III. THEORY OF STELLAR MAGNETIC FIELD GENERATION

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STELLAR DYNAMO THEORY

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ABSTRACT

Three topics of current interest are treated in this review:

a) The success and shortcomings of dynamo models for the solar cycle are explained, and oscillator models discussed briefly.

b) The intermittent (flux tube) nature of magnetic fields in convection zones leads to new conjectures about stellar dynamos. Arguments are given that the dynamo may operate in the overshoot region below a convective envelope. Mean-field theory for intermittent fields is illustrated.

c) I review nonlinear dynamo models and some attempts to interpret observational results concerning late-type active stars.

1. INTRODUCTION

Solar activity is no peculiarity of our parent star, no unique phenomenon. On the contrary, most late type stars show the signature of magnetic activity when observed in X-ray, UV, visible or radio wavelengths. Stellar activity cycles similar to the Sun's have been discovered by O.C. Wilson (1978) and since then many new exciting observational results have transformed "solar dynamo theory" into "stellar dynamo theory". This development opens - in principle at least - the opportunity to test competing dynamo models or other conjectures concerning the mechanisms of stellar activity using a whole set of examples with different physical parameters (e.g. rotation, effective temperature, age, multiplicity, luminosity, ...). The Sun remains the only star where the processes can be studied in detail, but predictions of any theory of solar activity have to meet the requirements set by observations of stars. Consequently, we are in a much better position than before: We can decide what is typical of stellar activity and what is peculiar, i.e. what can guide our search for basic principles and what is misleading. For comparison, theories for the formation of the planetary system still suffer from the fact that our system is the

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only one observed. Consequently, even the most unlikely process cannot be ruled out just because it may have happened once. Theory of solar/stellar activity is not in this situation: we have the Sun as a nearby laboratory for observation of the detailed plasma processes, interaction of magnetic and velocity fields - and we have active stars which enable us to test our hypotheses when the parameters are changed. The synthesis of the study of solar and stellar activity may therefore lead to a very fruitful stimulation of dynamo and related theories.

Recently, dynamo theory has reached the 'textbook stage', i.e. four monographs have been published, viz. Moffatt (1978), Parker (1979a), Krause and Rädler (1980), Vainshtein, Zel'dovich and Ruzmaikin (1980). The state of theory is described quite comprehensively in these books while the more recent developments have been reviewed by Cowling (1981), Gilman (1981), Stix (1981, 1982), Weiss (1981c, 1982) and Yoshimura (1982) from different points of view. It is clear from this wealth of literature that there is no need for another broad review of the field. In the following chapters, therefore, I will concentrate on three points of current interest and discuss them in some detail. Further information may be found in the books and review papers cited above.

2. THE SOLAR DYNAMO CONTROVERSY

The characteristic Ohmic decay time for large scale magnetic fields in the sun is $\tau_D = R_\odot^2 \cdot \eta_m^{-1}$ with the molecular magnetic diffusivity $\eta_m \sim 10^4 cm^2 s^{-1}$ for the deeper parts of the convective zone. τ_D is very large compared to the time scale of the solar cycle, i.e. 22 years. Consequently, induction processes have to be invoked to understand the temporal and spatial behaviour of solar magnetic fields. Two possibilities have found most interest in the literature so far:

a) Interaction of magnetic fields and velocity fields (e.g. differential rotation, turbulent convection) leading to dynamo action under certain circumstances: small seed fields grow and a sizable magnetic field is maintained against Ohmic dissipation ("dynamo models").

b) A primordial large scale magnetic field anchored in the radiative interior of the Sun (slowly decaying with time scale τ_D) is periodically deformed by a suitable oscillation and produces the activity phenomena through the interaction with differential rotation ("oscillator models", Layzer et al., 1979, Piddington, 1976, and reference therein).

While dynamo theory has been developed in considerable mathematical detail (cf. the 4 monographs mentioned in the introduction), the oscillator picture has not been worked out enough to make a detailed evaluation possible. On the other hand, the proponents of oscillator theories have criticised dynamo models (especially the turbulent dynamos). As far as the question of applicability of the theory to the solar case is concerned, the doubts seem to be partly justified: At least, the models have to be modified in order to include the filamentary structure of the field and the dynamical interaction with convection. This will be discussed in Sec. 3. The more formal criticism of the mean field concept has already been discussed by Stix (1981) and Cowling (1981).

Let us stress a few basic conceptual problems of oscillator models. In Piddington's version, a meridionally oscillating fossil dipole field is conjectured which is thought rooted in the radiative interior of the Sun. However, no sign of a steady (non-reversing) dipole component in the solar magnetic field has been found; on the contrary, polar fields reversals have been observed three times (1957/58, 1969/71, 1980/?) in full conformity with dynamo models (Howard and LaBonte, 1981; Stenflo, 1981). Apparently to avoid this problem, Layzer et al. (1979) talk about an "irregular large-scale field, largely confined to the non-convective core". But it is not clear what they mean: A large-scale field with small scale fluctuations? This would face the same problem as Piddington's, i.e. no permanent large-scale field is observed. Or a "highly tangled field" as the authors write in another section? But this is identical to a field with only very weak large-scale components, i.e. a chaotic field. Such a field would have decayed long ago due to its small typical length scale. However, even if we allow for this field as a source for toroidal fields produced through differential rotation, it is not clear how the large amount of order in the solar cycle (e.g. coherent polarity rules for both hemispheres) is possible.

Furthermore, for equipartition fields ($B \sim 10^4$ Gauß, a reasonable flux density for the deep parts of the solar convection zone, cf. Schüssler, 1977) the velocity of convective flows is comparable to the velocities associated with the oscillation (being essentially a standing Alfvén wave). Consequently, the influence of convection on the oscillation cannot be neglected, i.e. eddy viscosity has to be taken into account. Compare the Lorentz force as driving force for the oscillations with the viscous force (to order of magnitude) at the base of the convection zone:

$$\gamma \equiv \frac{F_{Lor}}{F_{visc}} \sim \frac{\dfrac{B_{pol} \cdot B_{tor}}{4\pi\rho d}}{\nu_t \cdot \dfrac{v_A}{d^2}} = \frac{B_{pol} \cdot B_{tor} \cdot d}{4\pi\rho \cdot \nu_t \cdot v_A} \tag{2.1}$$

with poloidal field B_{pol}, toroidal field B_{tor}, density ρ, layer thickness d, Alfvén velocity v_A and eddy viscosity ν_t. For equipartition field strength, i.e. $B_{tor} \sim 10^4$ Gauß, $v_A = \frac{B_{tor}}{(4\pi\rho)^{1/2}}$ is equal to the convective velocity $v_c \sim 60$ m/s and, to order of magnitude, $\nu_t \sim v_c \cdot d$. Consequently, we get from (2.1):

$$\gamma \sim \frac{B_{pol}}{B_{tor}} << 1 \quad , \tag{2.2}$$

since the poloidal field has to be very small compared to the toroidal field in a torsional oscillation, i.e. $\gamma \sim 10^{-2} \ldots 10^{-4}$. The viscous force due to convection dominates the magnetic force by a large factor and it is difficult to see how the torsional oscillation can be maintained.

On the other hand, there is no strong asymmetry between odd and even cycles; the energy dissipated during a cycle must be very small, because differential rotation as energy source (e.g. a tendency for $\Omega \sim r^{-2}$ due to convection) can only replenish the energy during, say, odd cycles which leads to asymmetry. But if the dissipation and the rate of energy replenishment is small, how can we reconcile this with the strong irregular fluctuations in the intensity of the cycles?

Thus, it seems not easy for the contemporary state of oscillator models to disprove the final statement of Plumpton and Ferraro (1955) after discussing torsional oscillations: "... a theory of sunspots based on torsional oscillations alone is likely to raise more difficulties than it resolves".

Let us now turn to the success and the problems of dynamo models for the solar cycle. Using the mean-field-approach, nearly all dynamo models lead formally to the same equation for the mean magnetic field $<\underline{B}>$:

$$\frac{\partial <\underline{B}>}{\partial t} = \underline{\nabla} \times (<\underline{u}> \times <\underline{B}> + \alpha(\underline{r})<\underline{B}>) - \underline{\nabla} \times (\eta_t(\underline{r})\ \underline{\nabla} \times <\underline{B}>) \qquad (2.3)$$

In the kinematical case, the fuctions $\alpha(\underline{r})$, $\eta_t(\underline{r})$ depend on the statistical properties of the fluctuating velocity field. η_t is the turbulent magnetic diffusivity, while α leads to a current parallel to the mean field (in the simplest case; generally, α and η_t are tensors).

Dynamo action is possible either through the combined effect of differential rotation (the $<\underline{u}> \times <\underline{B}>$-term) and the mean electromotive force of a random velocity field under the influence of rotation (the $\alpha<\underline{B}>$-term) or through the α-term alone. The first kind of models ($\alpha\omega$-dynamos) with dominating influence of differential rotation predominantly yield oscillatory solutions, which can be compared with the observations of the solar cycle (Stix, 1976, 1978).

The following list shows observational results which have been reproduced by models based on Eq. (2.3) essentially:

- polarity rules of sunspot groups
- migration of the activity belt towards the equator ("butterfly diagram")
- polarity reversals of polar fields
- phase relationship between poloidal and toroidal fields consistent with direction of migration of the activity belt (Stix, 1976) and angular velocity increasing with depth as is indicated by observations of rotational splitting of 5-min-oscillations (Deubner et al., 1979; Claverie et al., 1981)
- rigid rotation of magnetic sector structure and coronal holes (Stix, 1974; 1977)

- zonal velocities associated with the activity belts observed by Howard and LaBonte (1980a): Since this flow has an 11-year-periodicity it is difficult to interpret as evidence for a torsional oscillation producing solar activity for which we would expect a 22-year period. It can be explained quite naturally by a wave of the (quadratic!) Lorentz force associated with a dynamo wave (Schüssler, 1981; Yoshimura, 1981; see LaBonte and Howard, 1982, for a diverging point of view).

Although the above list indicates that at least Eq. (2.3) describes the physics of the solar cycle to a great extent, we must not ignore the problems and weak points of the theory.

- Although the concept of positive turbulent diffusion of the mean field seems reasonable for the Sun (Kraichnan, 1976; Parker, 1979a, p. 584 ff.) so far there are no reliable means of calculating η_t (and α!) for the solar convection zone (Knobloch, 1978) because the First-Order-Smoothing-Approximation (FOSA) is hardly applicable. Essentially, this is due to our incomplete understanding of turbulent convection, in particular of MHD turbulence of high electrical conductivity (Cowling, 1981; Rädler, 1982).
- In view of the intermittent structure of photospheric magnetic fields, a kinematical approach which ignores the dynamical effects of concentrated magnetic fields can be questioned. The dynamical influence of convection which probably leads to the intermittent structures is not included (see Ch. 3).
- The cyclic variation of magnetic structures like the appearance of ephemeral active regions/X-ray bright points (Martin and Harvey, 1979; Golub et al., 1979) or sunspot intensity (Albregtsen and Maltby, 1981) cannot be described by the evolution of the mean field but may, nevertheless, contain important information on processes related to the cycle.

There is still the possibility that the success of dynamo models is possible just through the great number of free parameters or even free functions which can be tuned to give agreement with observations. It is very important to reduce this freedom for models by

- further detailed observation of solar surface magnetic and velocity structures
- reliable theory of the differential rotation $\Omega(r,\theta)$. Existing models depend very sensitively on boundary conditions (what about a fast rotating core?) and unknown parameters like the Prandtl-number (e.g. Schmidt, 1982) as well as on our insufficient understanding of the interaction between rotation and convection (Durney and Spruit, 1979; see also P. Gilman's review in these proceedings and Durney, 1981).

Possible alternatives to the classical type of $\alpha\omega$-dynamos are discussed in the next chapter.

3. DYNAMO ACTION AND CONCENTRATED FIELDS

As we have seen above, the $\alpha\omega$-models can describe the solar cycle rather well by parametrizing all effects of (turbulent) convection through the functions $\alpha(\underline{r})$ and $\eta_t(\underline{r})$ which appear in the equation for the mean field. The spatial two-scale approach (large scale: mean field and differential rotation, small scale: turbulent convection) loses its sense if large-scale convection ("giant cells") exists. Although observational confirmation is still lacking (Howard and LaBonte, 1980b), many numerical and analytical studies (e.g. Graham, 1975; Gilman, 1979; Gilman and Glatzmaier, 1981; Depassier and Spiegel, 1981) point clearly towards the existence of these structures. Such cells would be most efficient for field regeneration because they can be maximally influenced by rotation due to their long turn-over time ($\sim$ 1 month). Mean field dynamo equations may nevertheless be useful if time averages are invoked (Stix, 1976) because there is a big discrepancy between the time scale of the cycle (viz. 11 years) and that of giant cells ($\sim$ 1 month). Another possibility is averaging over longitude which leads to an axisymmetric mean field (e.g. Braginskij, 1964). Alternatively, large-scale convection can appear as a mean flow in spatially averaged dynamo equations while the traditional α-term arising from smaller scales can be neglected (but not η_t!).

Gilman and Miller (1981) have done such a calculation in the framework of a numerical study of Boussinesq convection in a spherical shell, in which also the differential rotation (the other ingredient of $\alpha\omega$-dynamos) is derived consistently through the action of the Reynold stresses exerted by global convection. The (not unexpected) result was that α-effect and ω-effect were of same order of magnitude, contrary to the assumptions for $\alpha\omega$-dynamos. The reason is that for convective eddies living about one turn-over time the differential rotation as well as the α-effect produced by them is of the order of the convective velocity itself. Estimating α from mixing length theory (e.g. Krause,1967) already yielded values much too big for succesful simulation of the solar cycle with $\alpha\omega$-dynamos (Köhler, 1973). Consequently, as discussed by Gilman and Miller, they did not find the results of $\alpha\omega$-models but dynamos with randomly fluctuating fields, no oscillatory solutions. Recently, Gilman (1982) reports, that new calculations with reduced eddy viscosity give oscillating solutions near the threshold for dynamo action (weakly excited dynamo), but with toroidal fields migrating poleward, too small period and no clear dominance of solutions antisymmetric with respect to the equatorial plane (dipole parity, as suggested by the polarity rules). It is not clear whether a calculation for a compressible medium can remove these difficulties as expected by Gilman.

In some way, the influence of convection on the field has to be reduced, because the convective induction effect has to be smaller and the large-scale order in the solar cycle as expressed by the sunspot polarity rules is evidence against a dominating role of convection. How can the field escape? One possibility is that the fields are strong, at least of equipartition field strength, to be able to resist deformation by convective flows. For the photosphere of the Sun, this is indeed the case: The field is organized in strong flux tubes from big sunspots to the small facular points.

This "differentiation" in a magnetic and a convecting phase can be understood in terms of the high electrical conductivity of stellar plasmas which leads to an exclusion of magnetic fields from regions of closed streamlines (Galloway et al., 1978; Galloway and Weiss, 1981; Weiss, 1981a, b). These extensive studies of the nonlinear interaction of magnetic fields and cellular convection support the conjecture that the field throughout the convective zone of the Sun is organized in strong (at least equipartition field strength) flux tubes. Within and near such a tube convection is efficiently throttled leading to a decrease of the efficiency of field regeneration. Childress (1979) investigated an idealized case and showed that the α-effect may be reduced by a factor $R_m^{1/2}$, where R_m is the magnetic Reynolds number.

There are two paths on which the investigation of dynamos including the effects of concentrated fields proceeds: The "overshoot layer dynamo" which is throught to operate at the interface between the convective zone and the radiative interior and the "flux tube dynamo" which assumes the whole convective zone to be permeated by strong flux tubes. Those tubes are influenced by convection, rotation (and possibly meridional circulation) and lead to dynamo action through the action of Coriolis forces (the classical Parker loop) and reconnection plus dissipation. We shall discuss the two approaches in the following conceding that also "mixed models" are possible.

Dynamo Action in the Overshoot Layer

Assuming an organization of magnetic fields in equipartition flux tubes in order to resist convective deformation leads to a new problem: Strong flux tubes are buoyant and, therefore, are difficult to store within the convection zone for times comparable to the solar cycle (Parker, 1975). High turbulent viscosity reduces the rising velocity drastically (Unno and Ribes, 1976; Schüssler, 1977, 1979a; Kuznetsov and Syrovatskii,1979) but on the other hand leads to strong coupling between flux tubes and convective down- or updrafts which may raise the flux to the solar surface within the convective timescale, viz. one month for global convection ("giant cells"). The only region where magnetic flux tubes could possibly be stored for times comparable to the solar cycle seems to be the lower boundary of the convective zone. In fact, a couple of mechanisms have been proposed which lead to an accumulation of flux there:

a) Topological pumping in a flow pattern which consists of a (multiple) connected network of downdrafts and isolated updrafts like the solar granulation (Drobyshevski and Yuferev, 1974; Drobyshevski et al., 1980)

b) Asymmetry between upflow and downflow velocities in an overshoot layer (van Ballegoijen, 1982; see discussion below)

c) The flux ejection dynamo discussed by Parker (1982a) produces magnetic flux at the bottom of a layer of convection cells subject to strong cyclonic rotation.

d) Van Ballegoijen (1982) proposes that toroidal flux tubes are transported equatorward by a meridional circulation. Conservation of angular momentum leads to a flow along the tubes opposite to the direction of rotation. The Coriolis force resulting from this flow is directed inwards and a flow velocity of a few m/s could stabilize the tube against buoyancy.

e) Trapping of flux tubes in large regions of closed streamlines (Parker, 1982e). A meridional circulation could trap toroidal flux in the deep convection zone.

f) A gradient in turbulent velocity (spatial dependence of turbulent diffusivity, "diamagnetism", see Zel'dovich, 1956; Spitzer, 1957; Rädler, 1968) is able to transport flux to the bottom of the convection zone (Ruzmaikin and Vainshtein, 1978). This effect is also evident from an $\alpha\omega$-dynamo calculation performed by Roberts and Stix (1972; see their Fig. 2).

g) Aerodynamic lift (Parker, 1979b) due to rotation with angular velocity increasing with depth is directed downward.

h) Convective instability of horizontal flux tubes leads to sinking of at least part of the tube (Spruit and van Ballegoijen, 1982).

i) Stable stratification in an overshoot layer below the formal boundary of the convection zone reduces the buoyancy force.

We see, many effects - which are all described by the MHD equations - lead to a downward transport of flux.

How may the field near the boundary of the convection zone look like? The downward forces compress the field and presumably lead to a more or less homogeneous magnetic layer which nevertheless may retain some flux tube structure imposed by convection. The field must have at least equipartition field strength B_e to resist severe deformation by convective flows. For the deep convection zone of the Sun, $B_e \sim 10^4$ Gauß. A value of $\phi_t \sim 10^{24}$ mx seems reasonable for the total toroidal flux produced during one half-cycle; consequently the hypothetical deep flux system located at the lower boundary of the convective zone consists of a magnetic layer with a minimum thickness between $d \sim 3000$ km and $d \sim 20\,000$ km depending on whether the whole latitude range or only a belt within ± 30° from the equator (where active regions appear) is taken (Schmitt and Rosner, 1982). Such a layer with equipartition field would interfere strongly with convective energy transport and we are led to the conclusion that it cannot be situated in a region where the main part of energy transport is by convection, because significant changes of luminosity would have to be expected during the cycle. A favourable site, however, is the overshoot layer below the "active" convection zone where energy is transported <u>inward</u> by convection and <u>outward</u> by radiation (the net flux being outward, of course; see Roxburgh, 1978; Cushman-Roisin, 1982). This has been proposed by Spiegel and Weiss (1980), van Ballegoijen (1982) as well as Schmitt and Rosner (1982). Earlier, some-what related ideas have been put forward by Gokhale (1977). Van Ballegoijen (1982) estimates the depth of the overshoot zone and finds a few tenths of a scale height which is comparable with the depth d of the magnetic layer derived above (d $\sim$ 5 % - 30 % of scale height $\Lambda_e \sim 6\cdot 10^9$ cm for the Sun). He also argues that downward flow is faster than upward flow in the overshoot layer because mechanical energy is converted into thermal energy (convective energy flux against temperature gradient). This effect leads to a net downward drag force: Take, for simplicity, a cylindrical flux tube with radius a. A flow with velocity v_c and density ρ_e leads to a drag force F_D per unit length of the tube (see e.g. Batchelor, 1967, p. 245 f.):

$$F_D = \rho_e v_c^2 \, a \, C_D \tag{3.1}$$

with drag coefficient $C_D \sim 1$. Take a system of convective rolls with downflow velocity v_d and upflow velocity v_u, whose axis are perpendicular to that of the tube. Include vanishing net mass flux and compare the net downward (for $v_d > v_u$) drag force with the buoyancy force (assuming thermal equilibrium of the tube with the surrounding gas)

$$F_B = \frac{B^2 a^2}{8\Lambda} \ell$$

where ℓ is a length along the tube which contains an equal number of updrafts and downdrafts. A short calculation gives the condition for equilibrium:

$$\gamma \equiv \frac{1 - v_u/v_d}{1 + v_u/v_d} = \left(\frac{B}{B_e}\right)^2 \cdot \left(\frac{\pi a}{2\Lambda_e C_D}\right) \sim \left(\frac{B}{B_e}\right)^2 \cdot \left(\frac{a}{\Lambda_e}\right) \sim 0.1 \tag{3.3}$$

for $B^2 \sim B_e^2 = 4u\rho_e v_d^2$ and a $\sim 10^9$ cm, a flux of $3\cdot 10^{22}$ mx corresponding to an active region. From (3.3) follows $v_u/v_d \sim 0.8$, a value which seems not unreasonable for compressible convection (Graham, 1977). Because we have not taken into account the other downward forces listed above and the stable stratification we may safely conclude that fields of equipartition field strength can be stored in the overshoot layer. There they are amplified by differential rotation until buoyancy dominates or instabilities set in and flux tubes leave the region predominantly toroidal (Schüssler, 1980a) but possibly suffer from instabilities (Acheson and Gibbons, 1978) and dynamical fragmentation (Schüssler, 1979a; Tsinganos, 1980).

A caveat seems in order here: We have made our estimates so far using results (e.g. convective velocities) taken from mixing length models (Spruit, 1977). These velocities may be wrong and differ significantly from numerical simulations which take compressibility into account (Graham and Moore, 1978; Massaguer and Zahn, 1980).

Although we would expect an organization of the magnetic field in the overshoot layer in the form of individual flux tubes due to the interaction with convection, it has also been proposed that the field is more homogeneous ("diffuse") in the dynamo layer (J. Schmitt and Rosner, 1982), because only small-scale convection is thought to exist in the overshoot layer. These are favourable conditions for conventional dynamo theory, because First-Order-Smoothing would be applicable: Field fluctuations are small compared to the mean field. However, a kinematical theory is hardly applicable: Fields of equipartition field strength (or more) are likely to have a strong influence on the convective flow. Boundary layer methods as applied by Childress (1979) seem more appropriate.

A non-filamentary field can find a magnetohydrostatic equilibrium by changing the stratification and thus overcome the buoyancy problem. J. Schmitt and Rosner invoke double-diffusive instabilities to trigger the flux loss from the dynamo layer that leads to the formation of flux tubes and the surface appearance of active regions. Furthermore, slow magnetostrophic waves can be excited by unstable field configurations (Acheson and Gibbons, 1978). Those waves exhibit net helicity and can be important for the regeneration of the poloidal field (D. Schmitt, private communication).

Assuming a deep-seated solar dynamo and having evidence for a fast rotating solar core (Claverie et al., 1981), it is tempting to speculate that the differential rotation in the boundary layer between fast rotating core and slowly rotating convection zone is the main driving force for the dynamo. However, it is easily estimated that the rotational energy of a solar core rotating at ten times the surface value would be exhausted after only $\sim 10^7$ cycles, i.e. 10^8 years, a short episode compared to the main sequence lifetime of the Sun (from an 11- and 22-year cyclical variation of glacial varves in Australia, Williams (1981) concluded that the solar cycle has been in operation at least for the last $7 \cdot 10^8$ years). Furthermore, the accelerating torque exerted on the convection zone by the magnetic field would exceed the solar wind torque $D_s \sim 3 \cdot 10^{29}$ dyn cm/sterad (Pizzo et al., 1982) by a large factor which leads to a spin up of the envelope contrary to observation (Skumanich, 1972). We conclude that even a deep-seated solar/stellar dynamo must be driven by processes in the convective (and overshoot) region itself. Differential rotation and field regeneration (α-effect) have to be produced in situ by convection and rotation. Any influence of a possible "fossil" field located in the radiative interior on the generation of the solar cycle must be extremely small. Levy and Boyer (1982) come to a similar conclusion observing that a fossil field would induce a strong asymmetry between the two halves of the magnetic cycle which is not observed.

Flux Tube Dynamo

Assume an ensemble of strong (i.e. at least equipartition flux density) magnetic flux tubes permeating a non-magnetic convecting medium. Is such a system able to sustain a dynamo? In principle, this is a much more difficult question than the classical kinematic dynamo problem because of the possibility of a strong interaction between flux tubes and the convective pattern: While magnetic fields are excluded from regions of closed streamlines (Galloway, Proctor and Weiss, 1978, and references therein), existing flux concentrations can shape the convective pattern and possibly stabilize large cells like supergranules on the Sun (Parker, 1982b).

Two extreme pictures for the state of the flux tube/convection system are possible which lead to different consequences:

a) The "fibril state" with many (> 10^6) tubes which carry fluxes of 10^{17} ... 10^{18} mx and have diameters of a few hundred kilometers, similar to the observed photospheric flux concentrations.

b) The "flux rope state" with few ($\sim 10^2$) tubes with a flux of $\sim 10^{22}$ mx, equipartition field strength (or more) and diameters of some thousand kilometers.

While fibrils refer to the observed small flux concentrations, flux ropes contain the flux of a whole active region. Depending on which flux system we think is responsible for the dynamo, different physical processes are involved.

Due to their small diameter, fibrils are dynamically dominated by convective cells with scale Λ_e from a depth of 3000 km on down through the whole convection zone (Parker, 1982d). Consequently, a kinematic mean-field theory is adequate by averaging over a length scale λ_1 with $d \ll \lambda_1 \ll L$ where $L \sim 2 \cdot 10^{10}$ cm is the depth of the convection zone and $d \sim 10^7 \ldots 10^8$ cm is the fibril scale. The small-scale ($l \lesssim d$) interaction between flux tubes and convection is implied in such a model by assuming a differentiation between tubes and field-free convecting fluid (a related point of view using a three-scale analysis has been proposed by Weiss, 1981c). Defining $\underline{v}$ and $\underline{B}$ as average over λ_1 we obtain the mean momentum equation for a set of locally nearly aligned flux tubes (Parker, 1982c):

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = \frac{1}{\rho'} \frac{\partial}{\partial x_j} \left[-\delta_{ij} p + m \frac{B_i B_j}{4\pi}\right] + \text{other forces} \qquad (3.4)$$

where p is the total pressure, ρ' a local mean fluid density (reduced within the tubes) and $m = b/B$ the ratio of the flux density b of the tubes to the mean field, the compression factor. Magnetic curvature forces are enhanced by the factor m as compared to a "diffuse" field of strength B. This is the essential difference to the simulation of mean field equations by Gilman and Miller (1981): The field is much less distorted than would be expected from a diffuse field; consequently, induction mechanisms are less efficient and the field cannot be tangled by the velocity until equipartition of the <u>total</u> kinetic and magnetic energy.

The tubes are bound to the nonmagnetic medium by means of the drag force, not by the high electrical conductivity as would be the case for a continuum field. Due to the curvature and buoyancy forces, the tube can slip through the medium with a velocity $\underline{u}$ derived from the equilibrium between curvature force $\underline{F}_C$, buoyancy force $\underline{F}_B$ and drag force $\underline{F}_D$:

$$\underline{F}_D(\underline{u}) + \underline{F}_B + \underline{F}_C = 0 \qquad (3.5)$$

Because the fibrils are assumed to be locally aligned, this velocity has to be added to the mean fluid velocity $\underline{v}$:

$$\frac{\partial \underline{B}}{\partial t} = \underline{\nabla}\times[(\underline{u}+\underline{v}) \times \underline{B} + \underline{E}] + \eta_t \underline{\nabla}^2 \underline{B} \qquad (3.6)$$

where η_t is the turbulent diffusivity. $\underline{E}$ stands for an electromotive force possibly arising from interaction of the fluctuating components of magnetic field and velocities. Such effects can be

a) The classical α-effect ($\underline{E} = \alpha\underline{B}$) which, however, may be drastically reduced or even absent for strong flux tubes because motions can be suppressed in the tube and the interaction restricted to a small boundary layer.

b) Helical waves driven by magnetic buoyancy in a rotating medium. Instability leads to slow magnetostrophic waves which propagate only in one direction with respect to the rotational velocity. Consequently, net helicity can be produced (Gibbons, 1977). D. Schmitt (1982, private communication) shows that this effect leads to an effective $\alpha \sim 10$ cm s^{-1} for parameters relevant for the lower solar convection zone.

Parker (1982c) shows that the slip velocity $\underline{u}$ can be neglected against $\underline{v}$ for fibrils if

$$\frac{a}{\Lambda_e} \;,\; \frac{a}{L} << \frac{v^2}{v_A^2}$$

where v_A is the Alfvén velocity corresponding to the fibril field. Consequently, for sufficiently slender fibrils (3.6) takes the form of the "classical" kinematic mean field equation (2.3) by setting $\underline{u} = \underline{0}$.

As mentioned above, if giant cells exist, such a treatment only makes sense for temporal or longitudinal averaging, because only then these can be treated as part of the fluctuating velocities. In that case, α-effect and turbulent diffusion η_t result mainly from the giant cells but any calculation of these quantities has to take into account the fibril state of the field which leads to Eq. (3.4). First order smoothing may not be adequate then for determination of α and η_t. In particular, η_t may not necessarily be identical to the turbulent diffusivity of a scalar quantity. However, the global properties of a fibril dynamo may well be described by the mean field equation (3.6).

In view of the complications arising from flux tube dynamics when no averaging is performed, it is not surprising that no detailed fibril dynamo model has been presented in the literature so far. Only a few very idealized pictures have been proposed (e.g. Adams, 1977; Giovanelli, 1982).

The physics of "flux rope models" on the other hand, is quite different. Guided from the typical flux of active regions, the field is thought to be organized in a few ($\sim 10^2$) big flux tubes with fields of equipartition field strength ($\sim 10^4$ Gauß for the deep convection zone) and a flux $\phi \sim 10^{22}$ mx which leads to a diameter of about 10^9 cm. We have shown above that because of their large radius such tubes can avoid severe distortion from convective motions in the deep convective zone and can be maintained for a significant part of the solar cycle period in the overshoot layer below the solar convection zone. Following Babcock's (1961) original suggestion, Schmidt (1968) and Piddington (1976) have conjectured the existence of big flux tubes or "ropes", while Galloway and Weiss (1981), Spiegel and Weiss (1980), Golub et al. (1981) and Schüssler (1980b) have discussed some consequences of such a model for the theory of the activity cycle. The cyclic variation of sunspot brightness (Albregtsen and Maltby, 1981) and the related variation of appearance of X-ray bright points and ephemeral active regions (Maltby and Albregtsen, 1979; Golub et al., 1979) can be understood in terms of the age of the basic flux ropes. The flux rope models seem to lead necessarily to the idea of a dynamo operating in the overshoot layer which we have discussed above. The remarks made there are relevant for this model, too.

4. NONLINEAR DYNAMO THEORY AND APPLICATION TO STARS

Stimulated by the pioneering work of O.C. Wilson (1978) the investigation of stellar activity has become one of the most rapidly evolving fields in observational astrophysics (see reviews by Noyes, 1981, 1982; Zwaan, 1981, Dupree, 1981; Vaughan, this volume). The possibility to observe a large number of stars which differ in spectral type, evolutionary status, multiplicity, rotation rate, etc. is a big advantage for theory. Predictions from models can be compared with different stars and conclusions are not drawn solely from the solar case with its (possible) peculiarities. However, for predictions to be made, the models must have been worked out to a fair degree of sophistication, an important point for which is the inclusion of nonlinear effects (from the Lorentz force) which leads to a finite amplitude of dynamo action.

Nonlinear Dynamos

Several possibilities have been discussed in order to introduce nonlinearities into the dynamo formalism (all arising from the Lorentz force in the induction equation):

a) The cut-off-α-effect: The growing field $\underline{B}$ exhibits a stronger resistance against deformation by small-scale motions. The α-effect which results from this interaction therefore decreases as $|\underline{B}|$ increases. Models incorporating this kind of nonlinearity have been presented by Stix (1972), Rüdiger (1973), Jepps (1975), Ivanova and Ruzmaikin (1977), Yoshimura (1978), Kleeorin and Ruzmaikin (1981). The disadvantage of this procedure is the essentially arbitrary device of $\alpha = \alpha(|\underline{B}|)$.

b) Flows driven by the mean Lorentz-force: Inclusion of the force arising from the mean field $\underline{B}$, viz. $(\underline{\nabla} \times \underline{B}) \times \underline{B}/4\pi$ into the momentum equation for the mean velocity $\underline{v}$ leads to a flow which, according to Lenz's rule, limits the growth of the mean field. This approach has been pursued by Malkus and Proctor (1975), Proctor (1977), Hellmich (1978) for incompressible flow and Schüssler (1979b) for a compressible medium. A special case where only the magnetically induced change in differential rotation is considered has been given by Hinata (1982). These models suffer from the possibility that the mean Lorentz force from the fluctuating components of $\underline{B}$ (which is neglected) may possibly be much larger than the force arising from the mean field.

c) Magnetic buoyancy and convective transport: The amplitude of a dynamo working in a layer (as discussed in Sec. 3) can be limited by removal of magnetic flux out of the layer through the joint action of buoyancy and convection. Quantitative models are difficult to investigate because they involve uncertain details as diameter and flux density of magnetic flux tubes, turbulent viscosity and the dynamical interaction of flux tubes and convection. However, crude parametrizations have been tried beginning with Leighton's (1969) model, followed later by Köhler (1973), Yoshimura (1975), Schüssler (1980b), Durney and Robinson (1982) as well as Robinson and Durney (1982).

d) Hydromagnetic Dynamos: Models of this kind investigate simultaneously the generation of the velocity field (e.g. thermal convection driven by an unstable temperature gradient) and of the magnetic field (through dynamo action of the velocity field) allowing for nonlinearity through the Lorentz force in the momentum equation. Although suffering from not including possible small-scale effects, hydromagnetic dynamos have the advantage of treating generation and limitation of the field self-consistently. Childress and Soward (1972), Soward (1974) and Busse (1975) used perturbation methods (see also review by Soward, 1979) while full non-linear numerical solutions have been given by Baker (1978) for cartesian geometry as well as Gilman and Miller (1981) for a spherical shell, both employing truncated expansions.

It is well known that a non-linear system of equations can show a behaviour drastically different from the linear case. Beginning with early studies of aperiodic behaviour of a system consisting of two disc-dynamos (Allan, 1962), non-linear model equations for nonlinear dynamos have been investigated by several authors (Robbins, 1977; Jones, 1981; Ruzmaikin, 1981). Bräuer (1979, 1980) found subcritical solutions in an analytical study of a simplified nonlinear $\alpha\omega$-dynamo. Yoshimura (1978, 1979, 1980) introduced a time delay in the back-reaction term and received aperiodic behaviour which he used for an interpretation of the long-term variability of solar activity (Maunder minimum, "long cycle") and geomagnetic reversals. Fully developed MHD turbulence, inverse cascades and dynamo action have been investigated by Meneguzzi et al.(1981).

Observational Results

The main conclusions drawn from observations of active stars so far are the following:

a) The X-ray flux and the Ca^{+}-emission of late-type stars (taken as a measure of magnetic activity by analogy to the Sun) increase with

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the rotation rate Ω for stars of the same spectral type (Walter and Bowyer, 1981; Walter, 1981; Vaughan et al., 1981; Middelkoop, 1981).

b) Ca^+-Emission generally decreases with increasing age of the stars (Skumanich, 1972). In particular, there are indications for two distinct groups of active stars: Young, rapidly rotating stars with a high level of emission and strong temporal fluctuations but no cyclical activity; on the other hand, old, slowly rotating stars with a lower level of emission and solar-type cycles (Vaughan and Preston, 1980; Vaughan, 1980). The dependence of L_X/L_{bol} (X-ray luminosity divided by bolometric luminosity) on angular velocity is different for both groups (Walter, 1982). Solar-type cycle periods seem to be independent of Ω (Vaughan et al., 1981).

c) X-ray activity (L_X/L_{bol}) increases for stars with lower mass and deeper outer convection zones (Linsky, 1981). The high activity of close binaries (e.g. RS CVn systems) is attributed to synchronous rotation leading to high values for Ω, not to multiplicity itself (Walter and Bowyer, 1981). However, a variant of the Herzenberg (1958) two-sphere dynamo has been proposed by Dolginov and Urpin (1979) which takes into account the mutual induction between the members of close binaries.

Theoretical Interpretation

Comparison with models of magnetic field generation until now are all made in the framework of $\alpha\omega$-dynamos. Since both the α-effect (due to the Coriolis force $F_c \sim v_c \cdot \Omega$) and the differential rotation increase with Ω (Belvedere et al., 1980a) the dynamo number

$$N_D = \left(\frac{\alpha\Delta\Omega}{L}\right)^{1/2} \cdot \frac{L^2}{\eta_t} \qquad (4.1)$$

($\Delta\Omega$: angular velocity difference within dynamo region of scale L) which is a measure for the excitation of the dynamo increases with Ω for otherwise identical stars. The ultimate level of magnetic energy increases with N_D in all nonlinear dynamo calculations presented so far and we expect an increase of stellar activity (as measured in Ca^+-emission and X-ray fluxes) with increasing rotation rate. This expectation is in full accord with the measurements summarized above. A more quantitative prediction of the expected level of activity is far more difficult because of all the uncertainties in our understanding of dynamo processes discussed in Chapters 2 and 3. All four quantities appearing in Eq. (4.1) are very uncertain and their detailed dependence on Ω even more. In spite of these difficulties Durney and Robinson (1982) and Durney et al. (1981) have tried to predict quantitatively the stellar activity level using simple estimates of α, $\Delta\Omega$ etc. guided by mixing length theory. Although the results are in qualitative agreement with observations, because of the many assumptions involved it is not clear how much weight should be attached to them. The same remark applies to the interpretation of X-ray data in terms of $\alpha\omega$-dynamos by Belvedere et al. (1981, 1982).

While the increasing activity for increasing Ω is straightforward to interpret, the growth of X-ray emission towards later spectral types calls for a more involved explanation. Several proposals have been put forward:

a) The convection zone of later stars extend into regions of higher temperature, scale height and density. Mixing length theory then predicts an increasing influence of rotation on convection expressed by decreasing Rossby number

$$Ro = \frac{v_c}{2\Lambda \cdot \Omega} \tag{4.2}$$

even if Ω decreases (Durney and Latour, 1978). This may lead to a larger α-effect for later stars. Preferred nonaxisymmetric modes ("starspots") for rapid rotation possibly can be understood by growing anisotropy of the α-tensor (Rüdiger, 1978, 1980). Furthermore, since the buoyant rise of flux tubes takes place at some fraction of the Alfvén velocity $v_A = B \cdot (4\pi\rho)^{-1/2}$ the higher density and scale height at the base of the stellar convection zone leads to a decreasing efficiency of buoyancy limitation of the field amplitude (Belvedere et al., 1980b).

b) In a model for differential rotation which invokes latitude-dependent heat transport, the differential rotation first decreases sharply from spectral type F5 to G5 and then increases from G5 to M0 (Belvedere et al., 1980a). Consequently, the induction by differential rotation (ω-effect) increases for stars later than G5.

c) The surface gas pressure Pg of the stars grows with decreasing effective temperature. Consequently, the field strength of flux tubes with $\beta = 8\pi Pg/B^2 \sim 1$ increases and - for the same total magnetic flux - the magnetic energy is a good deal larger (a factor ~ 20 between G0 and M5, cf. Durney and Robinson, 1982, Table I). Magnetic heating of coronae by waves, currents or other mechanisms always strongly depends on the photospheric magnetic structure, i.e. flux tube geometry and peak field strength. For example, if the X-ray flux varies like B^2, the intensification of flux tubes due to stronger surface pressure alone could explain the increasing X-ray flux for later stars. The Ca^+ emission which at least for the Sun (Skumanich et al., 1975) is correlated with the total magnetic flux, seems to show a weaker dependence on the flux tube structure than the transition region (and coronal) emissions (Oranje et al., 1982). This may lead to the decrease of Ca^+-surface flux towards later stars while the X-ray flux increases. We should be very careful when interpreting the data in terms of dynamo efficiency while not knowing the surface structure of the fields and the implication for chromospheric and coronal heating.

There are different possibilities for the interpretation of the Vaughan-Preston gap in a plot of Ca^+ emission index S against (B-V), i.e. the bimodal distribution of active stars into old, slowly rotating stars with solar-type cycles (class C) and young, rapidly rotating stars with stronger but irregularly fluctuating activity (class I) (see also discussion in Durney et al., 1981):

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a) Two different dynamo mechanisms: one operating for rapid rotation, one for slow rotation. No attempt has been made so far to investigate this proposal in detail.

b) A rapid change of rotation rate Ω: A star of class I is presumed to have a nearly closed corona and no efficient magnetic braking can take place. But as Ω decreases slowly, activity gets weaker and coronal holes can open. Now magnetic braking through high-speed stellar winds is far more effective, Ω decreases more rapidly, activity gets even weaker: a positive feedback. However, no such episode of rapid change of Ω with time is apparent from the observations (Skumanich, 1972).

c) As Ω decreases, the dynamo excitation decreases too and the magnetic field configuration may change from a state with many higher modes to a state where only the fundamental mode of the dynamo appears. Nonlinear interaction of the higher modes leads to the irregular fluctuations for class I. However, even the single-mode dynamo will operate in the non-linear domain (see the long-term fluctuations of the solar cycle) and it is not clear whether linear results (e.g. critical Reynolds numbers for higher modes) can be used under those circumstances. Subcritical phenomena (Bräuer, 1980) may appear.

d) Model equations for dynamos often show chaotic solutions in certain regions of parameter space (Jones, 1981; Ruzmaikin, 1981). The crude flux tube dynamo of Schüssler (1980b) changes suddenly from periodic to chaotic solutions if the excitation exceeds a certain threshold value (see also Model 2 of Jepps, 1975). The same behaviour appears for too small turbulent diffusivity. High rotation rate Ω favours strong helicity fluctuations which may lead to small (or even negative) turbulent diffusivity (Kraichnan, 1976). This may not necessarily choke the dynamo but lead to more unrelated variations in different parts of the convective zone. Hysteresis effects (see also e) below) may lead to the gap.

e) Knobloch et al. (1981) propose a change in the pattern of convection as Ω is increased associated with a hysteresis effect. This leads to a discontinuous dependence of the dynamo-built magnetic field on Ω. The two patterns of convection are cellular convection for low Ω and cylindrical convection rolls parallel to the rotation axis (as a consequence of the Proudman-Taylor constraint) for high Ω.

Cycle periods have been discussed by Belvedere et al. (1980b) who solve the linear dynamo equations with differential rotation derived from latitude-dependent heat transport and by Robinson and Durney (1982) discussing nonlinear model equations. Marginal dynamo action for the linear case is achieved if the period of the dynamo wave

$$\tau_w \sim \frac{L}{(\alpha\Delta\Omega)^{1/2}} \tag{4.3}$$

is equal to the magnetic diffusion time

$$\tau_d \sim \frac{L^2}{\eta_t} \tag{4.4}$$

$\tau_W \sim \tau_d$ means then that the dynamo number $N_D \sim 1$ (see also Hinata, 1982, for the marginally unstable case). Because L (depth of convection zone) increases towards later stars, τ_d increases and, consequently, Belvedere et al. find that the cycle period grows for later stars. However, this trend is also influenced by their assumption of a constant Prandtl number which leads to a decreasing turbulent diffusivity for stars with decreasing mass. On the other hand, Robinson and Durney with their nonlinear model equations obtain periods decreasing towards later stars (for moderate nonlinearity) and only a weak dependence on spectral type for strong nonlinearity. A moderate dependence of cycle period on excitation has also been found by Jepps (1975) over a large range of parameters.

It seems clear from the discussion above that the evaluation of observations of stellar activity for theoretical purposes is still at its infancy. Instead of fitting parameters for ad-hoc models to get a "reasonable" agreement, observations should be used as a guide for the development of physical concepts. From such a point of view we can ask "questions to nature". One example is the idea of dynamo action in the inner overshoot layer (Chapter 3): If the solar dynamo and those of stars with convective envelopes work on this basis, we should expect a totally different behaviour of very low mass stars which may be fully convective; see, however, Cox et al. (1981). In fact, Liebert et al. (1979) found no indication of chromospheric emission for some very late M dwarfs. Further research along these lines could lead to a better understanding of the physics of magnetic field generation in late type stars.

5. CONCLUSION

Stellar dynamo theory is a rapidly evolving field of research. In my opinion, this is due to the fact that we can draw information from two different reservoirs: The Sun enables us to study the basic nonlinear interaction of turbulent convection and magnetic fields with high spatial and temporal resolution, while active stars show us the dependence of the mechanism on different parameters. Any model has a hard life when confronted with these two sets of observational results; while solar data exclude the simple "on the envelope" ideas, stellar results should keep us from constructing models which are extremely "tuned" to fit the features of the solar cycle. However, the Sun remains the Rosetta stone in the yet unfinished puzzle of stellar activity - it is hard to imagine the state of our understanding of stellar activity without the information drawn from the Sun, but it is quite probable that the idea of a magnetic origin of the phenomena would be a mere speculation among a number of other conjectures and "scenarios".

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DISCUSSION

GARCIA DE LA ROSA: Apart from the zone of magnetic flux concentration at the overshooting layer, wouldn't you expect to observe another such zone at the bottom of the supergranular cells? As mentioned in your talk for fibrils, the buoyant force is not important in comparison with the hydrodynamic drag, so as soon as a "flux rope" decays into smaller elements, these can be moved down to the bottom of the supergranules where an eventual concentration of fibrils can give rise to a small tube, large enough to be buoyant and to produce a small active region. This may be the explanation for the observed emergence of several small active regions after the decay of a large active region.

SCHÜSSLER: This may well be true. As far as topological pumping is concerned, there are important new results obtained by W. Arter, D. Galloway, and M. Proctor.

ENDAL: Has anyone examined how the existence of an overshooting layer at the bottom of the convective envelope would affect the pulsation properties of the sun?

SCHÜSSLER: I don't know. Is there an expert in the audience who can comment on that?

MULLAN: Details of the transition from convection to radiative zone at the base of the convection zone are very important for determining the rate of emission of g modes. These are focussed towards the center of the sun and may contribute seriously to energy transport if the g-mode amplitudes become large. Hence the idea of flux loops at the convective-radiative interface may be significant in the discussion of g modes.

ROSNER: Your estimate of an upper bound on the solar angular momentum loss is derived from measurements in the solar wind in (or near) the ecliptic. Observations of the inner corona show however that much mass loss occurs out of the ecliptic (the exact amount being very uncertain); hence your angular momentum loss estimate is likely to be similarly uncertain.

SCHÜSSLER: Quite true, but the polar regions have a much lower specific angular momentum. However, having an upper limit of 10^{-5} G for the field penetrating the envelope, I do not mind discussing about two orders of magnitude.

GOKHALE: (1) I am aware that it is unfair to expect any theoretical model to be consistent with too large a number of observational constraints. However, is there any turbulent dynamo model that incorporates the 10 day time scale of emergence and replacement of solar magnetic flux (as observed by Dr. Howard) *in addition* to the observational constraints in your "hit list"? (2) If we were to introduce the inertial forces due to the mass flow within the fluxtubes, would it affect substantially the conclusions of the model?

SCHÜSSLER: (1) This phenomenon in my opinion is a surface phenomenon of appearance, pulling down and reappearance of flux, probably confined to the granular–supergranular layer. It may not be very important for the dynamo. (2) No. Substantial mass flows only occur during the final phases of the rise of a fluxtube. Near the surface ($z < 10^4$ km) the tube gets unstable to loop formation, and downflows may occur as observed.

MULLAN: Is there a minimum size of fibrils deep in the convection zone? Near the surface, Meyer et al. (1977) found $F > 10^{19}$ Mx. Is there a corresponding result deep down?

SCHÜSSLER: The minimum fibril size is determined by diffusion and stability. Apart from the paper you mentioned there are no detailed calculations available yet.

SODERBLOM: I wish to make two points and raise a question. First, if the chromospheric emission of the Vaughan-Preston survey (i.e., Ca II H and K emission) is corrected for the decline of the underlying continuum, the chromospheric emission *decreases* in going down the main sequence, it does not increase. I am referring to the normalized flux: H+K flux $/L_{bol}$. Second, the Vaughan-Preston gap may not necessarily be real. Consideration of the effects that place stars in the diagram in which the "gap" is so evident leads one to conclude that the gap need not be real — it may be only a statistical fluctuation. My question concerns your remark that the center of the sun is rotating rapidly. The observations I have seen to date are quite controversial (to put it mildly). Are we sure that the core is in fact spinning rapidly?

SCHÜSSLER: L_{HK}/L_{bol} decreases, but L_x/L_{bol} increases or at least is constant. However, that may have more to do with the structure of the surface fields than with dynamo efficiency. The two other points clearly have to be clarified — mainly by the observers!

MESTEL: I am not arguing against the genuine solar dynamo and in favour of amplification models which depend on an external source of flux. And of course I agree that a hypothetical primeval field in the solar core will, if coupled with the base of the convective envelope, tend to iron out the inferred differential rotation between core and envelope. However, we are still I think unclear on the interaction of turbulence and rotation, and I recall a suggestion by Gough that perhaps the effect is to expel vorticity, analogous to the expulsion of primeval magnetic flux. Is it completely ruled out that there may be a state which is both kinematically and dynamically steady, with convective expulsion of vorticity balanced by magnetic coupling, and turbulent resistivity limiting the generation of toroidal field by non-uniform rotation?

SCHÜSSLER: Such a model is possible, but the turbulent expulsion of vorticity must be a very powerful mechanism then, because magnetic tensions are very effective.

STIX: There is another reason why the observed solar mean field should be confined to the convection zone (including the overshoot layer). The field is periodic with 22 years and so cannot penetrate into the radiative interior because of the skin effect. This means that whatever part of the field participates in the cycle should be disconnected from any interior field.

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