To the Editor, *The Mathematical Gazette*

TESSELLATIONS WITH PENTAGONS

Dear Sir.—I have just received the December 1971 issue of the Gazette, which I have found as interesting as usual. On pages 366 to 369 there is an article by Mr. J. A. Dunn on "Tessellations with Pentagons", which finishes with the query: "Are there other possibilities?" This can be answered by referring the author to an article by Mr. R. B. Kershner of the John Hopkins University, published in the October 1968 issue (vol. 75 No. 8) of the *American Mathematical Monthly*, pp. 839–844, entitled "On paving the plane", where all tessellations with congruent polygons having more than four sides are covered by the following two theorems, which are demonstrated in the article:

Theorem 1. A convex hexagon can pave the plane if and only if it is of one of the following three types:

1: $A+B+C = 2\pi$, $a = d$;
2: $A+B+D = 2\pi$, $a = d$, $c = e$;
3: $A = C = E = \frac{2}{3}\pi$, $a = b$, $c = e$, $e = f$.

Theorem 2. A convex pentagon can pave the plane if and only if it is of the following eight types:

1: $A+B+C = 2\pi$;
2: $A+B+D = 2\pi, a = d$;
3: $A = C = D = \frac{2}{3}\pi, a = b, d = c+e$;
4: $A = C = \frac{1}{2}\pi$, $a = b$, $c = d$;
5: $A = \frac{1}{3}\pi$, $C = \frac{2}{3}\pi$, $a = b$, $c = d$;
6: $A+B+D = 2\pi$, $A = 2C$, $a = b = e$, $c = d$;
7: $2B+C = 2D+A = 2\pi$, $a = b = c = d$;
8: $2A+B = 2D+C = 2\pi$, $a = b = c = d$.

It will be seen that not only are Mr. Kershner's results more precise and definite than Mr. Dunn's, but that he also gives two types of pentagon (his types 7 and 8) which are not included in Mr. Dunn's possibilities.

Mr. Dunn's first example is, of course, Mr. Kershner's type 1, and the former's second example is a special case of the latter's type 2. Mr. Kershner's types 3 to 6 agree with Mr. Dunn's general idea of three angles totalling 360° and the other two totalling 180°, but the pentagons are arranged in other ways, in one case (type 5) even *six* pentagons concurring at a vertex. In types 7 and 8 we have vertices where four pentagons meet without any two angles adding up to 180°.

The possibility of non-convex pentagons, mentioned by Mr. Dunn, would remain open.

Yours faithfully,
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