SUMMARY: THEORETICAL VIEWPOINT

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Rather than give a comprehensive review, I want to concentrate on a few areas where I was particularly impressed by the theoretical results discussed at this meeting.

The first such area is spiral structure theory. Kalnajs presented a clear and thoughtful review (and Lin gave us a fast Fourier transform of the review), but I mainly want to stress a very important point first made by Colin Norman.

Density wave theory was introduced in the early 1960's and quickly became the standard theory of spiral structure. It was based on the idea that the spiral arms were a wave pattern or normal mode of the galactic disk. For the past twenty years a number of theorists, primarily Kalnajs, Lin and Toomre, have been trying to understand these modes in detail. They have travelled a long and winding road, and now seem to be near their goal. In fact, this is the first meeting at which these three major protagonists all agree that they qualitatively understand the dominant normal modes in galactic disks.

Despite the sophisticated mathematics, the behaviour of these modes can be simply understood. Any normal mode has a corotation radius, where the wave pattern rotates at the same angular speed as the disk. Waves outside or inside corotation have positive or negative angular momentum respectively (this result is messy to prove but easy to rationalize: outside [inside] corotation the wave pattern angular speed is greater [less] than the disk angular speed so that the presence of the wave effectively increases [decreases] the angular momentum of the disk).

Consider a wave inside the corotation radius which propagates outward. It cannot pass corotation, since to do so its angular momentum would have to change sign. Hence the wave effectively reflects off the corotation circle and begins to propagate inward. The inward directed wave may reflect off the central bulge or propagate through the center of the galaxy; in either case a resonant cavity is set up whose

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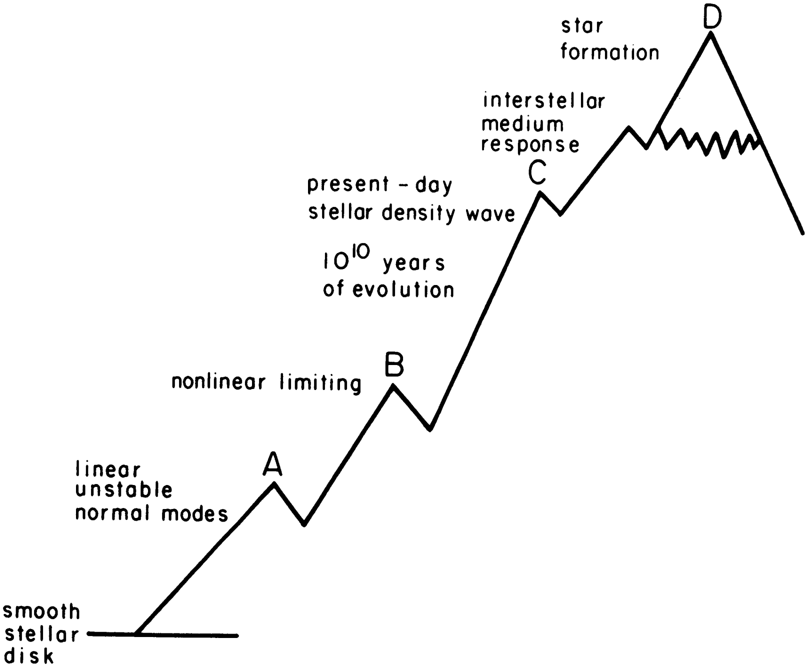
wall is the corotation circle. Moreover, tunneling through the wall amplifies the standing wave inside the cavity, since the generation of positive angular momentum waves outside corotation removes angular momentum from the cavity and thus strengthens the negative angular momentum waves inside. This is the amplification process variously called the "WASER" or "swing amplifier."

This argument (and most of the analytic work in spiral structure theory) is based on a "local" approximation, i.e. an approximation that the separation of the spiral arms is small compared to the galactic radius. Thus its relevance to realistic galaxies is limited. However, there are now a number of numerical codes which exactly calculate the linear normal modes of both gaseous disks (Bardeen, Haass and Iye) and stellar disks (Athanassoula, Kalnajs and Zang). Remarkably, all of the codes seem to show that these simple analytic arguments based on the local approximation work quite well, even for large scale waves, and can be used to predict fairly accurately the growth rates and shapes of unstable normal modes.

Thus, we now largely understand the linear normal modes of galactic disks, and in a sense the fundamental problem of density wave theory has therefore been solved. To put this accomplishment in perspective I want to use the mountain climbing analogy suggested several years ago by Toomre (see diagram).



We have now reached peak A. Next we must understand what limits the growth of the linear normal modes (saturation of the Lindblad resonances? shock damping? nonlinear effects in the stellar disk?). This is the climb to peak B. After a steady spiral pattern is established we must follow it for 10^{10} yr, as it evolves due to angular momentum transfer, star formation, dynamical friction from the bulge and halo, infall and a variety of other effects. The importance of angular momentum transfer was pointed out ten years ago by Lynden-Bell and Kalnajs (1972) and again by Kalnajs at this meeting; the importance of infall was also stressed here by Gunn, and Kormendy has discussed morphological evidence for evolution in disk galaxies. Nevertheless, this part of the climb, from B to C, has not received the attention it deserves. It is difficult but rewarding, since the present structure of spiral galaxies may largely be determined during this phase.

Finally, we must understand star formation in sufficient detail to relate the density and potential perturbations in the stellar disk to the bright stars that define the optical arms (the climb to D).

An important cause for optimism is the recent development of efficient and accurate N-body codes, by Aarseth, Hohl, James, Miller, Sellwood, Wilkinson, Zang and others. We can now follow realistic galaxy models for $\sim 10^{10}$ yr, and can thus check our understanding of galaxy dynamics at all stages of the climb up to peak C (Sellwood described some checks of linear normal mode calculations in his review).

To summarize, we should congratulate the density wave theorists, who are approaching the successful completion of twenty years of work on the stability of galactic disks, and wish them equal success in the study of the origin and evolution of spiral structure.

The second area in which I feel that real progress has been made is the theory of triaxial stellar systems. Of course, we have known that such systems exist ever since Hubble defined the class of barred spiral galaxies, and some specialized triaxial models were constructed by Freeman in the 1960's. But the first real clue to the importance of these systems came from N-body experiments conducted by Hohl and Miller (the best review of these was given at the last Besançon meeting on dynamics [Hohl 1975]). They found that rapidly rotating axisymmetric stellar systems are always strongly unstable to the formation of a bar-like or triaxial subsystem.

The second clue came from experiments on the collapse of non-rotating triaxial systems. Aarseth and Binney, Miller, and especially Wilkinson and James (1982) showed that triaxiality is conserved in a collapse, so that triaxial systems are naturally formed in a collapse from irregular initial conditions. The Wilkinson and James calculation also shows conclusively that the equilibrium system preserves its triaxiality, showing virtually no evolution over a large fraction of a Hubble time.

The conclusion is that triaxiality is a natural state for stellar systems, whether rapidly or slowly rotating, and that axisymmetry should be regarded as the exception, not the rule.

Accompanying these calculations has been Schwarzschild's (1979) development of a general algorithm for the construction of stellar systems. The application of this method to axisymmetric systems has been described by Richstone and de Zeeuw at this meeting, and Schwarzschild has used it to construct a non-rotating triaxial stellar system (which might be called the Schwarzschild ellipsoid). The density distribution in the Schwarzschild ellipsoid was chosen a priori to resemble the emissivity distribution in a real galaxy; hence the potential has no special features such as integrability or separability. Thus it is significant that Schwarzschild's algorithm rapidly converged on a self-consistent solution; this result lends support to the notion that triaxial stellar systems are easy to form in nature. In addition, Schwarzschild's work showed that the proper initial approach to the study of triaxial systems is morphological: one studies and classifies the families of orbits in order to get a qualitative grasp of how to combine these orbits to make up a self-consistent stellar system. We have heard about studies of this kind by Mulder and de Zeeuw.

In summary, we obviously do not understand triaxial systems yet, but we do understand that they are important; we have some preliminary theoretical results; and we have all the tools we need to finish the job. Ironically, many of the observations discussed at this meeting argue against strongly triaxial galaxies: Illingworth showed that faint ellipticals rotate like oblate spheroids with isotropic pressure tensors; Schweizer stressed that isophotal twists are often due to recent mergers; and studies of apparent axis ratio distributions are either inconclusive or weakly favour oblate systems.

The last topic that I shall discuss is the evidence for massive halos. At the last Besançon meeting, in 1974, there was a spirited controversy over whether the HI rotation curve of M31 was flat, and at this meeting there were similar controversies. However, over the last eight years Bosma, Rubin and others have accumulated impressive data showing that most spiral galaxies have flat rotation curves extending to at least one Holmberg radius, and hence that these galaxies have heavy halos containing at least 1-2 disk masses. The rotation curves provide the strongest evidence that such halos exist. However, there are many other clues.

One can combine M/L estimates for the disk and bulge with their photometric profiles and thus ask whether they contain enough mass to produce the observed rotation speed at radii $\lesssim 10$ kpc. The answer is generally "no" (van der Kruit, at this meeting; also Bahcall and Soneira 1980).

Dynamical models of the Magellanic Stream require that our Galaxy has a massive halo out to $\gtrsim 70$ kpc if there are no non-gravitational forces on the Stream (e.g. Lin and Lynden-Bell 1982; also Lynden-Bell at this meeting).

The Local Group timing (e.g. Gunn 1975) strongly suggests that the Galaxy and M31 have a total mass of $\sim 3\times 10^{12}$ $M_{\odot}$, far larger than their combined disk and bulge masses.

Studies of the dynamics of the Galactic globular cluster system (Frenk and White 1980) and the ellipticity of the Galactic bulge (Monet, Richstone and Schechter 1981) also suggest that an extended massive halo is present in the Galaxy.

Ostriker and Peebles (1973; see Sellwood's review) argued that either a halo or a hot disk component with mass comparable to the observed disk mass was needed for stability. This argument still lends strong support to the heavy halo hypothesis, although a number of speakers here have suggested that the second alternative, the hot disk, should be considered seriously.

There are also tests of the heavy halo hypothesis which are useful in principle but weak in practice. The selection effects in studies of binary galaxy dynamics (White *et al*. 1982) are so large that they offer no strong evidence for or against heavy halos. Similarly, satellite galaxy tidal radii cannot be used to constrain halo masses, since the observed radii are uncertain, the satellite galaxy M/L's are unknown, and the theory of tidal radii is poorly developed (although progress on the latter two problems was reported here by Illingworth and Freeman).

To summarize, the evidence for halos containing $\sim$1-2 disk masses is very strong but not conclusive. A number of new arguments support the heavy halo hypothesis and there is still no substantial evidence against it. On the negative side, the evidence for very heavy halos extending to several hundred kpc is still very slim, and we have made virtually no progress in understanding what the composition of the halo could be (10^6 $M_{\odot}$ black holes? 1 $M_{\odot}$ black holes? Jupiters? massive neutrinos? etc.).

To close, may I say that I hope we can look forward to a third Besançon symposium on dynamics in another eight years, and I hope that that meeting will be as exciting and enjoyable as this one has been.

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DISCUSSION

OORT : Our chairman, Dr. Toomre, asked for my reaction to the symposium. I have been impressed - and to some extent confused - by the wonderful account of the large progress made not only in observations of the structure of galaxies, but also in the understanding of these structures. A particularly impressive example was the discovery of the triaxiality of elliptical galaxies. My attention was first drawn to it in an introductory report by Freeman at the IAU General Assembly in Sydney, when I found it very surprising. For there seemed to be such good grounds for supposing that the apparent flattening was caused by rotation; until Illingworth found that there was no rotation, and Binney showed how the apparent flattening could be understood without rotation.
A second totally unforeseen development has been the realization of the enormous influence of mergers on the evolution process of galaxies which our chairman of this afternoon has had the boldness to introduce.
What has impressed me most is the boldness and the success with which one is now attempting to understand the formation as well as the evolution of galaxies.
It is wonderful to have such adventurous discoverers in the astronomical family.