Contents

	Preface		<i>page</i> ix
	Ack	nowledgments	xi
Part I Normal Mode Instabilities			
1	Preliminaries		3
	1.1	What Is Instability?	5
	1.2	Goals	6
	1.3	Tools	7
	1.4	Numerical Solution of a Boundary Value Problem	8
	1.5	The Equations of Motion	13
	1.6	Further Reading	17
	1.7	Appendix: A Closer Look at Perturbation Theory	17
2	Convective Instability		24
	2.1	The Perturbation Equations	24
	2.2	Simple Case: Inviscid, Nondiffusive, Unbounded Fluid	29
	2.3	Viscous and Diffusive Effects	38
	2.4	Boundary Effects: the Rayleigh-Benard Problem	41
	2.5	Nonlinear Effects	47
	2.6	Summary	48
	2.7	Appendix: Waves and Convection in a Compressible Fluid	48
3	Instabilities of a Parallel Shear Flow		53
	3.1	The Perturbation Equations	53
	3.2	Rayleigh's Equation	58
	3.3	Analytical Example: the Piecewise-Linear Shear Layer	61
	3.4	Solution Types for Rayleigh's Equation	68
	3.5	Numerical Solution of Rayleigh's Equation	68
	3.6	Shear Scaling	73

vi		Contents	
	3.7	Oblique Modes and Squire Transformations	75
	3.8	Rules of Thumb for a General Shear Instability	77
	3.9	Numerical Examples	77
	3.10	Perturbation Energetics	80
	3.11	Necessary Conditions for Instability	84
	3.12	The Wave Resonance Mechanism of Shear Instability	91
	3.13	Quantitative Analysis of Wave Resonance	97
	3.14	Summary	102
	3.15	Appendix: Classical Proof of the Rayleigh	
		and Fjørtoft Theorems	103
	3.16	Further Reading	106
4	Parall	Parallel Shear Flow: the Effects of Stratification	
	4.1	The Richardson Number	110
	4.2	Equilibria and Perturbations	111
	4.3	Oblique Modes	113
	4.4	The Taylor-Goldstein Equation	115
	4.5	Application to Internal Wave Phenomena	116
	4.6	Analytical Examples of Instability in Stratified	
		Shear Flows	117
	4.7	The Miles-Howard Theorem	127
	4.8	Howard's Semicircle Theorem	129
	4.9	Energetics	131
	4.10	Summary	133
	4.11	Further Reading	133
	4.12	Appendix: Veering Flows	134
	4.13	Appendix: Spatial Growth	134
5	Parallel Shear Flow: the Effects of Viscosity		137
	5.1	Conditions for Equilibrium	137
	5.2	Conditions for Quasi-Equilibrium: the Frozen Flow	
		Approximation	139
	5.3	The Orr-Sommerfeld Equation	140
	5.4	Boundary Conditions for Viscous Fluid	141
	5.5	Numerical Solution of the Orr-Sommerfeld Equation	143
	5.6	Oblique Modes	144
	5.7	Shear Scaling and the Reynolds Number	145
	5.8	Numerical Examples	146
	5.9	Perturbation Energetics in Viscous Flow	149
	5.10	Summary	152

		Contents	V11
6	Synthesis: Viscous, Diffusive, Inhomogeneous, Parallel Shear Flow		
	6.1	Expanding the Basic Equations	153
	6.2	Numerical Solution	156
	6.3	2D and Oblique Modes: Squire Transformations	158
	6.4	Shear and Diffusion Scalings	160
	6.5	Application: Instabilities of a Stably Stratified	
		Shear Layer	164
	6.6	Application: Analysis of Observational Data	169
	6.7	Summary	173
	6.8	Further Reading	173
7	Nonparallel Flow: Instabilities of a Cylindrical Vortex		174
	7.1	Cyclostrophic Equilibrium	175
	7.2	The Perturbation Equations	176
	7.3	Barotropic Modes ($m = 0$)	177
	7.4	Axisymmetric Modes ($\ell = 0$)	180
	7.5	Analytical Example: the Rankine Vortex	183
	7.6	Numerical Example: a Continuous Vortex	185
	7.7	Wave Interactions in Barotropic Vortices	187
	7.8	Mechanisms of Centrifugal and Convective Instabilities	190
	7.9	Swirling Flows	192
	7.10	Summary	194
	7.11	Further Reading	194
8	Instability in a Rotating Environment		195
	8.1	Frontal Zones	195
	8.2	Geostrophic Equilibrium and the Thermal Wind Balance	196
	8.3	The Perturbation Equations	197
	8.4	Energetics	198
	8.5	The Vertical Vorticity Equation	199
	8.6	Analytical Solution #1: Inertial and Symmetric Instabilities	200
	8.7	Analytical Solution #2: Baroclinic Instability	206
	8.8	Numerical Solution Method	215
	8.9	Instability in the Ageostrophic Regime	219
	8.10	Summary	220
	8.11	Further Reading	221
9	Convective Instability in Complex Fluids		222
	9.1	Conditional Instability in a Moist Atmosphere or	
		a Freezing Ocean	222
	9.2	Double Diffusive Instabilities	224

viii		Contents		
	9.3	Bioconvection	235	
	9.4	CO ₂ Sequestration	239	
10	Summ	ary	242	
	10.1	Equilibrium States	242	
	10.2	Instabilities	243	
Part II The View Ahead			245	
11	Beyon	d Normal Modes	247	
	11.1	Instability as an Initial Value Problem	247	
	11.2	Transient Growth in Simple Linear Systems	249	
	11.3	Computing the Optimal Initial Condition	251	
	11.4	Optimizing Growth at $t = 0^+$	253	
	11.5	Growth at Short and Long Times: a Simple Example	254	
	11.6	Example: The Piecewise Shear Layer	255	
	11.7	Mechanics of Transient Growth in a Shear Layer	256	
	11.8	Generalizing the Inner Product	257	
	11.9	Summary	258	
	11.10	Appendix: Singular Value Decomposition	259	
	11.11	Further Reading	261	
12	Instab	ility and Turbulence	262	
	12.1	Secondary Instabilities and the Transition to Turbulence	262	
	12.2	Turbulence-Driven Instabilities	264	
	12.3	Cyclic Instability	269	
	12.4	Further Reading	272	
13	Refini	ng the Numerical Methods	273	
	13.1	Higher-Order Finite Differences	273	
	13.2	Finite Differences on an Adaptive Grid	275	
	13.3	Galerkin Methods	277	
	13.4	The Shooting Method	281	
	13.5	Generalizations	285	
	13.6	Further Reading	286	
App	oendix A	A Homework Exercises	287	
Appendix B Projects				
References Index				