NONLINEAR COUPLING BETWEEN THE 110-YEAR PERIODIC MODULATIONS OF SOLAR DIFFERENTIAL ROTATION AND SOLAR CYCLE

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ABSTRACT. An observational example of nonlinear coupling between flows and magnetic field generated by dynamo action of the flows is found in the 110-year periodic modulations of the solar differential rotation and the solar cycle.

1. Introduction

One of the basic questions on the origin and dynamics of the solar cycle oscillatory magnetic field is the question of the nonlinear mechanism which is responsible for determination of the amplitude of the solar cycle. The amplitude is known to undergo long-term modulations with a variety of time scales. The existence of the Maunder Minimum in the 1600's when so few sunspots appeared on the surface of the Sun just after the telescopic discovery of sunspots shows that the solar activity rose to the present level of solar activity from a neglible level during the last 400 years. This was noticed when Wolf devised the sunspot relative number to describe the level of the solar cycle and studied history of sunspot appearance. He also noticed that the relative number time series underwent long-term modulations and postulated that the modulations could be understood as a beat of two or more periodic oscillators. Since then efforts have been made to understand the modulations by linear superpositions of harmonic oscillators in terms of Fourier analyses of the sunspot relative number time series.

2. A Nonlinear Dynamo

The origin of the solar magnetic field itself has been studied by various kinds of dynamo mechanisms. The basic concept of the dynamo mechanisms is that some kinds of flows of plasma or fluid with high electric conductivity amplify the magnetic field from an infinitesimal level and drive the solar cycle. The difference among the various dynamo mechanisms arises from the difference of flows that drive the dynamo. In a review paper in the present proceedings, we have described a case of the dynamo mechanism where the basic 11 year cyclic growing solutions of magnetic field of the solar cycle can natu-

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rally be obtained by a combined action of the flows of the differential rotation and the global convection (Yoshimura, 1972, 1975, 1983). The basic aspect of the dynamo processes can be understood by a linear concept in terms of topological deformation of magnetic field lines in a rotating system. In the context of the same dynamo theory, however, the level of the solar cycle is determined by a nonlinear mechanism in which Lorentz force of the generated growing magnetic field affects the dynamo driving flows and weakens the strength of the dynamo. The balance between the driving and the weakening forces of the dynamo determines the level of the solar cycle magnetic field oscillations. In the nonlinear dynamo, the modulations of the level of the solar cycle should, in principle, never be understood by a linear superposition of harmonic oscillators. The meaning of the periods of solar cycle and of its modulations must be understood in terms of concepts of a nonlinear theory. We have developed a nonlinear theory in the same context of the dynamo in which the magnetic field weakens the dynamo after some delay time (Yoshimura, 1978a, b, 1979). An essential aspect of the model is that the field needs some time to modify the flows. We have expressed this nonlinear process by a term in the nonlinear dynamo equation that describes the process by a set of delay time parameters. Using this parameterized nonlinear model, we obtained the following results. (i) If there was no time delay in the feedback process, the nonlinear oscillatory solutions showed limit cycle behavior with exactly constant period and amplitude. In this case, the concept of the period could be the same as that of a linear theory. (ii) If there was any time delay, however, the period and amplitude of the nonlinear oscillatory solutions were not constant and showed long-term pseudo-periodic modulations. When the time-delayed feedback process was expressed by one delay time parameter, one kind of pseudo-periodic modulations appeared. When the time delay feedback process was expressed by two delay time parameters, two kinds of pseudo-periodic modulations appeared. When the process was expressed by more than three kinds of delay time parameters, chaotic solutions with long-term rises and falls were obtained. We have interpreted that the third kind of modulations was related with the Maunder Minimum.

3. The 55-year Modulations of the Solar Cycle

In order to evaluate the utility of this nonlinear model to understand the real Sun, we analysed the original sunspot relative number time series. We found that the solar cycle had the first kind of modulations with the period-amplitude relation that was predicted by the theory (Yoshimura, 1979). The first kind of modulations had been found to have the following properties. First, the longer the delay time was, the longer the time scale of the modulations became. Second, the basic period of about 11 year of the solar cycle was shorter in the ascending phase of the modulations and longer in their descending phase. In other words, the solar cycle should have a kind of period-amplitude relation. This was confirmed by the data. The time scale of the discovered modulations was 55 years. One 55-year grand cycle consisted of five 11-year solar cycles. We called the modulations the 55-year grand cycle. The delay time that was necessary for the model to reproduce the 55-year grand cycle was about 20 years. The existence of the 55-year grand cycle required that either the differential rotation or the convection, or both, should be modified by the magnetic field after about 20 years. In this nonlinear model, however, we could not determine whether only the differential rotation or only the convection, or both, should be affected by the Lorentz force with the delay time of about 20 years.

4. The 110-year Modulations of the Solar Differential Rotation and the Solar Cycle

Recently, however, we found an evidence that the differential rotation was modulated with time scale on the order of 110 years with delay time of about 20 years following a 110-year modulation of the solar cycle that was found in the process of analysis of the differential rotation modulation. The 110-year modulation of the differential rotation was found by devising a new index of the differential rotation, which we call the angular momentum surface layer density M defined by integration of angular momentum density of rotation over the whole sphere;

$$M = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2\pi} \rho\ r\ cos\theta\ r\ cos\theta\ \Omega(\theta, r)\ r^2\ cos\theta\ d\phi d\theta,$$

where ρ and ϕ denote mass density and longitudinal coordinate and r = R_o with the radius of the Sun, R_o. When the angular velocity $\Omega\ (\theta)$ is expressed by $\Omega\ (\theta) = (\ A + B\ \sin^2\ \theta\ )$ with constants A and B, M can be expressed by $M = C\ (\ 4/3\ A + 4/15\ B\ )$, where $C = 2\ \pi\ \rho\ R_o^4$. Figure 1 shows the time series of M / C derived from the Mitaka data of sunspot group positions from 1943 to 1992 of the National Astronomical Observatory of Japan which was the Tokyo Astronomical Observatory of the University of Tokyo until 1988. The positions were digitized by Kambry (1991). The latitude dependence profile of rotation was obtained by a method which we call the running segment method. The method used a time interval of a fixed length of 11 years of the sunspot group position data to determine the angular velocity latitude dependence profile. The beginning year of the interval is displaced by an amount of one year (Yoshimura and Kambry, 1992a, b). Figure 1 also shows the yearly mean of the sunspot relative

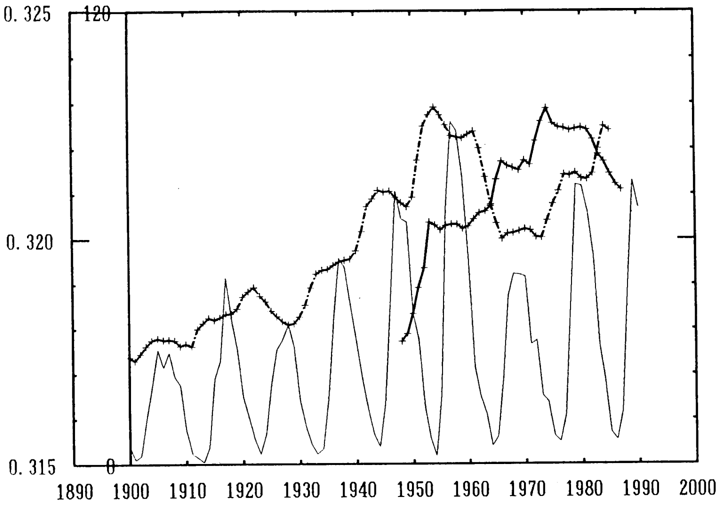

Fig. 1. Time series of the angular momentum surface layer density M / C denoted by the thick solid line, the single 11-year running mean of the yearly mean of the sunspot relative number denoted by the thick dotted line, and the yearly mean denoted by the thin solid line. The abscissa is year. The first ordinate is for M / C. The second ordinate is for the single 11-year running mean.

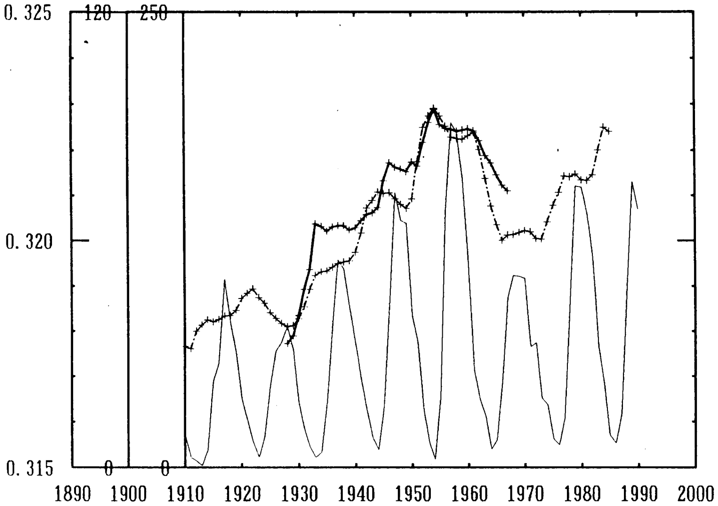

Fig. 2. The same diagram as Fig. 1 but with the M / C time profile displaced toward the past by an amount of 20 years. The two time profiles of M / C and of the single 11-year running mean of the yearly mean of sunspot relative number agree well with each other to a remarkable degree.

number, R, and its 11 year running mean, R_{11}. Figure 2 shows that, when the time series of M is displaced toward the past by an amount of 20 years, the time profiles of M and R_{11} agree well with each other to a remarkable degree. This means that the modulation of the solar differential rotation is real and suggests that the modulation is driven by Lorentz force of the magnetic field of the solar cycle and that the effect appears after delay time of 20 years. This was exactly what the nonlinear dynamo model required and predicted. In order to explore the meaning of these results, we extended the time interval of data by using the results of an analysis by Balthasar, Vázquez, and Wöhl (1986) of the sunspot group positions of the Greenwich Observatory published in form of the Greenwich Photoheliographic Results from 1874 to 1976. They derived values of A and B for each solar cycle from cycle 12 to cycle 20. Combining their results with those of the A and B values by Yoshimura and Kambry (1992a, c) for cycles 18-22, we derived a time series of M value for the entire period of the time interval of cycles 12-22 (Yoshimura and Kambry, 1992c, d). Figure 3 shows the time series of M / C by the thickest solid line. The thinnest solid line shows the yearly mean of the sunspot relative number. The secondly thinnest solid line shows the 11 year running mean of the yearly mean of the sunspot number which is shown in Figures 1 and 2. The dotted line shows the 11 year running mean of this 11 year running mean. We call the first running mean the single 11 year running mean and the second the double 11 year running mean of the sunspot number. The concept of the 110 year modulation of the solar cycle came from the repeated running average of the double 11 year modulation. The procedure and the result is shown in Figure 4. The procedure that reveals the 110 year modulation of the solar cycle is shown by four thin solid lines. The thin solid line with the highest peak designates the double 11 year running mean of the yearly mean of the sunspot number. The thin solid line with the second highest peak designates 20 year running mean of the double 11 year running mean. Successive lines are for 30 and 40 year running means. The thickest solid line is for 50 year running mean. This running averaging was done to extract components of long-term modulations without using Fourier analysis. The pseudo-period is not necessarily constant so that an ordinary Fourier analysis would produce multi-periods for such kinds of long-term modulations. By this successive running mean procedure, the three peaks of the long-term modulation appeared. The first peak of the 50 year running mean time profile consists of two 55-year grand cycles of I and II starting from around 1700. The second peak consists of two grand cycles of III and IV starting around 1810. We are not sure whether the third peak of the grand cycle V is a real peak or is still in a rising phase. Since two 55-year grand cycles constitute the modulation, we have called it the 110-year modulation.

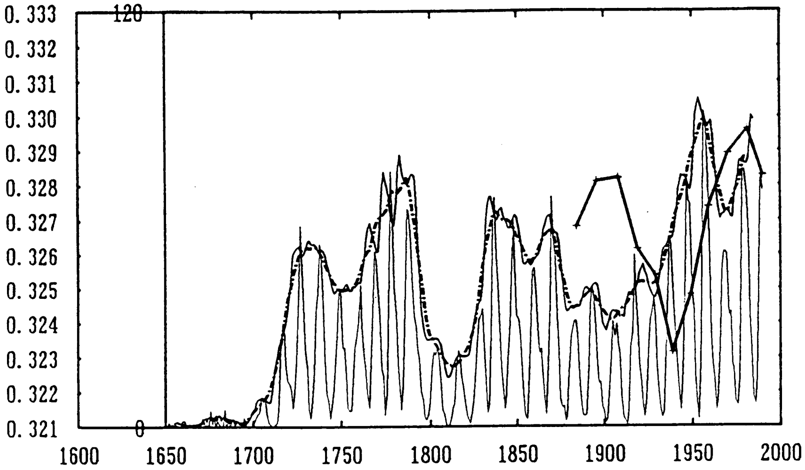

Fig. 3. Time series of the angular momentum surface layer density M / C denoted by the thick solid line, the single 11-year running mean of the yearly mean of the sunspot relative number denoted by the thin solid line, the double 11-year running mean denoted by the thick dotted line, and the yearly mean denoted by the thin solid line. The format of the diagram is the same as in Fig. 1.

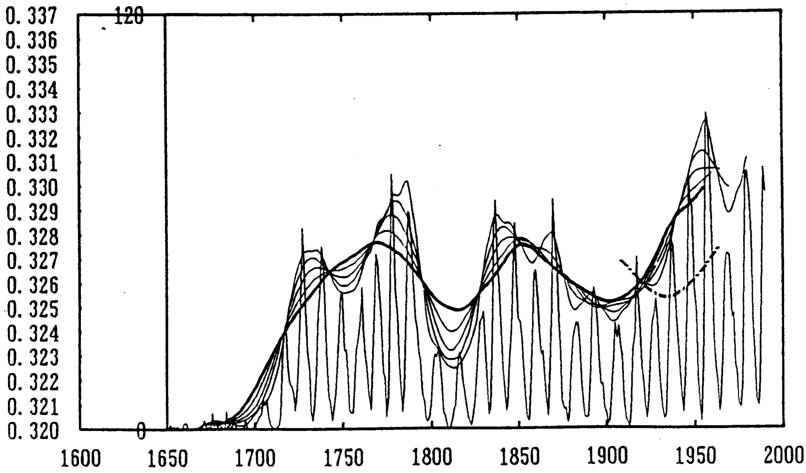

Fig. 4. Time series of 20, 30, 40, and 50 year running means of the double 11-year running mean of the yearly mean of the sunspot relative number. The yearly mean and 20, 30, and 40 year running means are denoted by thin solid lines. The 50 year running mean is denoted by the thick solid line. The 50 year running mean of M / C shown in Fig. 3 is denoted by the thick dotted line.

5. The Nonlinear Coupling

A similar 50 year running averaging was done for the time series data of M / C shown in Figure 3. The result is shown in Figure 4 by the dotted line. Although we have only 57 years of data after the 50 year running process, the time profile of M / C clearly shows the time-delayed similarity with that of the 110-year modulation time profile of the solar cycle in the minimum phase around 1900. The corresponding minimum phase of the 50 year running mean of M / C is around 1930. The longer delay time of about 30 years in this case is consistent with the concept of driving of the rotational modulation by the Lorentz force of the magnetic field of the solar cycle. When the solar cycle amplitude is low, its associated Lorentz force is weak and hence it takes longer time for the force to modulate the flows of the differential rotation that drives the solar dynamo and the solar cycle. By observing sunspots and sunspot groups for so many years, we have glimpsed a case of nonlinear coupling between flows and magnetic field in the real Sun. We need to keep observing sunspot and sunspot groups to understand the long-term dynamics of the Sun.

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