GENERALIZED GYLDEN–TYPE SYSTEMS IN UNIVERSAL DS–LIKE TR–VARIABLES

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Scheifele (1970) applied *Delaunay-Similar* (DS) elliptic Keplerian elements (with the *true anomaly* as the independent variable) to the J_2 Problem in Artificial Satellite Theory, making an element of the true anomaly. Deprit (1981) views Scheifele's TR-mapping as an extension of Hill's transformation from a 6-dimensional phase space to an enlarged, 8-dimensional one. To adapt this approach to elliptic-type two-body problems with a time-varying Keplerian parameter $\mu(t)$, Floría (1997, §3, §4) treated a *Gylden system* (Deprit 1983) and derived "Delaunay-Similar" variables via a TR-like transformation. Now we extend our treatment to *perturbed Gylden systems*, and modify the TR-map to deal with *any kind of two-body orbit*. We work out our generalization and the resulting variables within a *unified pattern* whatever the type of motion, in terms of *universal functions* (Stiefel & Scheifele 1971, §11; Battin 1987, §4.5, §4.6) and auxiliary angle-like parameters.

In *extended polar nodal variables* $(r, \theta, \nu, t; p_r, p_\theta, p_\nu, p_0)$, we formulate

$$\mathcal{H} \equiv \mathcal{H}(r, -, -, t; p_r, p_\theta, p_\nu, p_0; \varepsilon) = \mathcal{H}_0(r, t; p_r, p_\theta) + \sum_{j=0}^{2} V_j(t; p_\theta, p_\nu, p_0; \varepsilon)\, r^{-j} + p_0 .$$

In this *perturbed Gylden system*, $\mathcal{H}_0$ is the Gylden Hamiltonian. The perturbation can be expanded in powers of a small parameter ε. Using the Stumpff c_n and Battin U_n *universal functions*, and *universal parameters* $s(r, t; \Phi, L, G, N)$ and $f(r, t; \Phi, L, G, N)$, a generator based on Deprit (1981, Formula [23]) defines a TR–like mapping, from extended polar nodal variables to universal, generalized DS variables $(q_\Phi, q_L, q_G, q_N; \Phi, L, G, N)$, with

$$Q \equiv Q(r, t; \Phi, L, G, N; \varepsilon) = -2\,[L + V_0(t; G, N, L; \varepsilon)] + \frac{2\,[\mu(t) - V_1(t; G, N, L; \varepsilon)]}{r} - \frac{[\gamma^2 + 2 V_2(t; G, N, L; \varepsilon)]}{r^2},$$

$$\gamma \equiv \gamma(\Phi, L, G, N), \;\; \mu^* = \mu - V_1, \;\; q = [\mu^*(1-e)]\,/\,(2\Lambda), \;\; \Lambda = L + V_0,$$

$$\Gamma^2 \equiv \gamma^2 + 2 V_2 = \mu^* q(1+e), \qquad r = [q(1+e)]\,/\,[1 + e\cos f],$$

$$r = q + \mu^* e\, s^2 c_2\left(2\Lambda s^2\right) = q + \mu^* e\, U_2(s, 2\Lambda).$$

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This TR–like transformation converts Hamiltonian $\mathcal{H}$ into

$$\widetilde{\mathcal{H}} = \left(G^2 - \gamma^2\right) / \left(2\,r^2\right) + \Delta V_0 + (\Delta V_1/r) + \left(\Delta V_2/r^2\right)$$
$$- (\partial V_0/\partial t)\ [q\,s + \mu^*\,e\,U_3] + (\partial\,\mu^*/\partial\,t)\,s - (\partial V_2/\partial t)\ f/\Gamma\,,$$
$$\Delta V_j \equiv \widetilde{V}_j - V_j\,,\quad V_j \equiv V_j\,(t; G, N, L; \varepsilon)\,,\quad \widetilde{V}_j \equiv V_j\,(t; G, N, \widetilde{p}_0; \varepsilon)\,,$$

where $\widetilde{p}_0$ refers to p_0 in the *new* canonical variables. A *time transformation*, $dt = \widetilde{f}\,d\tau$, with $\widetilde{f} = r^2/\mathcal{G}$, $\mathcal{G} \equiv \mathcal{G}\,(\Phi, L, G, N)$, simplifies $\widetilde{\mathcal{H}}$. The *Hamiltonian* with this pseudo–time is $\mathcal{K} = \widetilde{\mathcal{H}}\widetilde{f}$. These Hamiltonians depend on the momenta (Φ, L, G, N) and on the *non–DS variables* (r, t, s), and should be expressed in the new DS–type variables. To this end, *implicit function results, inversion theorems and Fourier analysis of the two–body problem* must be applied. As in the case of classical DS variables, certain specifications of γ and $\mathcal{G}$, as functions of the new momenta, reduce the first term of $\widetilde{\mathcal{H}}$ and $\mathcal{K}$. Assumptions concerning the V_j–terms produce further simplifications.

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