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A CELLULAR CONSTRAINT IN SUPERCOMPACT HAUSDORFF SPACES

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1. Introduction. In this paper we prove a cardinal inequality for supercompact Hausdorff spaces which gives insight into the cellular structure of such spaces and yields new examples of compact Hausdorff non-supercompact spaces.

The notion of supercompactness was introduced by J. de Groot in [**6**]. A family of sets is *linked* if every two members have non-empty intersection. A family of sets is *binary* if every linked subcollection has non-empty intersection. A space X is *supercompact* if X possesses a binary closed subbase. By Alexander's lemma, a supercompact space is compact. Many compact spaces are supercompact, for example; all compact ordered spaces, all compact metric spaces [**11**] and [**3**] and all compact tree-like spaces [**2**] and [**9**]. Moreover, supercompactness is a productive property. Thus, all Tychonov cubes I^κ and all Cantor cubes 2^κ are supercompact. Also, every space has many supercompact extensions, known as *superextensions*—see A. Verbeek's book [**12**].

J. de Groot had asked whether all compact Hausdorff spaces were supercompact (at the times, A. Verbeek had an example of a compact T_1 non-supercompact space). It is now known that there are numerous compact Hausdorff non-supercompact spaces. The author [**1**] has shown that for X non-pseudocompact, βX is non-supercompact. The following two results were subsequently established in [**4**].

[A] (J. van Mill). *Let X be a supercompact Hausdorff space. If Y is a continuous image of a closed neighbourhood retract of X, then for all countably infinite subsets K of Y, all but countably many cluster points of K are the limit of some non-trivial sequence in Y (not necessarily in K).*

[B] (E. van Douwen). *Let X be a supercompact Hausdorff space. Then, no continuous image of a closed neighbourhood retract of X is homeomorphic to any compactification of a κ-Cantor tree* (cf. M. E. Rudin [**10**]) *where $\omega < \kappa \leqq c$.*

J. van Mill used [A] to give a different proof that βX supercompact implies X pseudocompact, furthermore he showed that an infinite supercompact Hausdorff space has a non-trivial convergent sequence. E. van Douwen used [B] to construct a compact Hausdorff non-supercompact space of cardinality ω_1 and a compact Hausdorff non-supercompact first countable space of cardinality c.

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All spaces considered are completely regular, infinite and Hausdorff. N denotes the countably infinite discrete space and βX denotes the Čech-Stone compactification of X. The *weight* of a space, $w(X)$, is the least cardinal of an open base for X. The *cellularity* of a space X, $c(X)$, is the supremum of $|\mathscr{G}|$, where $\mathscr{G}$ is a disjoint collection of open sets of X. The *density* of a space X, $d(X)$, is the least cardinal of a dense subspace of X. The *spread* of a space X, $s(X)$, is the supremum of $|D|$, where D is a discrete subspace of X. For a cardinal κ, κ^+ denotes the smallest cardinal larger than κ. ω is the first infinite ordinal, ω_1 is the first uncountable ordinal and c is the cardinality of the continuum.

2. Cellularity in supercompact spaces.

2.1 LEMMA. *Let X be a subspace of weight κ of a space Y. Let $\{V_\alpha: \alpha < \kappa^+\}$ $\{U_\alpha : \alpha < \kappa^+\}$ be open sets of Y such that*

(1) *For $\alpha < \beta < \kappa^+$, $\mathrm{Cl}_Y V_\alpha \cap \mathrm{Cl}_Y V_\beta$ is a compact set of X.*

(2) *For $\alpha < \kappa^+$, $\mathrm{Cl}_Y V_\alpha \subseteq U_\beta$.*

Then, there exists $\{U_n: n \geqq 0\} \subseteq \{U_\alpha: \alpha < \kappa^+\}$ (relabelled for convenience) such that sup $\{m$: *U_0 contains all 2-fold intersections of m $\mathrm{Cl}_Y V_n$'s, $n \geqq 1\} = \omega$.*

Proof. Let $\mathscr{B}$ be an open base for X, closed under finite unions, of cardinality κ. For $\alpha < \beta < \kappa^+$, choose $B_{\alpha\beta} \in \mathscr{B}$ such that

$$\mathrm{Cl}_Y V_\alpha \cap \mathrm{Cl}_Y V_\beta \subseteq B_{\alpha\beta} \subseteq U_\alpha \cap U_\beta.$$

For $2 \leqq m < \omega$, define $D_m = \{\alpha < \kappa^+: U_\alpha$ contains all 2-fold intersections of m other $\mathrm{Cl}_Y V_\beta$'s$\}$. It suffices to show that for each $m \geqq 2$, $|\{\alpha < \kappa^+: \alpha \notin D_m\}| \leqq \kappa$. For then, just choose $0 \in \bigcap \{D_m: m \geqq 2\}$ and the corresponding finite collections to make up the U_n's for $n \geqq 1$. To this end, the proof for D_3 is given, the general case is identical only longer.

Assume $F_0 = \{\alpha < \kappa^+: \alpha \notin D_3\}$ has cardinality κ^+. Choose $\alpha \in F_0$ and consider $\{B_{\alpha\beta}: \beta \in F_0 - \{\alpha\}\}$. There exists $F_1 \subseteq F_0 - \{\alpha\}$, $|F_1| = \kappa^+$, such that for distinct β and γ in F_1, $B_{\alpha\beta} = B_{\alpha\gamma}$. Choose $\beta \in F_1$ and consider $\{B_{\beta\gamma}: \gamma \in F_1 - \{\beta\}\}$. There exists $F_2 \subseteq F_1 - \{\beta\}$, $|F_2| = \kappa^+$, such that for distinct γ and δ in F_2, $B_{\beta\gamma} = B_{\beta\delta}$. Choose distinct γ and δ from F_2. Then

$$(\mathrm{Cl}_Y V_\alpha \cap \mathrm{Cl}_Y V_\beta) \cup (\mathrm{Cl}_Y V_\alpha \cap \mathrm{Cl}_Y V_\gamma) \cup (\mathrm{Cl}_Y V_\beta \cap \mathrm{Cl}_Y V_\gamma) \subseteq B_{\alpha\beta} \cap B_{\alpha\gamma} \cap B_{\beta\gamma} \subseteq U_\delta.$$

Hence $\delta \in D_3$, a contradiction. Consequently, $|F_0| \leqq \kappa$.

Remark. At this point, let us note that if $\mathscr{S}$ is a binary closed subbase for X, then without loss of generality, we may assume that $\mathscr{S}$ is closed under finite intersections. Also, a collection $\mathscr{S}$, closed under finite intersections, of closed subsets of a compact space, is a closed subbase if and only if for each closed set

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C contained in an open set U, there exists a finite subcollection $\mathscr{F}$ of $\mathscr{S}$ such that $C \subseteq \bigcup \mathscr{F} \subseteq U$.

Here is our main result.

2.2. THEOREM. *Let X be a supercompact Hausdorff space and let f be a continuous mapping from an open set O of X onto a compact Hausdorff space Y. If there exists a compact subset K of O such that $f(K) = Y$, then for all dense subsets D of Y, $c(Y - D) \leqq w(D)$.*

Proof. Let $\mathscr{S}$ be a binary closed subbase for X that is closed under finite intersections. Assume $w(D) = \kappa$ and choose a dense subset E of D with $|E| \leqq \kappa$. Striving for a contradiction, assume $c(Y - D) > \kappa$. Let $\{C_\alpha: \alpha < \kappa^+\}$ be a family of disjoint non-empty open subsets of $Y - D$. Pick $P_\alpha \in C_\alpha$ and since Y is regular, choose an open set W_α of Y such that $P_\alpha \in W_\alpha$ and $\mathrm{Cl}_Y W_\alpha \cap (Y - D) \subseteq C_\alpha$. Again using the regularity of Y, find $E_\alpha \subseteq E$ and open sets V_α and U_α of Y such that

$$P_\alpha \in \mathrm{Cl}_Y E_\alpha \subseteq V_\alpha \subseteq \mathrm{Cl}_Y V_\alpha \subseteq U_\alpha \subseteq \mathrm{Cl}_Y U_\alpha \subseteq W_\alpha.$$

For each $e \in E$, choose an $\bar{e}$ in K such that $f(\bar{e}) = e$. Choose

$$q_\alpha \in \mathrm{Cl}_K\{\bar{e}: e \in E_\alpha\} - (f|K)^{-1}(D).$$

This is possible because $f|K$ is a closed map onto Y and $(\mathrm{Cl}_Y E_\alpha - D) \neq \emptyset$. By continuity of $f|K$,

$$q_\alpha \in \mathrm{Cl}_K\{\bar{e}: e \in E_\alpha\} \subseteq (f|K)^{-1}(\mathrm{Cl}_Y E_\alpha) \subseteq f^{-1}(V_\alpha).$$

Notice that $\mathrm{Cl}_K\{\bar{e}: e \in E_\alpha\}$ is a closed set of X contained in the open set $f^{-1}(V_\alpha)$ of X. Using the fact that $\mathscr{S}$ is a subbase for X, get $S_\alpha \in \mathscr{S}$ such that $S_\alpha \subseteq f^{-1}(V_\alpha)$ and $q_\alpha \in \mathrm{Cl}_K[\{\bar{e}: e \in E_\alpha\} \cap S_\alpha]$. Let $F_\alpha = \{e \in E_\alpha: \bar{e} \in S_\alpha\}$. Therefore, $f(q_\alpha) \in \mathrm{Cl}_Y F_\alpha \cap (Y - D)$. Since $|E| \leqq \kappa$ and there are κ^+ α's, without loss of generality we may assume there exists $x \in \bigcap_{\alpha<\kappa^+} F_\alpha$ and thus $\bar{x} \in \bigcap_{\alpha<\kappa^+} S_\alpha$.

For $\alpha < \beta < \kappa^+$, $\mathrm{Cl}_Y V_\alpha \cap \mathrm{Cl}_Y V_\beta$ is a compact subset of D, for if not, then it would meet $Y - D$ which is impossible since $C_\alpha \cap C_\beta = \emptyset$. It follows from Lemma 2.1. that there exists $\{U_n: n \geqq 0\} \subseteq \{U_\alpha: \alpha < \kappa^+\}$ such that sup $\{m: U_0$ contains all 2-fold intersections of m $\mathrm{Cl}_Y V_n$'s, $n \geqq 1\} = \omega$. Because

$$\mathrm{Cl}_Y W_0 \cap \bigcup_{n \geqq 1} [\mathrm{Cl}_Y F_n \cap (Y - D)] = \emptyset,$$

there exists U open in Y such that

$$U \cap \mathrm{Cl}_Y W_0 = \emptyset \quad \text{and} \quad \bigcup_{n \geqq 1} [\mathrm{Cl}_Y F_n \cup (Y - D)] \subseteq U.$$

Notice that

$$\mathrm{Cl}_Y\{f(q_n): n \geqq 1\} \subseteq \mathrm{Cl}_Y U \subseteq \mathrm{Cl}_Y(Y - \mathrm{Cl}_Y W_0) \subseteq Y - W_0 \subseteq Y - \mathrm{Cl}_Y U_0.$$

Therefore,

$$\mathrm{Cl}_K\{q_n: n \geqq 1\} \subseteq (f|K)^{-1}\,\mathrm{Cl}_Y\{f(q_n): n \geqq 1\} \subseteq f^{-1}(Y - \mathrm{Cl}_Y U_0).$$

Hence, there exists $\{T_i: 1 \leqq i \leqq k\} \subseteq \mathscr{S}$ with

$$\mathrm{Cl}_K\{q_n: n \geqq 1\} \subseteq \bigcup_{i=1}^{k} T_i \subseteq f^{-1}(Y - \mathrm{Cl}_Y U_0).$$

Choose $k + 1$ $\mathrm{Cl}_Y V_n$'s such that U_0 contains all 2-fold intersections of the $\mathrm{Cl}_Y V_n$'s. By the pigeon hole principle, there exist $n \neq m$ and an $1 \leqq i \leqq k$ such that $\{q_n, q_m\} \subseteq T_i$ and $\mathrm{Cl}_Y V_n \cap \mathrm{Cl}_Y V_m \subseteq U_0$.

The Contradiction. $\{S_n, S_m, T_i\}$ is linked yet has empty intersection. $\bar{x} \in S_n \cap S_m$, $q_n \in S_n \cap T_i$ and $q_m \in S_m \cap T_i$. However,

$$S_n \cap S_m \cap T_i \subseteq f^{-1}(V_n) \cap f^{-1}(V_m) \cap f^{-1}(Y - \mathrm{Cl}_Y U_0)$$
$$\subseteq f^{-1}[V_n \cap V_m \cap (Y - \mathrm{Cl}_Y U_0)] = \emptyset.$$

In particular, if Y is a closed neighbourhood retract of X, a continuous image of a closed neighbourhood retract of X, a closed neighbourhood retract of a continuous image of a closed neighbourhood retract of X, etc., there exist a mapping f, an open set O and a compact set K such that the hypotheses of the theorem hold. Notice that [B] is a consequence of Theorem 2.2, since any compactification X of a κ-Cantor tree ($\omega < \kappa \leqq c$) is a compactification of N with $c(X - N) > \omega$ while $w(N) = \omega$. Both [A] and Theorem 2.2. show that for X non-pseudocompact, neither βX nor $\beta X - X$ (when X is locally compact) are supercompact, since it is easy to show that in this case both βX and $\beta X - X$ contain a neighbourhood retract homeomorphic to βN and $c(\beta N - N) > w(N)$. We now give two examples to illustrate the sharpness of the inequality.

2.3. *Example.* In the theorem, cellularity cannot be replaced by spread. J. van Mill [**8**] has shown (in particular) that there exists a supercompactification γN of N such that $\gamma N - N = 2^c$. Since 2^c contains a copy of βN, $s(2^c) = c$. Hence, $s(\gamma N - N) \nleq w(N)$.

2.4. *Example.* In the theorem, weight cannot be replaced by density. Let βN be a copy of βN in 2^c. Let $D = N \cup (2^c - \beta N)$. Then D is open in 2^c and therefore separable, i.e., $d(D) = \omega$. However, $c(2^c - D) = c(\beta N - N) = c$. Hence, $c(2^c - D) \nleq d(D)$.

Our concern now is to display numerous examples of compact Hausdorff non-supercompact spaces which are not covered by [A] or [B]. They will all be first countable compactifications of N, γN, such that $\gamma N - N$ is connected and locally connected. We use the following result of E. van Douwen and T. C. Pryzymusínski [**5**].

[C] *Let $\mathscr{B}$ be an open base for a compact Hausdorff space Y with $|\mathscr{B}| \leqq c$, $Y \in \mathscr{B}$ and $\emptyset \notin \mathscr{B}$. Assume there is a function $h: \mathscr{B} \to \mathscr{P}(N)$ such that*

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(0) $h(Y) = N$;

(1) $h(B)$ *is infinite for* $B \in \mathscr{B}$;

(2) *if* $A, B \in \mathscr{B}$ *are disjoint*, $h(A) \cap h(B)$ *is finite;*

(3) *if* $A \in \mathscr{B}$ *and if* $\mathscr{F} \subseteq \mathscr{B}$ *is finite, and if* $A \subseteq \bigcup \mathscr{F}$, *then* $h(A) - \bigcup \{h(B): B \in \mathscr{F}\}$ *is finite.*

Then there is a Hausdorff compactification γN *of* N *such that* $\gamma N - N$ *and* Y *are homeomorphic. Furthermore* γN *is first countable if* Y *is first countable.*

Indeed, as the authors show, the family $\{B \cup (h(B) - F): B \in \mathscr{B}, F \subseteq N$ finite$\} \cup \{\{n\}: n \in N\}$ is an open base for the required topology on $\gamma N = N \dot{\cup} Y$.

We make use of the following general construction. Let $f: X \to Y$ where X and Y are T_1 spaces. Define XfY to be the space with underlying set $X \times Y$ and topology as follows: Basic open neighbourhoods of (x, y) where $y \neq f(x)$ are of the form $\{x\} \times (U_y - \{f(x)\})$ where U_y is an open neighbourhood of y in Y. Basic open neighbourhoods of $(x, f(x))$ are of the form $[(U_x - \{x\}) \times Y] \cup [\{x\} \times U_{f(x)}]$ where $U_x(U_{f(x)})$ is an open neighbourhood of $x(f(x))$ in $X(Y)$. XfY is a T_1 space. Properties that XfY inherits from both X and Y include compactness, Hausdorffness, connectedness, local connectedness and first countability. Furthermore, the following cardinal equalities hold if $|Y| > 1$:

$$c(XfY) = |X| \cdot c(Y) \quad \text{and} \quad w(XfY) = |X| \cdot w(X) \cdot w(Y).$$

Recall that a space X is *sequentially separable* if it has a countable dense subset D such that every point of X is the limit of some convergent sequence of points from D.

2.4. PROPOSITION. *Let* X *be an infinite sequentially separable compact Hausdorff space with no isolated points and* Y *be a separable compact Hausdorff space with no isolated points. Then, there exists a Hausdorff compactification of* N, γN, *such that* $\gamma N - N$ *and* XfY *are homeomorphic* (*for any* f).

Proof. Let $\mathscr{U}$ be a base for X with $|\mathscr{U}| \leqq c$, $\emptyset \notin \mathscr{U}$ and $\mathscr{V}$ be a base for Y with $|\mathscr{V}| \leqq c$, $\emptyset \notin \mathscr{V}$. Since X is sequentially separable, note that $|X| \leqq c$. Define

$$\mathscr{B}_1 = \{\{x\} \times (V - \{f(x)\}): x \in X, V \in \mathscr{V}\} \quad \text{and}$$
$$\mathscr{B}_2 = \{[(U - \{x\}) \times Y] \cup [\{x\} \times V]: x \in U \in \mathscr{U} \quad \text{and} \quad f(x) \in V \in \mathscr{V}\}.$$

Let $\mathscr{B} = \mathscr{B}_1 \cup \mathscr{B}_2 \cup \{XfY\}$. $\mathscr{B}$ is a base for the open sets of XfY with $|\mathscr{B}| \leqq c$. Let D be a countable dense subset of X rendering X sequentially separable and E be a countable dense subset of Y. Let $g: N \to D$ be a bijection. For each $x \in X$ choose a non-trivial sequence $\{d_k^x: k < \omega\} \subseteq D$ such that (d_k^x) converges to x. Consider $N_x = \{n: g(n) \in \{d_k^x: k < \omega\}\}$. $\{N_x: x \in X\}$ is an almost disjoint family. For each $x \in X$, let $g_x: N_x \to E$ be a bijection.

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For $x \in X$ and $V \in \mathscr{V}$ define

$$h(\{x\} \times (V - \{f(x)\})) = \{n: n \in N_x \text{ and } g_x(n) \in E \cap V\}.$$

For $x \in U \in \mathscr{U}$ and $f(x) \in V \in \mathscr{V}$ define

$$h([(U - \{x\}) \times Y] \cup [\{x\} \times V]) = \{n: n \notin N_x \text{ and } g(n) \in D \cap U\} \cup \{n: n \in N_x \text{ and } g_x(n) \in E \cap V\}.$$

Define $h(XfY) = N$. It is now a straightforward exercise that $h: \mathscr{B} \rightarrow \mathscr{P}(N)$ satisfies all the hypotheses of [C] and thus there exists a compactification of N, γN, such that $\gamma N - N$ and XfY are homeomorphic.

Hence if X and Y are infinite Peano spaces (compact, connected, locally connected metrizable spaces) and $f: X \rightarrow Y$ is an arbitrary correspondence, then γN where $\gamma N - N = XfY$ is an example of a compact Hausdorff first countable non-supercompact (since $c(XfY) = c$) space.

We remark that two further examples can be found in the theory of lexicographic order. Consider the long line and the lexicographic ordered square (cf. S. Willard [**14**]). Both have cellularity $> \omega$ and both are remainders of N in some compactification. These compactifications are not supercompact.

Supercompactness is not preserved in closed subspaces—witness βN in I^c—and it is unknown whether it is preserved by continuous maps with Hausdorff range; moreover, it is unknown if $X \times X$ supercompact implies X is supercompact.

3. Supercompactness in the Vietoris topology. Our attention is now diverted to the exponential or finite topology of Vietoris [**13**]. The author would like to thank A. Arhangel'skii for his motivation in this direction. An excellent background on the Vietoris topology can be found in E. Michael's paper [**7**]. For a space X, Exp (X) denotes the collection of all nonvoid closed subsets of X. Let

$$\langle U_1, \ldots, U_n \rangle = \{F \in \text{Exp}\,(X): F \subseteq \bigcup \{U_i: 1 \leqq i \leqq n\} \text{ and for each } 1 \leqq i \leqq n,\ F \cap U_i \neq \emptyset\}.$$

Then $\{\langle U_1, \ldots, U_n \rangle$: for $1 \leqq i \leqq n$, U_i is open in $X\}$ serves as a base for the open sets of Exp (X). For $A \subseteq X$, let Exp $(A) = \{F \in \text{Exp}\,(X): F \subseteq A\}$. Then Exp (A) is open (closed) in Exp (X) if A is open (closed) in X. In [**7**], E. Michael has shown that Exp (X) is compact Hausdorff if and only if X is compact Hausdorff.

Let X be compact Hausdorff and let f be a continuous mapping from an open subspace O of X onto a compact Hausdorff space Y. Consider Exp (O) as a subspace of Exp (X). Then the map $\hat{f}$: Exp $(O) \rightarrow$ Exp (Y) defined by $\hat{f}(F) = \{f(x): x \in F\}$ is continuous. Furthermore, if K is a compact subset of O and f maps K onto Y, then $\hat{f}$ maps Exp (K) onto Exp (Y).

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3.1. PROPOSITION. *Let* Exp (X) *be a supercompact Hausdorff space and let* f *be a continuous mapping from an open set* O *of* X *onto a compact Hausdorff space* Y. *If there exists a compact subset* K *of* O *such that* $f(K) = Y$, *then for all dense subsets* D *of* Y, $c(Y - D) \leqq w(D)$.

Proof. Let $\hat{f}$: Exp $(O) \to$ Exp (Y) be as above. Assume D is a dense subspace of Y and define $\mathscr{C}(D) = \{$all compact subsets of $D\}$. Then $\mathscr{C}(D)$ is dense in Exp (Y). Also, $\hat{f}$(Exp $(K)) =$ Exp (Y). Hence, by Theorem 2.2., c(Exp $(Y) - \mathscr{C}(D)) \leqq w(\mathscr{C}(D))$. However, $c(Y - D) \leqq c$(Exp $(Y) - \mathscr{C}(D))$. To see this, note that if $\mathscr{U}$ is a collection of open sets of Y such that $\{U \cap (Y - D): U \in \mathscr{U}\}$ is cellular in $Y - D$, then $\{$Exp $(U): U \in \mathscr{U}\}$ is a collection of open sets of Exp (Y) such that $\{$Exp $(U) \cap ($Exp $(Y) - \mathscr{C}(D)): U \in \mathscr{U}\}$ is cellular in Exp $(Y) - \mathscr{C}(D)$. Also, if $\mathscr{B}$ is an open base for D, closed under finite unions, of cardinality $w(D)$, then

$$\{\langle \mathrm{Int}_Y(\mathrm{Cl}_Y B_1), \ldots, \mathrm{Int}_Y(\mathrm{Cl}_Y B_n)\rangle \cap \mathscr{C}(D): \{B_1, \ldots, B_n\} \subseteq \mathscr{B}\}$$

is an open base for $\mathscr{C}(D)$ of cardinality $w(D)$. Hence $w(\mathscr{C}(D)) = w(D)$. Consequently, $c(Y - D) \leqq w(D)$.

It now follows that spaces like Exp (βN), Exp $(\beta N - N)$ and Exp (γN) (where $\gamma N - N$ is the long line) are non-supercompact. A relevant open question at this point is whether the supercompactness of X is equivalent to the supercompactness of Exp (X).

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