

# Abstracts of Australasian PhD theses

## Corner transfer matrices of the Ising model in statistical mechanics

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The work of this thesis is mainly on the investigation of the "corner transfer matrices" of the Ising model in statistical mechanics. The thesis starts with a review of the Ising model and is followed by a brief discussion of various methods for investigating the Ising model. In particular, new techniques and approaches developed by Baxter form the basis for further research described in this thesis. Thus in Section I the review is introduced first.

In Section II, the new technique for investigating the zero field, eight-vertex model on the square lattice using the corner transfer matrices suggested by Baxter [1] are applied to the anisotropic, ferromagnetic, triangular Ising lattice in zero field below its critical temperature. The diagonal form of the corner transfer matrices of the triangular lattice shows essentially the same structure as that for the square Ising lattice. The spontaneous magnetization can be obtained easily from this method. It is hoped that this technique will give illuminating insights into the problems of critical phenomena.

In Section III, the variational approximation approach for the square lattice Ising model developed by Baxter [2] is studied. The accuracy and rate of convergence of this approach is tested by applying the method to the zero field Ising model on the square lattice. The problem is simplified to that of solving a relatively small system of non-linear

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equations. The estimates of the spontaneous magnetization and the critical temperature from the sequence of variational approximations are obtained. The results converge rapidly to the exact ones. They exhibit a cross-over phenomenon and satisfy a scaling relationship for the spontaneous magnetization. Since this method can be applied to many systems such as the square lattice Ising model with a magnetic field, where the exact solution is not available yet, one may expect to obtain good approximations to the thermodynamic functions of these models by using this method.

The work described in this thesis has been reported in [3] and [4].

### References

- [1] R.J. Baxter, "Corner transfer matrices of the eight vertex model: I. Low temperature expansions and conjectured properties", *J. Statist. Phys.* 15 (1976), 485-503.
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