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RANDOM FIXED POINT THEOREMS FOR CONTRACTIVE TYPE MULTIFUNCTIONS

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Abstract

Some new random coincidence point and random fixed point theorems for multivalued mappings in separable complete metric spaces are proved. The results presented in this paper are the stochastic versions of corresponding results of Chang and Peng and extend the result of the author.

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1. Introduction and preliminaries

In order to generalise the well-known contraction principle of Banach to multivalued functions and random fixed point theorems, many authors ([1, 5, 7–11]) introduced more general contractive inequalities. We intend to consider a class of generalised contractions that includes the classes considered in ([1, 5, 7–11]) and that enables us to prove a more general random fixed point theorem for multifunctions.

Throughout this paper (X, d) is a separable complete metric space, $\mathbb{R}^+ = [0, \infty)$ and (Ω, δ) is a measurable space. Let 2^X be the family of all subsets of X, $CB(X)$ denote the family of all nonempty closed bounded subsets of X and $CC(X)$ denote the family of all nonempty compact subsets of X. For any nonempty subsets A, B of X, we denote

$$d(x, A) = \inf\{d(x, a) : a \in A\} \quad (x \in X),$$
$$d(A, B) = \inf\{d(a, b) : a \in A, b \in B\},$$
$$H(A, B) = \max\left\{\sup_{a\in A} d(a, B), \sup_{b\in B} d(b, A)\right\}$$

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and $H(\cdot,\cdot)$ is called the Hausdorff metric on $CB(X)$.

A mapping $\mu : \Omega \to 2^X$ is called *measurable* if for any open subset C of X, $\mu^{-1}(C) = \{w \in \Omega : \mu(w) \cap C \neq \emptyset\} \in \delta$. A mapping $\xi : \Omega \to X$ is said to be *measurable selector* of a measurable mapping $\mu : \Omega \to 2^X$ if μ is measurable and for any $w \in \Omega$, $\xi(w) \in \mu(w)$. A mapping $f : \Omega \times X \to X$ is called a *random operator* if for any $x \in X$, $f(\cdot, x)$ is measurable. A mapping $T : \Omega \times X \to CB(X)$ is called a *multifunction* if for every $x \in X$, $T(\cdot, x)$ is measurable. A measurable mapping $\xi : \Omega \to X$ is called a *random fixed point* of a multifunction (*random operator*) $T : \Omega \times X \to CB(X)$ ($f : \Omega \times X \to X$) if for every $w \in \Omega$, $\xi(w) \in T(w, \xi(w))$ ($f(w, \xi(w)) = \xi(w)$). A measurable mapping $\xi : \Omega \to X$ is a *random coincidence point* of $T : \Omega \times X \to CB(X)$ and $f : \Omega \times X \to X$ if for every $w \in \Omega$, $f(w, \xi(w)) \in T(w, \xi(w))$.

For the remaining part of this section $S, T : \Omega \times X \to CB(X)$ are multifunctions, $f : \Omega \times X \to X$ is a random operator and $\xi_n : \Omega \to CB(X)$ is a measurable mapping for each $n = 0, 1, 2, \ldots$.

For a map $\xi_0 : \Omega \to X$, if there exists a sequence $\{\xi_n(w)\}$ such that $f(w, \xi_{n+1}(w)) \in S(w, \xi_n(w))$, $f(w, \xi_{n+2}(w)) \in T(w, \xi_{n+1}(w))$, $n = 0, 1, 2, \ldots$, then $O_f(\xi_0(w)) = \{f(w, \xi_n(w)) : n = 1, 2, 3, \ldots$ for each $w \in \Omega\}$ is the *orbit* for (S, T, f) at $\xi_0(w)$. If there exists a measurable map $\xi : \Omega \to X$ such that $f(w, \xi_n(w)) \to f(w, \xi(w))$ for all $w \in \Omega$, then $O_f(\xi_0(w))$ converges in X. If $O_f(\xi_n(w))$ converges in X, then X is called $(S, T, f, \xi_0(w))$*-orbitally complete.*

A function $\Psi(t_1, t_2, t_3, t_4, t_5) : \mathbb{R}^{+5} \to \mathbb{R}^+$ is said to satisfy the condition (Ψ), if it is nondecreasing in each variable and there exists an increasing function $\Phi(t) : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the conditions (1) and (2) such that

$$\Psi(t, t, t, at, bt) \le \Phi(t), \quad \text{for all } t \ge 0, \quad a + b = 3, \quad a, b = 1, 2.$$

LEMMA 1.1 ([2]). *Let (X, d) be a metric space, $A \subset X$ a nonempty compact subset and $B \subset X$ a closed subset. If $d(A, B) = 0$, then $A \cap B \neq \emptyset$.*

LEMMA 1.2 ([13, Theorem 1]). *Let $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ be an increasing function such that*

$$\Phi(t+) < t \quad \text{for all } t > 0 \quad \text{and} \tag{1}$$

$$\sum \Phi^n(t) \quad \text{is finite for all } t > 0. \tag{2}$$

Then there exists a strictly increasing function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$\Phi(t) < \phi(t) \quad \text{for all } t > 0 \quad \text{and} \tag{3}$$

$$\sum \phi^n(t) \quad \text{is finite for } t > 0. \tag{4}$$

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LEMMA 1.3 ([13]). (i) *If* $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ *is strictly increasing and satisfies* (2), *then* Φ *satisfies* (1).

(ii) *Let* $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ *be increasing and satisfying* (1). *If* $\sum \Phi^n(t_1)$ *is convergent for some* $t_1 > 0$, *then* (2) *holds.*

(iii) *Let* $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ *be increasing and satisfying* (1). *If* $t \leq \Phi(t)$ *then* $t = 0$.

LEMMA 1.4 ([11]). *If* $S_1, S_2 : \Omega \to CB(X)$ *are measurable multifunctions,* $s_1 : \Omega \to X$ *is a measurable selector of* S_1 *and* $\lambda : \Omega \to (1, \infty)$ *is a measurable function then there exists a measurable selector* $s_2 : \Omega \to X$ *of the multifunction* S_2 *such that* $d(s_1(w), s_2(w)) \leq \lambda(w) H(S_1(w), S_2(w))$.

LEMMA 1.5 ([11]). *If* $S : \Omega \to CB(X)$ *is a continuous multifunction then* $x \to d(x, S(x))$ *is a continuous real valued function.*

2. Main results

Recently, Mustafa [8] gave the stochastic generalisation of the results of Kaneko and Sessa [4] and proved the following theorem:

THEOREM 2.1. *Let* $S, T : \Omega \times X \to CB(X)$ *be two continuous multifunctions and let* $f : \Omega \times X \to X$ *be a random operator such that* $S(w, X) \cup T(w, X) \subseteq f(w, X)$ *and for a measurable map* $\xi_0 : \Omega \to X$, $f(w, X)$ *is* $(S, T, f, \xi_0(w))$*-orbitally complete, for every* $w \in \Omega$, *and*

$$\begin{aligned} H(S(w,x), T(w,y)) \leq \alpha(w) \max \big\{ & d(f(w,x), f(w,y)), \\ & d(f(w,x), S(w,x)), d(f(w,y), T(w,y)), \\ & [d(f(w,x), T(w,y)) + d(f(w,y), S(w,x))]/2 \big\} \end{aligned}$$

for all $x, y \in X$ *and for all* $w \in \Omega$, *where* $\alpha : \Omega \to (0, 1)$ *is a measurable map. Then there exists a random coincidence point of* S, T *and* f.

Let $S, T : \Omega \times X \to CB(X)$ and $F : \Omega \times X \to CC(X)$ be multifunctions such that

$$\begin{aligned} (5) \quad H(S(w,x), T(w,y)) \leq \Phi\big(\max \big\{ & d(F(w,x), F(w,y)), \\ & d(F(w,x), S(w,x)), d(F(w,y), T(w,y)), \\ & [d(F(w,x), T(w,y)) + d(F(w,y), S(w,x))]/2 \big\}\big) \end{aligned}$$

for all $x, y \in X$ and for all $w \in \Omega$, where $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ is an increasing function satisfying conditions (1) and (2).

As an improvement and generalisation of Theorem 2.1, we have the following

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THEOREM 2.2. *Let $S, T : \Omega \times X \to CB(X)$ and $F : \Omega \times X \to CC(X)$ be multifunctions such that*

(i) *$S(w, \cdot)$, $T(w, \cdot)$ are both continuous for all $w \in \Omega$;*
(ii) *$S(\cdot, x)$, $T(\cdot, x)$ are both measurable for all $x \in X$;*
(iii) *$S(w, X) \cup T(w, X) \subset F(w, X)$, $F(w, X)$ is closed;*
(iv) *S, T and F satisfy (5) for all $w \in \Omega$ and all $x, y \in X$.*

Then there exists a measurable map $s : \Omega \to X$ such that

$$F(w, s(w)) \cap S(w, s(w)) \cap T(w, s(w)) \neq \emptyset.$$

PROOF. By Lemma 1.2, there exists a strictly increasing function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying conditions (3) and (4). For any $x, y \in X$ and $w \in \Omega$, let us denote

$$A(x, y) \leq \max \{d(F(w, x), F(w, y)), d(F(w, x), S(w, x)), d(F(w, y), T(w, y)), \\ [d(F(w, x), T(w, y)) + d(F(w, y), S(w, x))]/2\}.$$

Then (5) can be reduced to

$$H(S(w, x), T(w, y)) \leq \Phi(A(x, y)).$$

Let $\varphi : \Omega \times X \to \mathbb{R}^+$ be the function

$$\varphi(w, x) = d(x, S(w, x)), (w, x) \in \Omega \times X.$$

Since by (ii) $w \to S(w, x)$ is measurable for all $x \in X$ we conclude that $\varphi(\cdot, x)$ is measurable (see [3, Theorem 3.5]) and since $x \to S(w, x)$ is continuous for all $w \in \Omega$, we deduce from Lemma 1.5 that $\varphi(w, \cdot)$ is continuous for all $w \in \Omega$. Hence $\varphi : \Omega \times X \to \mathbb{R}^+$ is a Caratheodory function. Therefore, if $u : \Omega \to X$ is a measurable mapping we also have that $w \to \varphi(w, u(w))$ is measurable (see [12]). If ξ_0, $\xi_1 : \Omega \to X$ are measurable mappings and we consider the multifunction $S(\cdot, \xi_0(\cdot)) : \Omega \to CB(X)$, then we deduce from the Kuratowski-Ryll Nardzewski Selection Theorem [6] that there is a measurable selector $s_1 : \Omega \to X$ such that $s_1(w) \in S(w, \xi_0(w))$ for all $w \in \Omega$. Applying Lemma 1.4 we find a measurable function $s_2 : \Omega \to X$ such that $s_2(w) \in T(w, \xi_1(w))$, $w \in \Omega$.

For any measurable map ξ_0: $\Omega \to X$, since $S(w, X) \subset F(w, X)$, there exist measurable maps, say $\xi_0, \xi_1 : \Omega \to X$ such that $F(w, \xi_1(w)) \cap S(w, \xi_0(w)) \neq \emptyset$. Let $s_1(w) \in F(w, \xi_1(w)) \cap S(w, \xi_0(w))$, then we have

$$d(s_1(w), T(w, \xi_1(w))) \leq d(s_1(w), S(w, \xi_0(w))) + H(S(w, \xi_0(w)), T(w, \xi_1(w))) \\ \leq \Phi(A(\xi_0(w), \xi_1(w))).$$

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(a) If $A(\xi_0(w), \xi_1(w)) = 0$, then $d(F(w, \xi_0(w)), S(w, \xi_0(w))) = 0$. By Lemma 1.1, $F(w, \xi_0(w)) \cap S(w, \xi_0(w)) \neq \emptyset$. Taking $s(w) \in F(w, \xi_0(w)) \cap S(w, \xi_0(w))$, we have

$$\begin{aligned} d(s(w), T(w, \xi_0(w))) &\leq d(s(w), S(w, \xi_0(w))) + H(S(w, \xi_0(w)), T(w, \xi_0(w))) \\ &\leq \Phi\Big(\max\big\{0, 0, d(F(w, \xi_0(w)), T(w, \xi_0(w))), \\ &\qquad\qquad d(F(w, \xi_0(w)), T(w, \xi_0(w)))/2\big\}\Big) \\ &\leq \Phi(d(F(w, \xi_0(w)), T(w, \xi_0(w)))) \\ &\leq \Phi(d(s(w), T(w, \xi_0(w)))). \end{aligned}$$

By Lemma 1.3 (iii) $d(s(w), T(w, \xi_0(w))) = 0$, since $T(w, \xi_0(w))$ is closed, $s(w) \in T(w, \xi_0(w))$. Therefore in this case the conclusion of Theorem 2.2 is proved.
(b) If $A(\xi_0(w), \xi_1(w)) > 0$, then, by (3) we have

$$d(s_1(w), T(w, \xi_1(w))) \leq \Phi(A(\xi_0(w), \xi_1(w))) < \phi(A(\xi_0(w), \xi_1(w))).$$

Consequently, we can find an $s_2(w) \in T(w, \xi_1(w))$ such that

$$d(s_1(w), s_2(w)) \leq \phi(A(\xi_0(w), \xi_1(w))). \tag{6}$$

Since $T(w, X) \subset F(w, X)$, for $s_2(w) \in T(w, \xi_1(w)) \subset F(w, X)$, there exists a measurable map, say $\xi_2 : \Omega \to X$ such that $s_2(w) \in F(w, \xi_2(w))$. This implies that we can find an $s_2(w) \in F(w, \xi_2(w)) \cap T(w, \xi_1(w))$ such that (6) holds.

On the other hand, by the assumption we have

$$d(S(w, \xi_2(w)), s_2(w)) \leq H(S(w, \xi_2(w)), T(w, \xi_1(w))) \leq \Phi(A(\xi_2(w), \xi_1(w))).$$

If $A(\xi_2(w), \xi_1(w)) = 0$, by the same reason as stated in the proof of (a) we can prove that the conclusion of Theorem 2.2 is true.

If $A(\xi_2(w), \xi_1(w)) > 0$, repeating the reasoning as mentioned above, we can find measurable maps $\xi_3, s_3 : \Omega \to X$ such that $s_3(w) \in F(w, \xi_3(w)) \cap S(w, \xi_2(w))$ and

$$d(s_3(w), s_2(w)) \leq \phi(A(\xi_2(w), \xi_1(w))).$$

Inductively, we can define two sequence $\{\xi_n(w)\}, \{s_n(w)\} \subset X$ such that

$$\begin{cases} s_{2n+1}(w) \in F(w, \xi_{2n+1}(w)) \cap S(w, \xi_{2n}(w)) \\ s_{2n+2}(w) \in F(w, \xi_{2n+2}(w)) \cap T(w, \xi_{2n+1}(w)) \end{cases} \qquad n = 0, 1, 2, \ldots \tag{7}$$

and

$$\begin{cases} d(s_{2n+1}(w), s_{2n+2}(w)) \leq \phi(A(\xi_{2n}(w), \xi_{2n+1}(w))) \\ d(s_{2n+3}(w), s_{2n+2}(w)) \leq \phi(A(\xi_{2n+2}(w), \xi_{2n+1}(w))) \end{cases} \qquad n = 0, 1, 2, \ldots . \tag{8}$$

Now, we prove that $\{s_n(w)\}$ is a Cauchy sequence in X. In fact, for any positive integer n, we have

$$\begin{aligned} A(\xi_{2n}(w), \xi_{2n+1}(w)) \le \max \big\{&d(s_{2n}(w), s_{2n+1}(w)), d(s_{2n}(w), s_{2n+1}(w)), \\ &d(s_{2n+1}(w), s_{2n+2}(w)), \\ &[d(s_{2n}(w), s_{2n+2}(w)) + d(s_{2n+1}(w), s_{2n+1}(w))]/2\big\} \\ \le \max\{&d(s_{2n}(w), s_{2n+1}(w)), d(s_{2n+1}(w), s_{2n+2}(w))\}. \end{aligned}$$

Using the same argument we can prove that

$$A(\xi_{2n+2}(w), \xi_{2n+1}(w)) \le \max\{d(s_{2n+1}(w), s_{2n+2}(w)), d(s_{2n+2}(w), s_{2n+3}(w))\}.$$

Consequently, in general, for $n = 1, 2, \ldots$, we have

$$\begin{aligned} (9) \quad d(s_{n+1}(w), s_{n+2}(w)) &\le \phi(A(\xi_n(w), \xi_{n+1}(w))) \\ &\le \phi(\max\{d(s_n(w), s_{n+1}(w)), d(s_{n+1}(w), s_{n+2}(w))\}). \end{aligned}$$

If $d(s_{n+1}(w), s_{n+2}(w)) > d(s_n(w), s_{n+1}(w)) \ge 0$, then, by (9) and Lemma 1.3, we have

$$d(s_{n+1}(w), s_{n+2}(w)) \le \phi(d(s_{n+1}(w), s_{n+2}(w))) < d(s_{n+1}(w), s_{n+2}(w))$$

a contradiction. Therefore, $d(s_{n+1}(w), s_{n+2}(w)) \le d(s_n(w), s_{n+1}(w))$. Hence

$$(10) \quad d(s_n(w), s_{n+1}(w)) \le \phi(d(s_{n-1}(w), s_n(w))) \le \cdots \le \phi^{n-1}(d(s_1(w), s_2(w))).$$

If $d(s_1(w), s_2(w)) = 0$, that is, $s_1(w) = s_2(w)$, for all $w \in \Omega$, denoting $s(w) = s_1(w) = s_2(w)$, then $s(w) = s_1(w) \in F(w, \xi_1(w)) \cap S(w, \xi_0(w))$, $s(w) = s_2(w) \in F(w, \xi_2(w)) \cap T(w, \xi_1(w))$. Hence $s(w) \in F(w, \xi_1(w)) \cap T(w, \xi_1(w))$. Similarly using the proof in (a) we can prove that $s(w) \in S(w, \xi_1(w))$. Hence the conclusion of Theorem 2.2 is proved.

If $d(s_1(w), s_2(w)) > 0$, in view of condition (4), we know that $\sum \phi^{n-1} d(s_1(w), s_2(w))$ is convergent. It follows from (10) that $\sum d(s_n(w), s_{n+1}(w))$ is convergent too. This implies that $\{s_n(w)\}$ is a Cauchy sequence in X. Since X is complete, there exists a measurable map $s^* : \Omega \to X$ such that $s_n(w) \to s^*(w)$. Since $s_n(w) \in F(w, \xi_n(w)) \subset F(w, X)$ and $F(w, X)$ is closed, this shows that $s^*(w) \in F(w, X)$. Hence there exists measurable map $s : \Omega \to X$ such that $s^*(w) \in F(w, s(w))$. By (5) and (7) we have

$$\begin{aligned} d(s^*(w), S(w, s(w))) &\le d(s^*(w), s_{2n+2}(w)) + d(s_{2n+2}(w), S(w, s(w))) \\ &\le d(s^*(w), s_{2n+2}(w)) + d(s_{2n+2}(w), T(w, \xi_{2n+1}(w))) \\ &\quad + H(T(w, \xi_{2n+1}(w)), S(w, s(w))), \end{aligned}$$

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$$
\begin{aligned}
d(s^*(w), S(w, s(w))) \le d(s^*(w), s_{2n+2}(w)) \\
+ \Phi(\max\{d(s^*(w), s_{2n+1}(w)), d(s^*(w), S(w, s(w))), \\
d(s_{2n+1}(w), s_{2n+2}(w)), \\
[d(s^*(w), s_{2n+2}(w)) + d(s_{2n+1}(w), S(w, s(w)))]/2\}).
\end{aligned}
$$

Letting $n \to \infty$, we have $d(s^*(w), S(w, s(w))) \le \Phi(d(s^*(w), S(w, s(w))))$. By Lemma 1.3 (iii) we have $d(s^*(w), S(w, s(w))) = 0$. Since $S(w, s(w))$ is closed, we have $s^*(w) \in S(w, s(w))$. Similarly, we can prove that $s^*(w) \in T(w, s(w))$. Therefore we have $s^*(w) \in F(w, s(w)) \cap S(w, s(w)) \cap T(w, s(w))$. This completes the proof. □

REMARK. Theorem 2.1 is a special case of Theorem 2.2 with F being a single-valued mapping and $\Phi(t) = \alpha(w)t$, where $\alpha : \Omega \to (0, 1)$ is a measurable mapping and $t \in \mathbb{R}^+$.

COROLLARY 2.3. *Let $T_i : \Omega \times X \to CB(X)$, $i = 1, 2, \dots$, be multifunction such that*

(i) *$T_i(w, \cdot)$, $T_j(w, \cdot)$ are continuous for all $w \in \Omega$, $i \neq j$;*
(ii) *$T_i(\cdot, x)$, $T_j(\cdot, x)$ are measurable for all $x \in X$, $i \neq j$;*
(iii) *For all i, j, $i \neq j$*

$$
H(T_i(w, x), T_j(w, y)) \le \Phi(\max\{d(x, y), d(x, T_i(w, x)), d(y, T_j(w, y)), [d(x, T_j(w, y)) + d(y, T_i(w, x))]/2\}) \tag{11}
$$

for all $x, y \in X$ and for all $w \in \Omega$, where $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ is an increasing function satisfying conditions (1) *and* (2).

Then the random fixed point sets $\{\xi : \Omega \to X : \xi(w) \in T_i(w, \xi(w))\}$, $i = 1, 2, \dots$, are nonempty, closed and equal to each other.

PROOF. For the sake of convenience we prove the conclusions of Corollary 2.3 only for the case $i = 1$ and $j = 2$. By Theorem 2.2, there exists a measurable map $s : \Omega \to X$ such that $s(w) \in T_1(w, s(w)) \cap T_2(w, s(w))$. Now we prove that the random fixed point sets of T_1 and T_2 are equal to each other. In fact, if measurable map $u : \Omega \to X$ is a random fixed point of T_1, that is $u(w) \in T_1(w, u(w))$, then we have

$$
\begin{aligned}
&d(u(w), T_2(w, u(w))) \\
&\quad \le H(T_1(w, u(w)), T_2(w, u(w))) \\
&\quad \le \Phi\big(\max\big\{d(u(w), u(w)), d(u(w), T_1(w, u(w))), d(u(w), T_2(w, u(w))), \\
&\qquad [d(u(w), T_2(w, u(w))) + d(u(w), T_1(w, u(w)))]/2\big\}\big) \\
&\quad \le \Phi(d(u(w), T_2(w, u(w)))).
\end{aligned}
$$

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By Lemma 1.3 (iii), we have $d(u(w), T_2(w, u(w))) = 0$. Since $T_2(w, u(w))$ is closed, $u(w) \in T_2(w, u(w))$. Using the same argument we can prove that if a measurable map $v : \Omega \to X$ is a random fixed point of T_2 then v is also a random fixed point of T_1. Hence

$$\{\xi : \Omega \to X : \xi(w) \in T_1(w, \xi(w))\} = \{\xi : \Omega \to X : \xi(w) \in T_2(w, \xi(w))\}.$$

Let $\{\xi_n(w)\} \subset \{\xi : \Omega \to X : \xi(w) \in T_1(w, \xi(w))\}$ and $\xi_n(w) \to \xi(w)$ as $n \to \infty$. Since $\xi_n(w) \in T_1(w, \xi_n(w))$ and $T_1(w, \xi_n(w)) \to T_1(w, \xi(w))$ as $n \to \infty$. We have

$$\begin{aligned} d(\xi(w), T_1(w, \xi(w))) &\le d(\xi(w), \xi_n(w)) + d(\xi_n(w), T_1(w, \xi(w))) \\ &\le d(\xi(w), \xi_n(w)) + H(T_1(w, \xi_n(w)), T_1(w, \xi_1(w))) \xrightarrow{n\to\infty} 0, \end{aligned}$$

that is, $\xi(w) \in T_1(w, \xi(w))$. Therefore, $\{\xi : \Omega \to X : \xi(w) \in T_1(w, \xi(w))\}$ is closed. This completes the proof. □

Let $S, T : \Omega \times X \to CB(X)$ and $F : \Omega \times X \to CC(X)$ be multifunctions such that

$$(12) \qquad \begin{aligned} H(S(w,x), T(w,y)) \le \Psi\big(&d(F(w,x), F(w,y)), d(F(w,x), S(w,x)), \\ &d(F(w,y), T(w,y)), d(F(w,x), T(w,y)), \\ &d(F(w,y), S(w,x))\big) \end{aligned}$$

for all $x, y \in X$ and for all $w \in \Omega$, where $\Psi(t_1, t_2, t_3, t_4, t_5) : \mathbb{R}^{+5} \to \mathbb{R}^+$ satisfies condition (Ψ).

THEOREM 2.4. *Let $S, T : \Omega \times X \to CB(X)$ and $F : \Omega \times X \to CC(X)$ be multifunctions such that*

(i) *$S(w, \cdot)$, $T(w, \cdot)$ are both continuous for all $w \in \Omega$;*
(ii) *$S(\cdot, x)$, $T(\cdot, x)$ are both measurable for all $x \in X$;*
(iii) *$S(w, X) \cup T(w, X) \subset F(w, X)$, $F(w, X)$ is closed;*
(iv) *S, T and F satisfy* (12) *for all $w \in \Omega$ and all $x, y \in X$.*

Then there exists a measurable map $s : \Omega \to X$ such that

$$F(w, s(w)) \cap S(w, s(w)) \cap T(w, s(w)) \neq \emptyset.$$

PROOF. Let

$$\begin{aligned} t^* = \max\big\{&d(F(w,x), F(w,y)), d(F(w,x), S(w,x)), d(F(w,y), T(w,y)), \\ &[d(F(w,x), T(w,y)) + d(F(w,y), S(w,x))]/2\big\}. \end{aligned}$$

Without loss of generality we can assume that

$$d(F(w,x),\,T(w,y)) \ge d(F(w,y),\,S(w,x)).$$

Then

$$t^* \ge \max\{d(F(w,x),\,F(w,y)),\,d(F(w,x),\,S(w,x)),\,d(F(w,y),\,T(w,y))\},$$
$$t^* \ge [d(F(w,x),\,T(w,y)) + d(F(w,y),\,S(w,x))]/2 \ge d(F(w,y),\,S(w,x)),$$
$$2t^* \ge d(F(w,x),\,T(w,y)) + d(F(w,y),\,S(w,x)) \ge d(F(w,x),\,T(w,y)).$$

Using condition (Ψ) and (12) we have

$$\begin{aligned} H(S(w,x),\,T(w,y)) &\le \Psi(t^*,t^*,t^*,2t^*,t^*) \le \Phi(t^*) \\ &= \Phi\big(\max\big\{d(F(w,x),\,F(w,y)),\,d(F(w,x),\,S(w,x)), \\ &\qquad d(F(w,y),\,T(w,y)), \\ &\qquad [d(F(w,x),\,T(w,y)) + d(F(w,y),\,S(w,x))]/2\big\}\big). \end{aligned}$$

Therefore, F, S and T satisfy all conditions of Theorem 2.2. The conclusion of Theorem 2.4 follows from Theorem 2.2 immediately. □

From Theorem 2.4 we can obtain the following

COROLLARY 2.5. *Let* $T_i : \Omega \times X \to CB(X)$, $i = 1, 2, \ldots$, *be multifunctions such that*

(i) $T_i(w,\cdot)$, $T_j(w,\cdot)$ *are continuous for all* $w \in \Omega$, $i \ne j$;
(ii) $T_i(\cdot,x)$, $T_j(\cdot,x)$ *are measurable for all* $x \in X$, $i \ne j$;
(iii) *For all* i, j, $i \ne j$

$$\begin{aligned} H(T_i(w,x),\,T_j(w,y)) \le \Psi(&d(x,y),\,d(x,\,T_i(w,x)),\,d(y,\,T_j(w,y)), \\ &d(x,\,T_j(w,y)),\,d(y,\,T_i(w,x))), \end{aligned}$$

for all $w \in \Omega$ *and all* $x, y \in X$, *where* $\Psi(t_1,t_2,t_3,t_4,t_5) : \mathbb{R}^{+5} \to \mathbb{R}^+$ *satisfies condition* (Ψ).

Then the random fixed point sets $\{\xi : \Omega \to X : \xi(w) \in T_i(w,\xi(w))\}$, $i = 1, 2, \ldots$, *are nonempty, closed and equal to each other.*

REMARK. Our results are stochastic versions of the corresponding results of Chang [2].

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