STOCHASTIC STELLAR ORBITS AND THE SHAPES OF ELLIPTICAL GALAXIES

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Abstract Using Melnikov's method to study the appearance of stochastic orbits in perturbed Stäckel potentials, a correlation is found between the observed shapes of elliptical galaxies and the occurence of mainly regular orbits. Some other potential perturbations giving rise to large regions of stochastic orbits, on the other hand, appear to be inconsistent with observations.

Motivation: Most orbits in potentials near those of elliptical galaxies appear to be *regular* box and tube orbits that respect three integrals of motion (Schwarzschild 1979). A simple argument why stochastic orbits cannot occur in large numbers is that they would make the density contours rounder than the potential contours, while Poisson's equation requires the opposite in cases of interest for elliptical galaxies.

There is a family of completely integrable potentials (Stäckel potentials) in which all orbits are regular and belong to the four orbital families that also comprise most orbits in Schwarzschild's model (de Zeeuw 1985). While the proximity of these integrable potentials must clearly play a role for the dynamics of elliptical galaxies (i.e. for the apparent regularity of their phase-spaces, by the KAM-theorem), the fact that the overwhelming majority of potentials near those of elliptical galaxies are non-integrable implies that these galaxies are very unlikely to have an exactly separable potential.

Hence it is important to study perturbations of Stäckel potentials. Some questions of interest are: (i) Is the regular orbital structure of the Stäckel potentials stable to perturbations, i.e. is the region of stochastic orbits thess introduce small? (ii) For which types of perturbations - if not for all - is this true, and what can be learned from this for elliptical galaxies?

Method: In integrable potentials which support several orbit families, small perturbations generally destroy the non-classical integrals of motion because the surfaces on which the different families of orbits touch (the so-called homoclinic surfaces) become infinitely tightly wrapped by the perturbation. In this way, layers of stochastic orbits are introduced in the vicinity of these surfaces. This phenomenon may be shown to occur by Melnikov's method, in which one follows the wrapping and intersection of the perturbed homoclinic surfaces to first order in the perturbation. A new, canonically invariant formulation of Melnikov's method has been derived, which makes application to three-dimensional galaxy potentials

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possible. In this case, the homoclinic surface is, like any other torus, a three-dimensional phase-space surface, and generally contains a two-parameter family of orbits asymptotic to a one-parameter family of unstable quasi-periodic orbits. Then one has to evaluate a 2-vector of Melnikov integrals along, and as a function of, the unperturbed homoclinic orbits. With the aid of these integrals one may estimate the importance of the resulting stochastic layer, i.e. its width in phase-space. For details see Gerhard (1985, 1986).

Applications to Stäckel potentials: We consider perturbations of Stäckel potentials, which are separable in ellipsoidal coordinates λ, μ, ν. The commonly studied cases correspond to inhomogeneous mass models with homogeneous cores; they then contain the following unstable periodic orbits: z-axial and closed y-loop orbits (for energy $E > E_1$) and y-axial orbits ($E > E_2 > E_1$) (de Zeeuw 1985). Only the z-axial orbits are doubly unstable; for them the theory has to be slightly modified. The y-axial and closed y-loop orbits sire one-dimensional families of unstable quasi-periodic orbits on two-dimensional tori, and the Melnikov integrals must be evaluated as functions of the two-parameter families of orbits asymptotic to them. To this end the equations of motion must be solved and the integrals evaluated alongside. Since this is a difficult numerical problem the computations have sofar been restricted to two-dimensional potentials (describing motion in the equatorial plane of a triaxial galaxy). Then the only unstable periodic orbits are the y-axial orbits, and the Melnikov vector reduces to a single integral. The results of these calculations (Gerhard 1985) are summarized next.

Results: For a particular planar Stäckel potential with density $\rho \propto r^{-2}$ at large radii, several classes of perturbations were studied. The main result is that those perturbations that are consistent with observations of early-type galaxies approximately preserve the regular orbital structure of the integrable potential and lead to only small stochastic layers. Specifically, this was found for (i) $\cos m\phi$ perturbations with $m = 0, 2, 1, 4$ (axisymmetric, elliptic, lopsided, box-shaped) in order of the importance of the resulting stochastic layer, (ii) potential perturbations with moderate ellipticity gradients, and (iii) small figure rotation.

In contrast, other perturbations lead to large stochastic regions and a rapid breakdown of the regular structure of phase-space; e.g. $\cos m\phi$ perturbations with $m = 3$ or ≥ 5. These results suggest that (i) the integrable Stäckel potentials are sufficiently close to galaxy potentials in the studied energy range that the latter may be considered as perturbations of the former, and (ii) that the triaxial-symmetric shapes of ellipticals are determined by the requirement that self-consistent equilibrium models exist rather than by special initial conditions.

References

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