Digital Filtering of Stellar Spectrograms

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1. *INTRODUCTION*

This paper deals with the possibility of recovering the information contained in a spectrogram, in the case of line widths larger than the mean period of the granulation noise of the photographic plate. Suppose the spectrogram to be known by means of its microphotometric one-dimensional record, *e.g.* x(t), where x is the transmittance of the plate at abscissa (wavelength) t. The measurement of the position and shape of a line of the spectrogram, easy when the amplitude of the line exceeds the granulation noise, becomes very hard to do if the S/N ratio approaches unity. Nevertheless, as the power spectrum of the noise has normally a bandwidth larger than the spectrum of the signal, it is possible to reject all the noise power beyond the highest frequency in the signal spectrum, by means of suitable filters. This technique does not lose any useful information, since by definition the information is the signal, while on the contrary it betters the S/N ratio by diminishing the noise power. Of course, as both the signal and noise spectra are continuous, it is not practically possible to perform the filtering without signal losses. However, the technique may be advantageous in all cases, such as highly rotating stars, in which the spectrum of the signal has a cutoff much sharper than the noise cutoff. A simple criterion of applicability is the base width of the lines: if the base width is larger than the granulation noise period, the filtering may result in an improvement of the S/N ratio.

2. *THE FILTER*

The filter used is the following operator:

$$y(t) = F^{-1}[A(f)] . w(t) * x(t) \tag{1}$$

where

y is the filtered transmittance
x is the unfiltered transmittance
t is the wavelength
f is the Fourier conjugate variable of t
F is the Fourier transform operator
w is a convenient ripple smoothing function
A is the wanted filter shape $(A(f) = F[y(t)]/F[x(t)])$

The operator of formula (1) may be put easily in a discrete form, suitable for digital computation Of course, the transmittances x (and y) must be given in a discrete form also: this job may be done by means of a digitizing microphotometer. The discrete case formulae are:

$$\left.\begin{array}{l}
y_k = \sum\limits_s a_s . x_{k-s} \qquad k = N, N-M; \; s = -M, M; \; a_s = a_{-s} \\
a_s = \dfrac{1}{M} \sum\limits_n A_n . \cos\left(\dfrac{s.n.\pi}{M}\right) . w_s . N(s, n) \qquad s = 0, M; \; n = 0, M \\
w_s = 0.42 + 0.5 . \cos\left(\dfrac{s.\pi}{M}\right) + 0.08 . \cos\left(\dfrac{2.s.\pi}{M}\right) \\
N(s, n) = 1 \qquad s \neq M \qquad n \neq 0; M \\
\qquad\quad 1/2 \qquad s \neq M \qquad n = 0; M; \; s = M \qquad n \neq 0; M \\
\qquad\quad 1/4 \qquad s = M \qquad n = 0; M
\end{array}\right\} \tag{2}$$

$$\begin{array}{lll}
t = k\Delta t & (\Delta t \text{ digit}) & k = 0, N \\
f = n\Delta f & (\Delta f \text{ digit}) & n = 0, M \\
A_n = A(f = n . \Delta f) & & \\
f_n = n . \Delta f = \dfrac{n}{M} \cdot \dfrac{1}{2\Delta t} & & n = 0, M
\end{array} \tag{3}$$

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3. *THE RESULT*

From a rigorous point of view, one should follow the procedure given here:

(*a*) Compute the power spectrum of the unfiltered spectrogram, x(t).

(*b*) Compute the power spectrum of the granulation noise, at a mean exposure value, of the plate used.

(*c*) Compute the expected signal spectrum, by subtracting the two spectra.

(*d*) Put A(f) equal to the optimum filter shape matching the noise and the signal spectrum.

(*e*) Filter the x(t) by means of the operator (1), in which A(f) is now specified by the preceding steps.

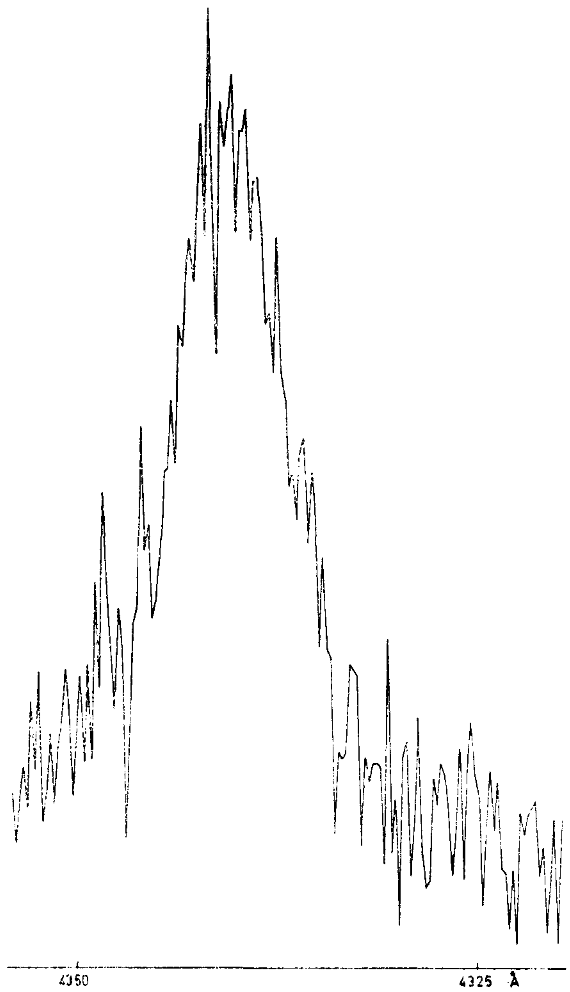

Fig. 1

Unfiltered record of Hγ line, recovered up to the folding frequency and amplified by a factor of 10.

In practice, it is more convenient to compromise between efficiency and simplicity, by means of the criterion at the end of the introduction. If it is valid, the spectrum of the signal approximates a step function with respect to the spectrum of the noise. Consequently, it is possible to use directly the set (2), in which one puts

$$\begin{cases} A_{0,1\ldots C} = 1 \\ A_{c+1,\ldots M} = 0 \end{cases}$$

where f_C is the wanted cutoff frequency of the low-pass filter used. Normally, f_C must be chosen in the range 5–25 per cent of the folding frequency. The value of M, which determines the cutoff of the filter, is not very critical. A value of M between 10 and 100 would match most of the practical cases.

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By means of the simplified procedure we treated the spectrogram of a highly rotating star. The parameters are:

$\Delta t = 1$ mm (on a 0.25 Å/mm microphotometric record)
$M = 50$
$C = 6$ ($f_C = 12$ per cent f_{MAX})

The results are given in Figures 1 and 2. They show the unfiltered record of the Hγ line (ten times smaller in the original record) and the filtered record, in which are clearly distinguishable "new" features, practically undetectable in the unfiltered record owing to their $S/N \approx 0.25$. The identifications of the features were then confirmed by means of a two spectrogram cross correlation (see Fig. 2), by which we can get a fairly positive conclusion.

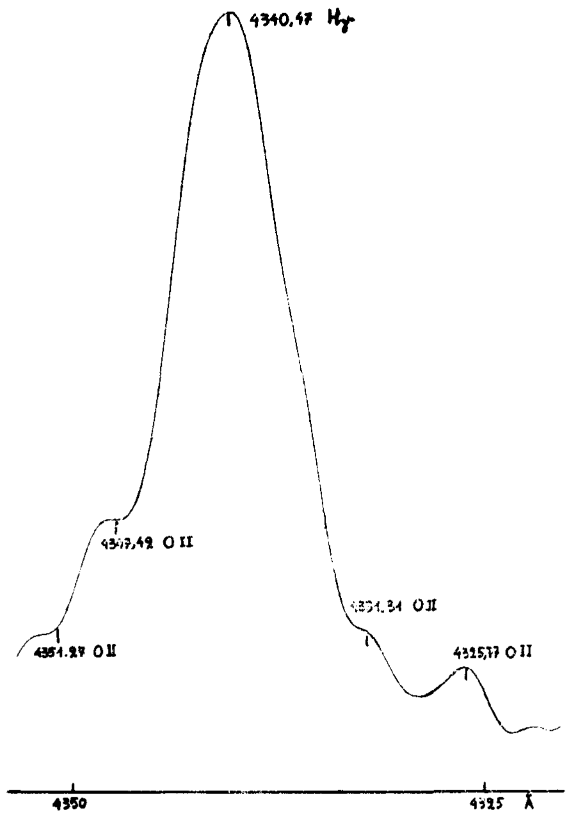

Fig. 2
Filtered record of the same line, recovered up to the frequency $f_c = 12$ per cent of the folding frequency ($M = 50$).

The work was done by programming the set (2) in Fortran IV, and by using an IBM 7044 machine with secondary plotting unit facilities. The overall computation, with 5.10^3 input x_k samples, requires about 1 min, and the writing and plotting times do not exceed, normally, 10 min.

DISCUSSION

G. B. PARRENT: In the low-pass filtering operation, is the filter you sketched essentially the filter you used, as opposed to a matched filter? I'm a little worried about spurious resolution.

G. SEDMAK: It is not a matched filter. I supposed the S bandwidth to be much smaller than the (S + N) bandwidth, so that a simple low-pass filter would improve the S/N ratio.

G. B. PARRENT: It can be a matched filter. Do you trace the grain noise once to calculate its power spectrum? Does your filter not introduce extra lines in the spectra? One would expect that it should, with such a sharp cutoff.

G. SEDMAK: By means of the function w(t) I eliminate the ripple of the filter and introduce the possibility of creating a practical new answer. It is more likely to lose lines than to create them.

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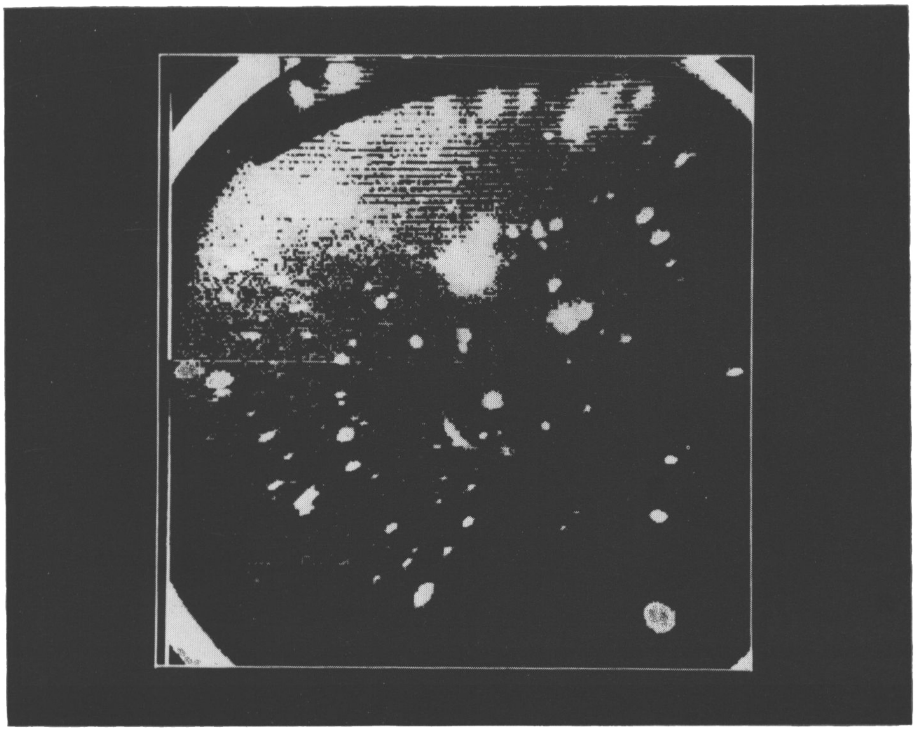

DEUTSCHMAN Fig. 1
A sample camera 3 picture.

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G. B. PARRENT: Is the noise power spectrum of the grain contained in w(t)?

G. SEDMAK: No. w(t) is the function that eliminates the ripple intrinsic to the analytical operator, not to the noise of the operation or granularity noise.

R. B. DUNN: My digital people always tell me to make a very sharp cut filter at approximately one-half the sampling frequency. In fact they used to call me the perfect filter; I'd cut it off exactly when it got to Dunn! Should we not do that now?

G. B. PARRENT: Think of a filter in frequency space that cuts off sharply. The effect on the data is to convolve the data with something that will ring and have extra lobes, and this will tend to introduce structure in the smoothed and corrected trace, which is in fact incorrect detail. Even a gentle slope like that will produce lobes at least on the order of $(\sin t/t)^2$, so you get lobes that are 17 dB down.

R. B. DUNN: That's right, and isn't some of the reason for doing this digitally to get yourself a perfect filter?

G. B. PARRENT: Yes, but you don't do it if your digitally designed one introduces side-lobes.

G. SEDMAK: This is not an ideal filter, but a practical filter. The ideal filter is very difficult to build, as the actual noise spectrum is practically unknown. Not to mention the mathematics!

G. B. PARRENT: If you do this correction of the filter with a sort of *ad hoc* multiplier, then indeed you tend to lose lines.

G. SEDMAK: I said that the method is applicable when the mean period of the granularity noise is of the order of the folding frequency of the (S + N) spectrum, and this is higher than the highest in the S spectrum.

K. NANDY: With what microphotometer slit-width was the spectrum traced?

G. SEDMAK: These samples were made with $\Delta t = 1$ mm (on a 0.25 Å/mm microphotometric tracing) and with a filter for which $M = 50$.

K. NANDY: If you widen the slit you will reduce the noise.

G. SEDMAK: When you have $M = 50$ the cutoff is sufficient. A larger Δt will only imply a smaller total bandwidth.

K. NANDY: Apart from lines, can you detect any other features such as discontinuities in the continuum, *e.g.* the Balmer discontinuity?

G. SEDMAK: I have not done such a test on spectrograms. I believe that the filtering can improve the S/N of the measure if the bandwidth of the discontinuity is smaller than the total (S + N) bandwidth, in frequency space.

J. TINBERGEN: I should like to recommend R. N. Bracewell's book *The Fourier Transform and its Applications* (McGraw-Hill, 1965).

NOTES ADDED BY G. SEDMAK AFTER THE COLLOQUIUM

(i) This work is a trivial application of filter theory, the simplest and first that one can use to improve the S/N.

(ii) The correct procedure would be the production of the optimal matched filter, which is not exactly determined on account of the rather unpredictable dependence of the noise upon the signal.

(iii) Consequently, one must do what is useful for one's particular problem. Low-pass filtering, as I did, when one is dealing with the shape, and the S and (S + N) bandwidths make this possible. Low-pass pre-filtering and high-pass filtering, or carefully chosen band-pass filtering, when one deals with the measure of point to point amplitudes, and the S and (S + N) bandwidths make this possible.

(iv) For ordinary practical work on stellar spectrograms, simple low-pass filtering is fairly good, as the ringing it may introduce does not affect the measure, neither of position, since it is symmetrical in phase, nor of equivalent widths, since it has zero mean value.