

Part 13

The Shapes and Extents of Dark Halos

The shapes of simulated dark matter halos

Volker Springel

*Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Straße 1,
85740 Garching bei München, Germany*

Simon D. M. White

*Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Straße 1,
85740 Garching bei München, Germany*

Lars Hernquist

*Harvard-Smithsonian Center for Astrophysics, 60 Garden Street,
Cambridge, MA 02138, USA*

Abstract. Dark matter halos formed in CDM simulations are known to have triaxial shapes, with substantial distortions from spherical symmetry. However, conflicting claims have been made with respect to radial variations of halo shape, or trends with mass. We propose a new, more robust method to determine halo shapes based on the gravitational potential, thereby avoiding measurement uncertainties that may be the cause of these discrepancies. We find a strong preference towards prolate halo shapes, with mean minor-to-major axis-ratios close to ~ 0.5 . Spherical halos are generally very rare, but there is a trend for smaller mass systems to be less elongated. We also compare dark halo shapes found in hydrodynamical simulations that include radiative cooling, star formation, and feedback processes, with those measured in equivalent collisionless simulations. Dissipation makes dark halos substantially rounder at small radii, to the extent that their shape distribution matches that of elliptical galaxies.

1. Introduction

Already early numerical work on structure formation in CDM cosmologies established that the average shape of the mass distribution of dark matter halos is quite strongly aspherical (Frenk et al. 1988; Dubinski & Carlberg 1991; Warren et al. 1992), with typical minor-to-major axis ratios in the range $\langle c/a \rangle \sim 0.5 \pm 0.2$. Some of these studies also suggested that there appears to be a preference for prolate configurations, and that there may be systematic variations of typical halo shapes with mass, redshift, and cosmology.

However, while substantial effort has been invested in the study of spherically averaged properties of halos, the triaxial nature of dark matter halos received comparatively little attention, and has only recently been revisited again by a small number of studies (e.g. Thomas et al. 1998; Bullock 2002; Jing & Suto 2002). While these studies all agree on the triaxial nature of dark halos, they

in part differ in their quantitative predictions for axis-ratio distributions, and in particular, for trends of halo shapes with radius or halo mass. For example, while Thomas et al. (1998) found that high-mass clusters are rounder than low-mass ones, Bullock (2002) found the opposite trend. Or while Bullock (2002) sided with Frenk et al. (1988) in finding that inner parts of halos appear to be rounder than outer parts, Jing & Suto (2002) found just the reverse, in agreement with early results by Warren et al. (1992). We suspect that the origin of these discrepancies may largely lie in different methods used to define and measure halo shapes, as we will discuss in more detail below.

New observational data is beginning to tightly constrain the outer halo shapes, using for example X-ray contours in galaxy clusters, gravitational lensing, or dynamical tracers of galactic halo shapes, as for example provided in polar ring galaxies (Sackett et al. 1994). These observations provide interesting new constraints on the CDM theory, highlighting the need for more accurate numerical predictions for dark halo shapes. To this end, we here propose a new, more robust method to define the shapes of simulated halos, based on the gravitational potential instead of the mass distribution.

In the inner parts of halos, baryonic processes may significantly alter the shape of the dark matter distribution. This has been demonstrated explicitly with simulations of isolated toy-models of halos that include a dissipative component (Dubinski 1994), but has not yet been studied in self-consistent simulations of hierarchical galaxy formation. For the first time, we here compare the shapes of dark matter halos in large hydrodynamical simulations that include radiative cooling, star formation and feedback processes, with those found in equivalent dark-matter-only simulations.

2. Measurement of halo shapes

Essentially all measurements of halo shapes in CDM simulations carried out to date characterized the shape of the dark matter distribution using simple second moments of the mass distribution. This can be done by computing a moment of inertia tensor

$$\mathbf{I} = \sum_n m_n \vec{x}_n \vec{x}_n^T, \quad (1)$$

where the sum extends over dark matter particles of the halo. Diagonalizing \mathbf{I} provides eigenvalues I_{xx}, I_{yy}, I_{zz} , which can be used to define principal axis ratios, e.g. according to $c/a = \sqrt{I_{zz}/I_{xx}}$. We adopt the convention $a \geq b \geq c$.

If the true shape of a halo can be obtained from a spherically symmetric configuration by linear distortions in two orthogonal directions, then its isodensity contours form a set of self-similar triaxial ellipsoids. The above method will then be able to recover the corresponding principal axis ratios, provided a set of particles that is enclosed by one of the isodensity contours is selected for Eqn. (1). This points to the first practical complication of this method; since the shape of the triaxial isodensity contours is initially not known, an iterative method needs to be used to solve for them. A second complication arises from the distance-squared weighting of particles in the sum of Eqn. (1). For typical halos, this means that particles in the outermost included shell monopolize the sum, which may be undesirable. Some authors have

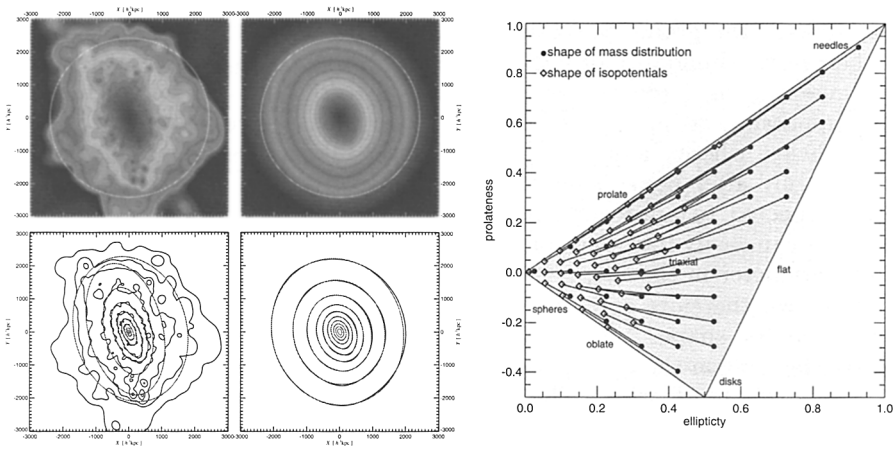


Figure 1. The dark matter density (top left) and gravitational potential (top middle) in a 2D-cut through the center of a cluster of galaxies formed in a Λ CDM cosmological simulation. The circle indicates the formal virial radius of the system. The two panels on the bottom left show isodensity and isopotential contours, respectively, fitted with ellipses. On the right, we show the relation between halo-shapes measured for mass distribution and gravitational potential, shown here in the prolateness-ellipticity plane for a distorted NFW-halo.

therefore changed the weighting of particles, for example by effectively projecting them onto the unit-sphere, others preselected certain particles close to fiducial isodensity contours (Jing & Suto 2002). Much of the quantitative differences between different authors' results presumably originates in the slightly different methodological approaches taken by each of the studies to define their shape measurements.

However, a common feature of all variants of the moment-of-inertia method is that they ignore the fact that the true shapes of dark matter isodensity contours are far from being triaxial ellipsoids. This is exemplified in Fig. 1, where the top left panel shows a two-dimensional cut through the dark matter density field of a galaxy cluster. For the most part, the complicated shape of the isodensity contours is not caused by noise in the sampling of the density field (this halo contains about 4×10^5 particles inside the virial radius), but rather reflects the grainy phase-space structure of hierarchically formed halos and their large abundance of dark matter subhalos, the most massive of them sometimes causing large "excursions" of isodensity contours. This makes it nearly impossible to directly fit ellipsoids to isodensity contours in the outer parts of halos, where the halo is generally least relaxed (see lower left panel of Fig. 1). Only after heavy smoothing of the density field, isodensity contours can be expected to approach something more akin to ellipsoids, but such a smoothing can easily introduce biases towards rounder shapes, and would be an ad-hoc parameter in the shape measurement procedure.

Here, we therefore propose to use the gravitational potential of a halo to measure its shape. Since the potential at every point depends in a collective fashion on the global mass distribution, it is rather insensitive to local fluctuations of the density. This is immediately apparent in the top middle panel of Fig. 1, where we show the potential in the same 2D-cut used for the adjacent density slice. The isopotential surfaces are much smoother, more robustly defined, and allow a direct fit with triaxial ellipsoids. Note that two-dimensional cuts through ellipsoids are always ellipses. We found that isodensity contours in 2D-cuts like the one shown in Fig. 1 are indeed very well fit by ellipses, with no evidence for systematic "disky" or "boxy" distortions. This opens up an economical way to fit full 3D isopotentials; instead of computing a fine 3D-grid of the potential that covers the whole halo, it is sufficient to do so in three orthogonal 2D planes through the halo center, and then to simultaneously fit the resulting isopotential contours with intersections of an ellipsoid with these planes.

An important point to keep in mind, however, is that the magnitudes of shape distortions of mass and potential are different. In general, any given shape distortion of the mass distribution will produce a weaker one in the potential. If the distortion of the halo mass distribution is self-similar with radius, the shapes of isopotentials will be most elongated in the center, but less so than that of the mass, and become rounder with increasing radial distance.

Given a model for the halo, it is however possible to "translate" between the shape of the potential, and that of the mass. In Fig. 1, we show the shapes of a family of NFW-halos (Navarro, Frenk & White 1996) that we distorted by varying amounts along two orthogonal axes. Each of the shapes of these halos is marked as a point in the two-dimensional space of prolateness and ellipticity,

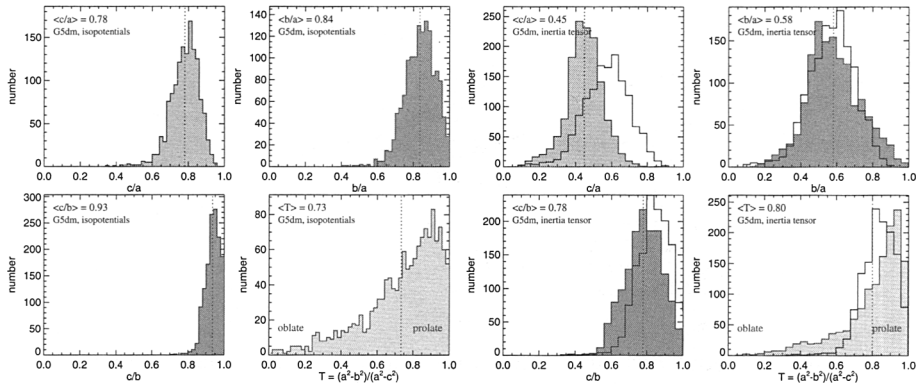


Figure 2. The four panels on the left give the distributions of axis-ratios of the gravitational potential of dark halos in simulation G5dm. The panels on the right show the same quantities, but for the shapes of the dark mass distribution. The latter was determined in two different ways; either by iteratively solving for the eigenvalues of a moment-of-inertia tensor for the particles inside an ellipsoid (filled histograms), or by mapping the measured shape of the isopotential to a predicted dark halo shape (thin histograms).

here defined as $p = (a^2 + c^2 - 2b^2)/(a^2 + b^2 + c^2)$ and $e = (a^2 - c^2)/(a^2 + b^2 + c^2)$. We have then numerically measured the shapes of isopotentials that enclose a volume equal to the “virial volume”, and also plotted these shapes as symbols, connecting them to their corresponding mass-shape with a line. The result is a mapping between shapes measured for isopotentials and for the mass, which under the assumption that real halos are reasonably close to distorted NFWs can be used to translate between these different shapes. We will later apply this as a consistency check between our shape measurements based on the potential and those based on the moment-of-inertia method.

3. Shape of dark halos in collisionless simulations

In this section, we analyze two collisionless simulations of the Λ CDM model, run in a periodic box of size $100 h^{-1}$ Mpc using two different mass resolutions corresponding to 216^3 and 324^3 particles. We refer to these models as ‘G4dm’ and ‘G5dm’, respectively, motivated by the names of the hydrodynamical simulations studied by Springel & Hernquist (2003). Their simulations also contained baryons, and followed additional physical processes of radiative cooling, star formation, and associated feedback processes. We will later use two of their simulations, ‘G4’ and ‘G5’, for comparing the shapes of dark halos formed in dissipative simulations with the shapes found in the corresponding dark matter-only simulations G4dm and G5dm. In addition, we will consider adiabatic versions of these two models where only ordinary hydrodynamics was included. All simulations were run with a modified version of the GADGET-code

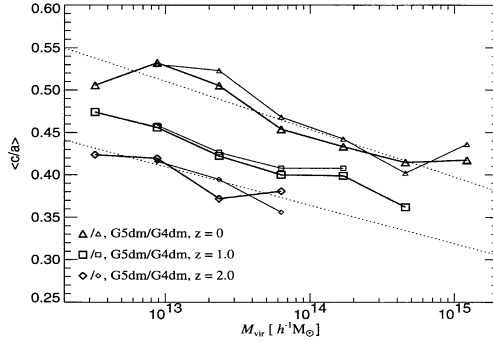


Figure 3. Redshift and mass dependence of the mean minor-to-major axis-ratio of the shape of the dark halo mass distribution, here measured using the moment-of-inertia method. We also compare results for simulations of different resolution.

(Springel, Yoshida & White 2001), and used the same large-scale modes in the realization of the initial conditions, such that halos can be directly compared on a halo-by-halo basis as a function of numerical resolution or included physics.

In Fig. 2, we show the distribution of axis-ratios for the 150 most massive halos in the G5dm simulation at $z = 0$. The four panels on the left give the distribution of minor-to-major, intermediate-to-major, and minor-to-intermediate axis-ratios for the shape of the halo potential, measured at a few radial distances per halo, logarithmically spread within the virial radius. We also show the triaxiality parameter $T = (a^2 - b^2)/(a^2 - c^2)$. While the shape of the potential is triaxial in general, the two minor axes are typically quite close in length. In fact, prolate distortions (“cigar-like”), where two axes are close in length and the third one is longer, are strongly preferred over oblate configurations (“pills”), where one axis is significantly shorter than the other two.

The magnitude of the shape distortions is quite large, as is perhaps best appreciated if one considers the corresponding distortions of the shape of the mass distribution. We show this in the right four panels of Fig. 2, measured in two different ways. The filled histograms show measurements of the mass-shape using the iterative moment-of-inertia method described earlier. The overplotted thin histograms are translations of the potential shape into mass distortions using the mapping provided by Fig. 1. While there is broad consistency between these two different approaches, there is also a small bias of the results based on the moment-of-inertia tensor towards more elongated shapes. We find that this is likely the result of halo substructure, which can “pull away” the iteratively found solution to more elongated shapes. The typical shape distortions in the minor-to-major axis-ratio are of order $\sim 0.5 \pm 0.2$, consistent with earlier studies. Note that spherical halos essentially do not exist – they are very rare, and more an exception than the rule if found.

In Fig. 3, we compare mean axis-ratios as a function of halo mass and epoch. We also compare results obtained both for the lower and higher resolution runs,

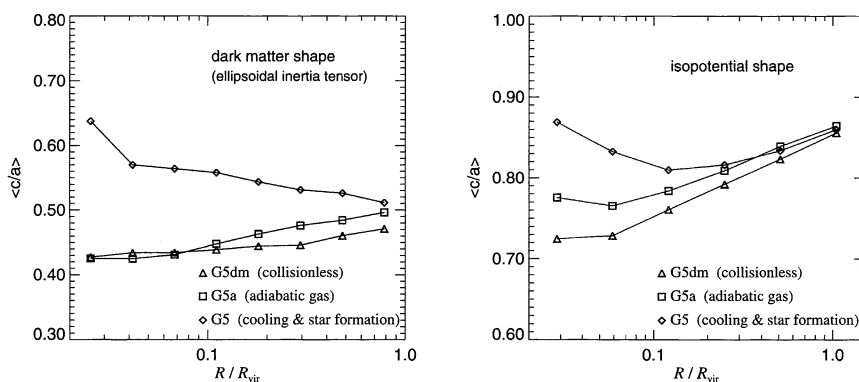


Figure 4. Radial dependence of the halo shape. We compare results for purely collisionless simulations with those of hydrodynamic simulations that either follow only adiabatic gas physics, or also model radiative cooling and star formation. The shape of the mass distribution is shown on the left, that of the potential on the right.

G4dm and G5dm, highlighting that all the results we discuss here are quite robust with respect to numerical resolution. Interestingly, more massive halos tend to be more elongated, perhaps as a consequence of their more recent average formation times. Also, there is a trend with redshift in the sense that halos at higher redshift tend to be more elongated. Our results appear to agree well with the empirical scaling $\langle c/a \rangle \sim M^{-0.05} (a+z)^{-0.2}$ given by (Bullock 2002, dotted), although our shape distributions do not quite match.

4. The influence of baryons on dark halo shapes

Our analysis thus far focused on halos formed in collisionless simulations that only followed dark matter. However, in more realistic Λ CDM models about $\sim 15\%$ of the matter should consist of ordinary baryons, interacting by collisional hydrodynamics, and even more importantly, having the ability to lose internal energy radiatively. This allows them to collapse far beyond the dark matter, forming dense baryonic structures at the centers of halos. It is clear that this can change the total gravitational potential strongly, not only by means of the centrally concentrated baryonic mass, but also by changing the dark matter mass distribution itself, for example by scattering dark matter particle orbits that pass through the center, causing changes in their orbital mix.

In Fig. 4, we show the average shape of isopotentials and of the dark mass distribution as a function of radius. We compare results for the dark matter-only simulations with those from simulations that include gas, both for the adiabatic case, and for the case where dissipation and star formation are included. Note that the potential here refers to the dark mass component only. For the dissipationless simulations, the shape of the dark mass of halos is nearly constant with radius, with a weak trend to become slightly more elongated towards the center. Adding a contribution of adiabatic gas makes the shapes

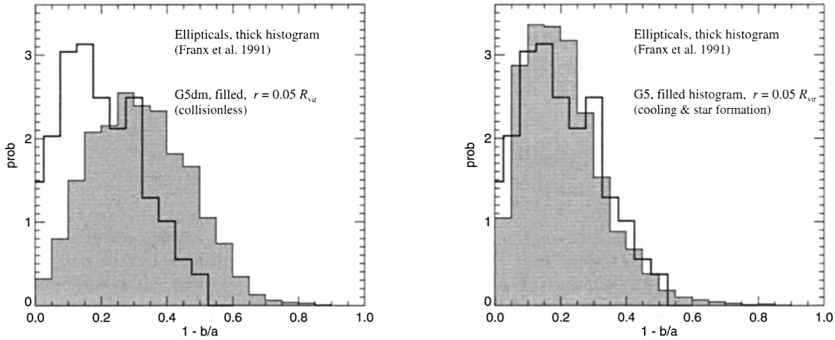


Figure 5. Ellipticities of the 2D-projected dark mass distribution compared to the ellipticities of isophotes of elliptical galaxies. We show results both for collisionless simulations (left panel) and for dissipative hydrodynamical simulations (right panel). In both cases, the shape of the mass distribution was inferred from measurements of the potential.

slightly rounder, an effect which can be attributed to the presence of isotropic gas pressure. However, dissipative simulations clearly have a much stronger effect; they lead to a rounding of halos in the centers. This effect is very prominent both in the mass distribution and the isopotentials.

It is interesting to relate the central shape of dark matter halos to the shapes of elliptical galaxies. The latter are known to be fairly round, although not exactly so. The 2D-shape distribution of isophotal shapes as measured by Ryden (1992) is shown as the thin histogram in Fig. 5, and compared with the central shapes of 2D projections of the dark mass distribution of halos in G5dm. There is a substantial mismatch between the shapes of collisionless dark halos and the stellar systems of elliptical galaxies (see also Warren et al. 1998). This is not necessarily a problem, as it is in principle possible to even embed a spherical stellar system in a triaxial dark matter, but it may still be hard to arrange this in galaxy formation theory. However, if we make the same comparison using the dark matter shapes found in dissipative simulations, a stunning agreement between the distributions is obtained. It is presently unclear whether this may be of help to constrain galaxy formation theory, but it certainly suggests that there is no immediate conflict between highly elongated dark matter halo shapes found in purely collisionless CDM simulations and the comparatively round stellar systems of elliptical galaxies. Also, this result highlights the importance of baryonic processes for the central properties of dark matter halos.

5. Conclusions

We have investigated the shapes of dark matter halos formed in cosmological simulations of the Λ CDM model, using a new, more robust method based on the gravitational potential. We find that isopotential surfaces appear to be well fit by triaxial ellipsoids. In agreement with previous studies, we find that dark

halos have in general triaxial shapes, but they show a clear preference for prolate shape distortions. The dark mass component has typical minor-to-major axis ratios of $\sim 0.5 \pm 0.2$. Nearly spherical halos are rare. For collisionless simulations, the shape of the mass becomes slightly more elongated towards halos centers, a trend that is however reversed if a dissipative baryonic component is included. Halos of given mass are rounder at lower redshift, while at a fixed epoch, more massive halos tend to be more elongated.

A dissipative baryonic component significantly alters the shape of the central dark matter distribution, making it rounder. In fact, the distribution of central shapes of the dark matter then matches that of the 2D ellipticity distribution of the light in elliptical galaxies very well, while that obtained for pure dark-matter-only simulations has much too elongated shapes on average. This highlights the importance of baryonic physics for the inner structure of halos, and cautions against over-interpreting the current problems of the CDM theory on small scales as being deadly for the theory, because these baryonic effects have been more often neglected than included in the computations done to date.

References

- Bullock, J. S. 2002, in *The Shapes of Galaxies and their Dark Matter Halos*, ed. P. Natarajan (Singapore: World Scientific), p.109
- Dubinski, J. 1994, *ApJ*, 431, 617
- Dubinski, J., Carlberg, R. 1991, *ApJ*, 378, 496
- Frenk, C. S., White, S. D. M., Davis, M., Efstathiou, G. 1988, *ApJ*, 327, 507
- Jing, Y. P., Suto, Y. 2002, *ApJ*, 574, 538
- Navarro, J. F., Frenk C. S., White S. D. M. 1996, *ApJ*, 462, 563
- Ryden, B. S. 1992, *ApJ*, 396, 445
- Sackett, P. D., Rix, H.-W., Jarvis, B. J., Freeman K. C. 1994, *ApJ*, 436, 629
- Springel, V., Hernquist, L. 2003, *MNRAS*, 339, 289
- Springel, V., Yoshida, N., White, S. D. M. 2001, *New Astronomy*, 6, 79
- Thomas, P. A., et al. 1998, *MNRAS*, 296, 1061
- Warren, M. S., Quinn, P. J., Salmon, J. K., Zurek, W. H. 1992, *ApJ*, 399, 405