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Cross Recurrence Plots Analysis of the North–South Sunspot Activities

Dmitri I. Ponyavin[1] **and Nadejda V. Zolotova**[1]

[1]Institute of Physics, University of St.Petersburg, St.Petersburg, 198504, Russia, Email: ponyavin@geo.phys.spbu.ru

Abstract. A new technique of nonlinear interrelations between time series developed by Marwan & Kurths, (2002) has been applied to the sunspot data. By using this tools we have investigated synchronization and phase difference in annual sunspot areas — time series available for Northern and Southern Hemispheres of the Sun.

1. Introduction

Recurrence is fundamental property of dissipative dynamical systems typical in nature. Recurrence plots (RP) firstly introduced by Eckmann et al., (1987) simply display recurrent behavior of natural processes in phase space. The main step is the calculation of the $N \times N$ — matrix.

$$\mathbf{R}_{ij} = \Theta(\varepsilon - \|\mathbf{x}_i - \mathbf{x}_j\|), \qquad i, j = 1, \ldots, N \tag{1.1}$$

where ε is a predefined cut-off distance, $\|\cdot\|$ is a norm and $\Theta(\cdot)$ is the Heaviside function. The values *one* and *zero* in the matrix can be simply visualized by the colours *black* and *white*. The recurrence plot exhibits characteristic patterns for typical dynamical behavior. The main of RP advantages is its relevance for short and nonstationary data.

Extension of the RP technique is the method of cross recurrence plots (CRP) described by Marwan & Kurths, (2002). In contrast to the conventional RP, two time series are simultaneously embedded in the same phase space. The test for closeness of each point of the first trajectory with each point of the second trajectory results in a $N \times M$ array.

$$\mathbf{CR}_{ij} = \Theta(\varepsilon - \|\mathbf{x}_i - \mathbf{y}_j\|), \qquad i = 1, \ldots, N, \quad j = 1, \ldots, M \tag{1.2}$$

The visualization of this is called the *cross recurrence plots*. Marwan et al., (2002) focused on the bowed "main diagonal" in the CRP. Regarding the conventional RP (1.1), one always finds a main diagonal in the plot because the (i, i)-states are identical. The RP can be considered as a special case of the CRP (1.2), which usually does not have a main diagonal as the (i, i)-states are not identical. But, if the sets are similar, e.g. only rescaled, a more or less continuous line in the CRP that is like a distorted main diagonal can occur. According to Marwan et al., (2002) this line contains information on the rescaling and is called "line of synchronization". Thus, the line of synchronization lies closer to main diagonal, that concerned processes are better synchronized.

2. Application to the sunspot data

In this paper we have analyzed the North-South synchronization of sunspot activity using CRP toolbox package available from http://www.agnld.uni-potsdam.de/ marwan/toolbox/.

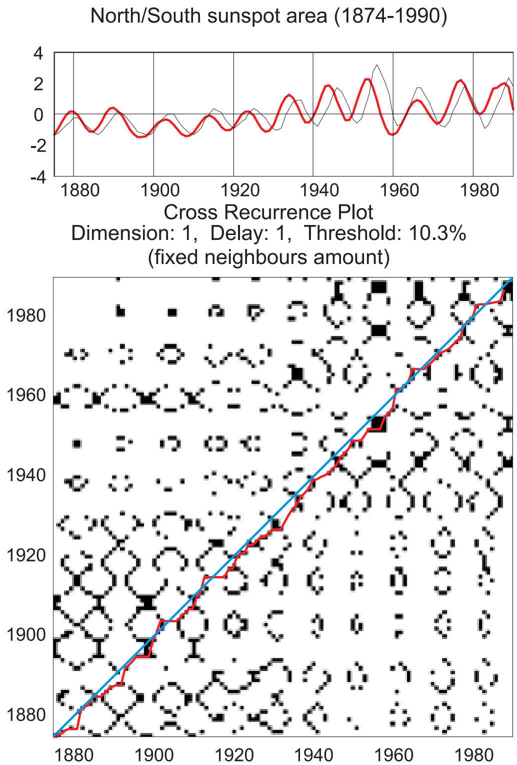

Figure 1. CRP of smoothed annual sunspot areas for Northern and Southern hemispheres. Dimension: 1, delay: 1, threshold: 10.3 percent, fixed amount of nearest neighbors.

The sunspot areas used in this study are annual averages available since 1874 (see Figure 1). CRP patterns were obtained with following parameters: embedding dimension equals to one, without using time delay and fixed amount of nearest neighbors. Neighborhood covers 10.3 percent of all phase space vectors.

CRP demonstrate high level of synchronization between Northern and Southern hemispheres. However line of synchronization lies mostly beneath the main diagonal. This means that processes of sunspot formation in the Northern hemisphere are mainly in late with respect to the Southern hemisphere. The line of synchronization exhibits variations that may have effects on the long-term oscillations in the north-south asymmetry of the solar magnetic field (Knaack et al., (2004)).

3. Conclusions

By means of cross recurrence plots one can easily reveal synchronization and phase differences between in the state of northern and southern sunspot activities. But their causes are still not clear so far.

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