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Erratum to "Full and reduced C^*-coactions". Math. Proc. Camb. Phil. Soc. 116 (1994), 435–450.

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Proposition 2·5 of [**5**] states that a full coaction of a locally compact group on a C^*-algebra is nondegenerate if and only if its normalisation is. Unfortunately, the proof there only addresses the forward implication, and we have not been able to find a proof of the opposite implication. This issue is important because the theory of crossed-product duality for coactions requires implicitly that the coactions involved be nondegenerate. Moreover, each type of coaction — full, reduced, normal, maximal, and (most recently) exotic — has its own distinctive properties with respect to duality, making it crucial to be able to convert from one to the other without losing nondegeneracy.

While it is generally believed that all coactions of all types are nondegenerate, in this note we summarise what little is actually known about nondegeneracy of C^*-coactions. We also hope to caution the reader that the error in [**5**] has propagated widely, and sometimes invisibly, in the literature. For example, a *normal* coaction is nondegenerate if and only if the associated *reduced* coaction is nondegenerate ([**5**, proposition 3·3], which is independent of proposition 2·5). So since reduced coactions of *discrete* groups are automatically nondegenerate ([**1**, corollaire 7·15]), it is often mistakenly assumed (as in [**6**] and [**2**]) that every *full* coaction of a discrete group is also nondegenerate. An equivalent assumption (as in [**3**, section 2·4]) is that every C^*-algebra that carries a coaction of a discrete group is the closed span of its spectral subspaces.

An overview of the definitions and basic results concerning full coactions, their normalizations, and their reductions can be found in [**2**, appendix A].

THEOREM 1. *Let* (A, δ) *be a full coaction of a locally compact group* G. *Then among the following conditions, we have the implications* (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3):

(1) (A, δ) *is nondegenerate;*
(2) *the normalization* (A^n, δ^n) *is nondegenerate;*
(3) *the reduction* (A^r, δ^r) *is nondegenerate.*

LEMMA 2. *Let* (A, δ) *and* (B, ε) *be full coactions of a locally compact group* G. *If* (A, δ) *is nondegenerate and there exists a* δ - ε *equivariant surjection* $\varphi : A \to B$, *then* (B, ε) *is also nondegenerate.*

Proof. By equivariance, $\varepsilon(B) = \varepsilon(\varphi(A)) = (\varphi \otimes \mathrm{id})(\delta(A))$, so

$$\begin{aligned}\overline{\operatorname{span}}\, \varepsilon(B)(1 \otimes C^*(G)) &= \overline{\operatorname{span}}\, (\varphi \otimes \mathrm{id})(\delta(A))(1 \otimes C^*(G)) \\ &= \overline{\operatorname{span}}\, (\varphi \otimes \mathrm{id})\big(\delta(A)(1 \otimes C^*(G))\big) \\ &= (\varphi \otimes \mathrm{id})\big(\overline{\operatorname{span}}\, \delta(A)(1 \otimes C^*(G))\big).\end{aligned}$$

Since (A, δ) is nondegenerate, $\overline{\text{span}}\ \delta(A)(1 \otimes C^*(G) = A \otimes C^*(G)$, so

$$\overline{\text{span}}\ \varepsilon(B)(1 \otimes C^*(G)) = (\varphi \otimes \text{id})(A \otimes C^*(G)) = B \otimes C^*(G).$$

Thus (B, ε) is nondegenerate as well.

Proof of Theorem 1. Since A^n is by definition a quotient of A, and δ^n is defined so that the quotient map is δ - δ^n equivariant, (1) $\Rightarrow$ (2) is immediate from Lemma 2.

By [**5**, definition 3·5], the reduction (A^r, δ^r) of an arbitrary coaction (A, δ) coincides with the reduction $((A^n)^r, (\delta^n)^r)$ of the normalization (A^n, δ^n). By [**5**, proposition 3·3], (A^n, δ^n) is nondegenerate if and only if $((A^n)^r, (\delta^n)^r)$ is, and (2) $\Leftrightarrow$ (3) follows.

We reiterate that it is still an open question whether or not (2) implies (1). We are grateful to Iain Raeburn for pointing out the error in the original proof, and to Alcides Buss for drawing our attention to a problem with our initial attempt to correct it.

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