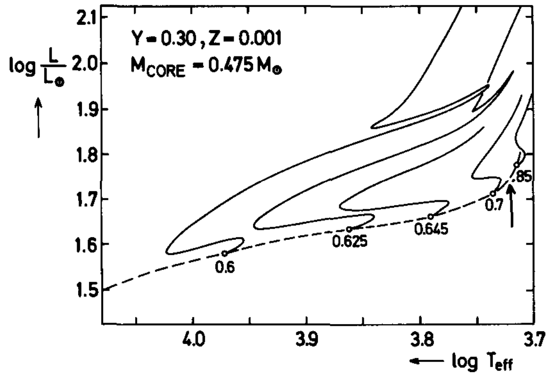

Figure 10. The zero age horizontal branch (broken line) and the post-horizontal branch evolution for different total masses (solid lines). The numbers at the horizontal branch give the stellar mass in solar units. The core mass is the same for all models. The models on the horizontal branch therefore can be considered as having their origin in stars of the same original mass but having undergone mass loss of different strength in their evolved stage. Without mass loss stars of globular clusters in the observed age-range would populate only the very right-hand side of the horizontal branch (after Iben, 1972).

been made but recently Kraft (1981) together with his co-workers have put forward some rather new ides. They investigated CH and NH bands in 139 giants in globular clusters and in field stars. It turned out that the C and N abundances are not in agreement with theory. Stars in two of the clusters have different abundances than those in the other two clusters and in the field stars. The authors come to the conclusion that in globular clusters carbon enrichment might be caused by accretion of material which has been expelled in the early phase by the supernova explosions of massive stars. Carbon enriched material blown off the exploding stars normally escapes, but in denser clusters gravity might hold back some of it which could then pollute the stars in the cluster.

Wood and Carro (1981) made attempts to relate period changes of Mira stars like in A Aql, R Hya, W Dra to changes in the stellar interior during thermal pulses, but the situation seems not to be so clear.

But is our picture of the post helium flash evolution correct? Did we compute the helium flash properly? There is a Ph.D. thesis at Oxford University by Wickett (1977). Unfortunately only the results are published. The author claims that, during the flash, convection is insufficient to carry away the energy and that the star explodes. As far as I know, nobody in stellar evolution theory has tried to clarify this point until now. Recently in another Ph.D. thesis by P.W. Cole (under the supervision of R.G. Deupree (Boston)) the problem was taken up again. (Cole, Deupree, 1980, 1981). It is the first attempt to deal with convection in stars with a twodimensional code. Unfortunately the code they used had only two meshpoints in ϑ-direction. The convection they can compute is something more akin to circulation. With the onset of the flash the energy was carried away by "nonradial" convection and inertia forces became important. This has to be clarified.

I have tried to make some estimates using Thomas' flash calculation (Thomas, 1970). At the shell of maximum temperature he finds

$$L_r = 4\times10^{44},\ P = 4.02\times10^{21},\ \varrho = 2.19\times10^{5},\ r = 6\times10^{8};$$

(all quantities in c.g.s. units). We now use two equations of mixing length theory:

$$F_k = c_p\varrho\, T v(\nabla-\nabla')\ \frac{\ell}{2H_p}\ ,\ v^2 = \frac{g\ell^2}{8H_p}\ (\nabla-\nabla')$$

(see for instance Hofmeister et al., 1964a) where F_k is the convective energy flux, ℓ the mixing length which we put equal to H_p, $H_p = P/g\varrho$ is the pressure scale height, g the gravitational acceleration, $\nabla =$ $= d\ln T/d\ln P$, $\nabla' \approx \nabla_{ad}$ describes the temperature variation in the rising or falling elements and $\delta = (\ln\varrho/\ \ln T)_P$. From the two equations we obtain

$$\nabla - \nabla' = \frac{32}{3}\frac{\psi}{\alpha^2}\frac{v^2}{P}$$

where we have made use of the approximate relation $\delta \approx 3/4\psi$ where ψ is the degeneracy parameter which has the value ≈ 3 in the region of interest. With the numerical values given above and with $\alpha = 1.5$ we obtain $\nabla - \nabla_{ad} \approx \nabla - \nabla' = 0.0116$. This means that the temperature gradient is only slightly superadiabatic. It is not clear from this estimate whether this effect is important or not. A slightly superadiabatic gradient means a higher temperature which would increase the strength of the flash, raising ∇ even higher. This, in principle, could enhance the flash in such a way that the inertia terms indeed become important. Up to now one generally computed the helium flash by assuming that whenever convection occurs, the temperature gradient will be the adiabatic one, since convection can easily carry away the energy generated there. This is not necessarily always true.

In the last decade with the improvement of computational facilities and with the development of better Henyey codes it has become rather easy to compute stellar evolution by varying uncertain parameters and trying to investigate their influence. I am not sure whether this was always very effective. The new picture of overshooting and the consequences as they were investigated in Wasserman's thesis, the disturbing results on the effects of diffusion in thin shells as obtained by the Israeli group and the results of the two Ph.D. thesis' in Oxford and Boston questioning the effectivity of convection in the helium flash might cause *qualitative* changes in our picture. A large amount of stellar evolution models published and used to explain observations might turn out to need revision. Maybe we stellar evolution people should look into these problems ourself and not leave all the difficult problems to our graduate students.

I thank Dr. H.-C. Thomas for helpful discussions.

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