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EXTERIOR POWERS OF FIELDS AND SUBFIELDS

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Dedicated to Professor Goro Azumaya on the occasion of his 60th birthday

Introduction

Let D be a division algebra of finite dimension n^2 over it's center F. Suppose D has an involution, τ, of the first kind, of symplectic type (e.g. [1], p. 169). By the theory of the pfaffian, τ symmetric elements have degree less than $n/2$ over F. On the other hand, Tamagawa has shown (unpublished) that involutions like τ are closely related to minimal symmetric idempotents in $D \otimes_F D$. This author began by examining and trying to generalize these relationships. But before any theory seemed possible for division algebras, a theory relating subfields and symmetric idempotents was required. This investigation gave rise to the results presented here, especially the main theorem in Section Two.

We begin by considering a finite separable field extension L/F and it's tensor power $T_m(L/F) = L \otimes_F \cdots \otimes_F L$ (m times). Already from Tamagawa's work, it is clear that not all minimal symmetric idempotents are of interest to us, but only those corresponding to symplectic involutions. It turns out that what we are actually interested in is an F algebra $E_m(L/F)$ closely related to the exterior power $\Lambda^m L$. This is the exterior power of the title. In Section Two we prove a correspondence theorem relating idempotents of $E_m(L/F)$ and subfields of L. Specifically, there is a one to one correspondence between subfields $L' \subseteq L$ of codimension m and certain minimal-like idempotents of $E_m(L/F)$. This correspondence theorem generalizes the facts concerning the pfaffian mentioned above.

Assuming L/F is a separable field extension is unnaturally restrictive. It better serves our purpose to assume that L is a separable commutative algebra over F, and that F is a finite direct sum of fields. We do so assume in all of this paper. An F module V may not be a free F module,

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but it is free if the dimension of V is the same when viewed over any of the components of F. Calling this dimension n, we say V has constant dimension n over F.

§1.

This first section will contain the fundamental properties of the exterior power F algebra $E_m(L/F)$. As mentioned above, F will always be a finite direct sum of fields and L a commutative separable F algebra of constant dimension n. We may write $F = F_1 \oplus \cdots \oplus F_r$ where the F_i's are fields, and then $L = L_1 \oplus \cdots \oplus L_r$ where L_i is an algebra over F_i. Our assumptions are exactly that for all i, L_i is a separable commutative F_i algebra of dimension n.

Denote by $T_m(L)$ or $T_m(L/F)$ the tensor power $L \otimes_F \cdots \otimes_F L$ (m times). $T = T_m(L)$ is, of course, a separable commutative F algebra. The symmetric group, S_m, acts on T in a natural way. Denote by $\Sigma_m(L)$ or $\Sigma_m(L/F)$ those elements of T fixed by S_m (i.e. the symmetric elements). $\Sigma_m(L)$ is easily seen to be a commutative separable F algebra of constant dimension. Set $V = \Lambda^m L$ to be the m^{th} exterior power of L over F. V is just a module over F. There is a canonical surjection $\Phi: T \to V$ such that the kernel of Φ is spanned by all elements of the form $\{a_1 \otimes a_2 \otimes \cdots \otimes a_m \mid a_i = a_j$ for some $i \neq j\}$. The following lemma shows that V is a module over $\Sigma_m(L)$.

LEMMA 1.1. *Let W be the kernel of Φ.*

a) $\Sigma_m(L) W \subset W$

b) *V is naturally a $\Sigma_m(L)$ module.*

Proof. Of course, part a) implies b). To prove a), note that $\Sigma_m(L)$ is spanned by elements of the form $t = \sum_{\sigma \in S_m} a_{\sigma(1)} \otimes \cdots \otimes a_{\sigma(m)}$ where the $a_i \in L$. So without loss of generality, it suffices to show that $t(b \otimes b \otimes b_3 \otimes \cdots \otimes b_m) \in W$ for any $b, b_3, \cdots, b_m \in L$. But $t(b \otimes b \otimes b_3 \otimes \cdots \otimes b_m)$ is a sum of terms of the form

$$(a_i b \otimes a_j b + a_j b \otimes a_i b) \otimes a_k b_3 \otimes \cdots \otimes a_r b_m$$

and the above expression is in W. Q.E.D.

Denote by $E_m(L)$, or $E_m(L/F)$, the image of $\Sigma_m(L)$ in $\mathrm{End}_F(V)$. $E_m(L)$ is our "exterior power" of the algebra L. It is, of course, a commutative separable F algebra. Denote by $\Psi: \Sigma_m(L) \to E_m(L)$ the natural map used to define $E_m(L)$. Since $\Sigma_m(L)$ is commutative and separable over F, there is a unique idempotent we call $e \in \Sigma_m(V)$ such that Ψ induces an isomorphism $e\Sigma_m(L) \cong E_m(L)$. In this way we will view $E_m(L)$ as a subset of $\Sigma_m(L)$ and $T_m(L)$.

We observe in the next lemma that the algebra $E_m(L)$ behaves well with respect to direct sums and base change. We will omit the proof as it is quite direct.

LEMMA 1.2. a) *Suppose* $F = F_1 \oplus F_2$. *If* $E_m(L) = E_m(L)_1 \oplus E_m(L)_2$ *and* $L = L_1 \oplus L_2$ *are the corresponding decompositions of* $E_m(L)$ *and* L, *then* $E_m(L_i/F_i) \cong E_m(L)_i$ *in a natural way.*

b) *Suppose* $F' \supseteq F$ *is an extension of* F *and also a finite direct* sum *of fields. Set* $L' = L \otimes_F F'$. *Then* $E_m(L'/F')$ *is well defined and* $E_m(L/F) \otimes_F F' \cong E_m(L'/F')$.

An important special case of the above construction occurs when F is an algebraically closed field. Assuming this, $L = F \oplus \cdots \oplus F$ (n times). Denote by $e(1), \cdots, e(n)$ the full set of minimal idempotents of L. $T_m(L)$ has, as minimal idempotents, all the elements $e(i_1, \cdots, i_m) = e(i_1) \otimes \cdots \otimes e(i_m)$. Set $e[i_1, \cdots, i_m]$ to be the corresponding symmetrized element. That is,

$$e[i_1, \cdots, i_m] = \sum_{\sigma \in S_m} e(i_{\sigma(1)}, \cdots, i_{\sigma(m)}) .$$

It is almost immediate that the symmetric algebra $\Sigma_m(L)$ has the $e[i_1, \cdots, i_m]$'s as a basis over F. Finally consider $E_m(L)$ and the map $\Psi: \Sigma_m(L) \to E_m(L)$ defined above. $V = \Lambda^m(L)$ has the set of all $e(i_1) \wedge \cdots \wedge e(i_m)$ as a basis, where the i_j's are all distinct. It follows that $\Psi(e[i_1, \cdots, i_m]) = 0$ if and only if $i_j = i_k$ for some $j \neq k$. Stated differently, we have part a) of the next lemma.

LEMMA 1.3. a) *Suppose F is algebraically closed as above. Considered as a subspace of* $T_m(L)$, $E_m(L)$ *has, as a basis, the set of all* $e[i_1, \cdots, i_m]$*'s where the* i_j*'s are all distinct.*

b) *For general* L *and* F, $E_m(L/F)$ *has dimension* $\binom{n}{m}$ *over* F.

Proof. Part a) has been shown and part b) follows from a) using 1.2. We will end this section by giving an alternate description of $E_m(L/F)$. Let us fix some notation. Suppose R is any commutative ring and $f(x) \in R[x]$ a polynomial. Denote by $R\{f(x)\}$ the R algebra $R[x]/(f(x))$. We call the image of x in $R\{f(x)\}$ a generic element of $R\{f(x)\}$. If $R = F$ is a direct sum of fields, then a polynomial $f(x) \in F[x]$ is said to be separable if it's image is separable over each of the components of F. Clearly, if $f(x) \in F[x]$ is a monic separable polynomial of degree n, then $F\{f(x)\}$ is a separable algebra of constant degree n over F. Conversely, if L is a separable commutative F algebra of constant dimension n, then $L \cong F\{f(x)\}$ for some monic separable $f(x)$ of degree n.

One should think of $F\{f(x)\}$ as the algebra obtained by formally adjoining a root of $f(x)$ to F. In fact, if $u_1 \in F_1 = F\{f(x)\}$ is a generic element, then, of course, u_1 is a root of $f(x)$. One can write $f(x) = (x - u_1)f_1(x)$ where $f_1(x) \in F_1[x]$ is a monic separable polynomial of degree $n - 1$. We continue this process inductively. Assume F_i, $f_i \in F_i[x]$ and $u_i \in F_i$ have been defined. Set $F_{i+1} = F_i\{f_i(x)\}$, choose $u_{i+1} \in F_{i+1}$ *a* generic element, and factor $f_i(x) = (x - u_{i+1})(f_{i+1}(x))$ where $f_{i+1}(x) \in F_{i+1}[x]$. It is clear that each f_i is monic separable over F_i of degree $n - i$, and that F_i is separable over F of constant dimension $n(n-1)\cdots(n-i+1)$. We focus on F_m and think of it as the algebra obtained by formally adjoining m roots of $f(x)$ to F.

THEOREM 1.4. *Suppose $L = F\{f(x)\}$ and let S_m be the symmetric group on* $\{1, 2, \cdots, m\}$.

a) *Setting $\sigma(u_i) = u_{\sigma(i)}$ defines an action of S_m on F_m.*

b) *$E_m(L/F)$ is isomorphic to the fixed ring of S_m on F_m.*

Proof. Let $T_m(L)$, $\Sigma_m(L)$ and $\Psi: \Sigma_m(L) \to E_m(L)$ be as above. Once again we let $e \in \Sigma_m(L)$ be the idempotent such that Ψ induces an isomorphism on $e\Sigma_m(L)$, and set $N = eT_m(L)$.

We claim that N is isomorphic naturally to F_m. We will prove this in a series of steps, as follows. First, we show that N has dimension $n(n-1)\cdots(n-m+1)$ over F. But using 1.2, we may assume F is an algebraically closed field. Using our previous notation, N has, as an F basis, the set of all idempotents $e(i_1, \cdots, i_m)$ where the i_j's are distinct. The dimension of N is now clear.

To continue with the proof of our claim, we consider the following. Denote by η_i: $L \to T_m(L)$ the natural embedding of L onto the i^{th} factor of the tensor power $T_m(L)$. We show now that if $a \in L$ generates L over F, and $r \neq s$, then $(\eta_r(a) - \eta_s(a))e$ is a unit in N. As before, we may assume F is an algebraically closed field. Write $a = \alpha_1 e(1) + \cdots + \alpha_n e(n)$ $(\alpha_i \in F)$ and note that $\alpha_r \neq \alpha_s$ if $r \neq s$. It suffices to show that $(\eta_r(a) - \eta_s(a))e(i_1, \cdots, i_m)$

$\neq 0$ for all m tuples $(i_1, \cdots, i_m)$ with the i_j's distinct. But $(\eta_r(a) - \eta_s(a)) \cdot e(i_1, \cdots, i_m) = (\alpha_k - \alpha_t)e(i_1, \cdots, i_m)$ where $k = i_r$, $t = i_s$. This part is done.

To finish the claim, we will embed F_m in N. The embedding is constructed inductively. $F_1 = L$ we embed via η_1, and we denote the image of F_1 by N_1. The image of $u_1 \in L$, in N_1, we designate as x_1. More generally we designate by x_i the element $\eta_i(u_1)e \in N$. Define a map $F_2 \to N$ by sending u_2 to x_2. To show that this map is well defined, it suffices to show that x_2 is a root of $f_1'(x) \in N_1[x]$, f_1' being the image of $f_1 \in F_1[x]$. Now $0 = f(x_2) = (x_2 - x_1)f_1'(x_2)$. Since $x_2 - x_1$ is a unit, $f_1'(x_2) = 0$. In a similar way, if φ: $F_i \to N$ has been defined with $\varphi(u_i) = x_i$, and $N_i = \varphi(F_i)$, then we let $f_i' \in N_i[x]$ be the image of $f_i \in F_i[x]$, and note that $0 = f(x_{i+1}) = (x_{i+1} - x_1)(x_{i+1} - x_2) \cdots (x_{i+1} - x_i)f_i'(x_{i+1})$ and so $f_i'(x_{i+1}) = 0$. Now φ extends to F_{i+1} by setting $\varphi(u_{i+1}) = x_{i+1}$. By this inductive process, we have defined a map φ: $F_m \to N$ such that $\varphi(u_i) = x_i$. φ is surjective because the x_i's generate N, and φ is injective because F_m and N have equal dimensions. This proves our claim.

Using the isomorphism φ, we can transfer the action of S_m on N over to an action on F_m, and this latter action is exactly as is described in this theorem. Finally the fixed ring of S_m on N is exactly $e\Sigma_m(L/F) = E_m(L/F)$ and so the theorem is proved. Q.E.D.

The above theorem gives some handle on the nature of $E_m(L/F)$ when L is a field. For example, one has this next corollary.

COROLLARY 1.6. *Suppose L/F is a separable extension of fields of degree n and the Galois group of L/F is S_n. Then $E_m(L/F)$ is a field, for any $1 \leq m \leq n$.*

§ 2. The correspondence theorem

The corollary which ended the last section showed that the existence of idempotents in $E_m(L/F)$ implies that L/F is "not so bad". In this section we develop a much more detailed relationship between idempotents in $E_m(L/F)$ and the structure of L/F. Specifically, we are concerned with F subalgebras of L of the following type. We say that the F subalgebra $L' \subset L$ has codimension m if and only if L has constant dimension m over L'.

We are interested in a special class of idempotents of $E_m(L/F)$. Consider $E_m(L/F) \subseteq T_m(L/F)$ and recall the maps η_i: $L \to T_m(L/F)$ defined in

Section One. If $e \in E_m(L/F)$ is an idempotent, one can view $T_m(L/F)e$ as a module over L via multiplication by $\eta_1(a)$ for $a \in L$. We say e is *regular* if $T_m(L/F)e$ has constant dimension as an L module. Of course, if L is a field then all idempotents of $E_m(L/F)$ are regular. The effect of assuming regularity is to force such e to reflect the structure of L/F and not to arise from idempotents of L or F. In order to make our next definition, note that for regular $e \in E_m(L/F)$, $eE_m(L/F)$ has constant dimension over F. The *rank* of e we define to be the dimension of $eE_m(L/F)$ over F.

Consider the case F is an algebraically closed field. Any idempotent of $E_m(L/F)$ is a sum of the primitive idempotents $e[i_1, \cdots, i_m]$, where the i_j's are distinct integers between 1 and n. Changing notation, set $e[i_1, \cdots, i_m] = e[A]$ where A is the m element set $\{i_1, \cdots, i_m\}$. Thus any idempotent $e \in E_m(L/F)$ can be written as $e[A_1] + \cdots + e[A_r]$, where the A_i are distinct m element subsets of $\{1, \cdots, n\}$. The integer r is the rank of e.

LEMMA 2.1. *With F algebraically closed, $e = e[A_1] + \cdots + e[A_r]$ is regular if and only if $n \mid rm$ and each j, $1 \leq j \leq n$, appears in exactly rm/n of the A_i's.*

Proof. Write $L = Fe(1) + \cdots + Fe(m)$. Once again, consider $L \subset T_m(L/F)$ by identifying L and $\eta_1(L)$. For an e as above, $T_m(L/F)e$ has constant L dimension if and only if the F modules $T_m(L/F)ee(j)$ have equal dimension for all j. Call this dimension s_j. Note that $e[A]e(j) \neq 0$ if and only if $j \in A$. Thus each j appears in exactly s_j of the A_i's. Suppose the s_j's are all equal to, say s. Then by counting, s must be rm/n. This lemma is now clear. Q.E.D.

For the rest of this section we assume that m divides n. Still treating the case F is algebraically closed, we have that if $e \in E_m(L)$ is regular, then $r \geq n/m$. We say e is *basic* if e is regular and has rank exactly n/m. The next lemma is an immediate consequence of the last.

LEMMA 2.2. *Assume $m \mid n$ and that F is an algebraically closed field. Then $e = e[A_1] + \cdots + e[A_r]$ is basic if and only if the A_i's form a partition of the set $\{1, \cdots, n\}$.*

Consider, now, general L and F. If $e \in E_m(L/F)$ then the above lemma and 1.2 show that e has rank greater than or equal to n/m. As in the special case, we say e is basic if it is regular of rank equal to n/m.

The main result of this paper is a bijection between basic idempotents in $E_m(L/F)$ and F subalgebras $L' \subset L$ of codimension m. This bijection is defined as follows. For a basic idempotent $e \in E_m(L)$, set $\Psi(e)$ to be the subalgebra of L equal to $\{a \in L \mid \eta_1(a)e = \eta_2(a)e\}$. Note that since e is symmetric, $\Psi(e)$ is also equal to $\{a \in L \mid \eta_r(a)e = \eta_s(a)e\}$ where $r \neq s$.

THEOREM 2.3. *Ψ induces a bijection between basic idempotents of $E_m(L)$ and F subalgebras of L of codimension m.*

Proof. Assuming e is basic, we first observe that basic idempotents behave well with respect to direct sums and base change. That is, if $F = F_1 \oplus F_2$ and $E_m(L) = E_m(L_1/F_1) \oplus E_m(L_2/F_2)$, then the basic idempotents of $E_m(L)$ are exactly the elements of the form $e_1 \oplus e_2$ where $e_i \in E_m(L_i/F_i)$ is basic. One can quickly compute that $\Psi(e_1 \oplus e_2) = \Psi(e_1) \oplus \Psi(e_2)$. Similarly if $F' \supseteq F$ and $e \in E_m(L/F)$ is basic, then $e \otimes 1 \in E_m(L \otimes F'/F')$ is basic and $\Psi(e \otimes 1) \cong \Psi(e) \otimes_F F'$.

The next step is to show that if $e \in E_m(L/F)$ is basic, then $\Psi(e)$ is as claimed. Using the first paragraph, one sees that we may assume F is an algebraically closed field. In this case, we can write $e = e[A_1] + \cdots + e[A_r]$ where the A_i's are a partition. An easy calculation shows that $\Psi(e) = \{\alpha_1 e(1) + \cdots + \alpha_n e(n) \mid \alpha_i = \alpha_j$ whenever i, j are in the same $A_k\}$. That $\Psi(e)$ has codimension m is now clear.

Conversely, suppose $L' \subset L$ is an F subalgebra of codimension m. Viewing L as an L' algebra we can define $T_m(L/L')$, $\Sigma_m(L/L')$, and $E_m(L/L')$. Denote by $f \in \Sigma_m(L/L')$ the idempotent such that $f\Sigma_m(L/L') = E_m(L/L')$. $T_m(L/L')$ only differs from $T_m(L/F)$ in that the former is a tensor power over a larger coefficient ring. Thus there is a natural F algebra surjection $\varphi\colon T_m(L/F) \to T_m(L/L')$. Note that φ preserves the action of S_m, and so induces a surjection $\Sigma_m(L/F) \to \Sigma_m(L/L')$. Denote by V and V' the m^{th} exterior powers of L taken with respect to F and L' respectively.

Just as before, there is a natural map $\rho\colon V \to V'$. Using φ we consider V' to be a $\Sigma_m(L/F)$ module and note the immediate fact that ρ is a $\Sigma_m(L/F)$ module map. It follows that ρ and φ induce a surjection $\mu\colon E_m(L/F) \to E_m(L/L')$. When $E_m(L/F)$ and $E_m(L/L')$ are considered subsets of $T_m(L/F)$ and $T_m(L/L')$ respectively, μ is just the restriction of φ. Let $e \in E_m(L/F)$ be the unique idempotent such that μ induces an isomorphism $eE_m(L/F) \cong E_m(L/L')$. Since V' has dimension one over L', $E_m(L/L')$ has dimension n/m over F and so e has rank n/m. φ induces an isomorphism from $eT_m(L/F)$ to $fT_m(L/L')$, and so e is regular and hence basic. That $\Psi(e) \supseteq L'$

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follows precisely from the fact that $T_m(L/L')$ is a tensor power over L'. Finally, $\Psi(e) = L'$ using dimensions.

It remains to show that Ψ is injective. Suppose that $e' \in E_m(L/F)$ is basic and $e \in E_m(L/F)$ is constructed, as in the above paragraph, using $L' = \Psi(e')$. Let $W \subset T_m(L/F)$ be the F submodule spanned by all elements of the form $\eta_r(a) - \eta_s(a)$ for $a \in L'$ and $r \neq s$. Then W is exactly the kernel of φ and e' annihilates W. If W' is the annihilator of W, then φ is injective on W' and $e, e' \in W'$. Since $e' \in E_m(L/F)$, $\varphi(e') \in E_m(L/L') = f\Sigma_m(L/L')$ and so $\varphi(e')f = \varphi(e')$. Pulling back to W' we have $e'e = e'$. But e and e' have the same rank so $e = e'$. Q.E.D.

Consider the case L is a field. Then every idempotent of $E_m(L/F)$ has rank $\geq n/m$. Thus the basic idempotents are the minimal ones. Note that a minimal idempotent will not be basic, unless some basic idempotent occurs.

COROLLARY 2.4. *Let L be a field. Then either L/F has no subfields of codimension m, or there is a bijection between such subfields and the set of minimal idempotents of $E_m(L/F)$.*

Supposing $e \in E_m(L/F)$ to be basic, then $eE_m(L/F)$ has dimension n/m over F. Therefore it is natural to expect that $eE_m(L/F)$ is isomorphic to $\Psi(e) \subset L$.

THEOREM 2.5. *$E_m(L/F)e \subset \eta_1(L)e$, and if this last algebra is identified with L, then $E_m(L/F)e = \Psi(e)$.*

Proof. To begin, let us show that $E_m(L/F)e \subset \eta_1(L)e$. As usual, it suffices to show this when F is an algebraically closed field. We have $e = e[A_1] + \cdots + e[A_r]$ where the A_i's partition $\{1, \cdots, n\}$. For each i, $1 \leq i \leq n$, let $r(i)$ be such that $i \in A_{r(i)}$, and set $B_i = A_{r(i)} - \{i\}$. Write $f_i = \sum_\sigma e(i) \otimes e(\sigma(a)) \otimes \cdots \otimes e(\sigma(b))$ where the summation is over all permutations of B_i. An easy exercise shows that $f_1, \cdots, f_n$ form a basis of $\eta_1(L)e$. Since $e[A_j]$ is the sum of f_i for $i \in A_j$, the inclusion is clear.

As $E_m(L/F)e$ and $\Psi(e)$ have equal dimension, we finish this proof by showing $E_m(L/F)e \subset \Psi(e)$. But if $\eta_1(a)e \in E_m(L/F)e$, then $\eta_1(a)e$ is fixed by S_m and so $\eta_1(a)e = \eta_2(a)e$. That is, $a \in \Psi(e)$. Q.E.D.

It is instructive to consider a special case of 2.3. Suppose L is a field and Galois over F with group G. For each $\sigma \in G$, $L \otimes_F L$ contains a unique idempotent $e(\sigma)$ such that, for all $a \in L$, $(a \otimes 1)e(\sigma) = (1 \otimes \sigma^{-1}(a))e(\sigma)$. Furthermore, $L \otimes_F L = \sum_\sigma Le(\sigma)$.

An easy argument shows that for any $\sigma_2, \cdots, \sigma_m \in G$, $T_m(L/F)$ has a unique idempotent $e(\sigma_2, \cdots, \sigma_m)$ such that

$$\eta_1(a)e(\sigma_2, \cdots, \sigma_m) = \eta_j(\sigma_j^{-1}(a))e(\sigma_2, \cdots, \sigma_m)$$

for all $2 \leq j \leq m$ and all $a \in L$. The $e(\sigma_2, \cdots, \sigma_m)$'s comprise all the minimal idempotents of $T_m(L/F)$. Therefore the action of S_m on $T_m(L/F)$ induces a permutation action of S_m on the $e(\sigma_2, \cdots, \sigma_m)$'s. We briefly describe this later action. Viewing S_m as the permutation group of the set $\{1, \cdots, m\}$, consider $S_{m-1} \subset S_m$ to consist of those permutations which fix 1. S_{m-1} acts on the $e(\sigma_2, \cdots, \sigma_m)$'s via permutation of the σ_j's. If $\tau \in S_m$ is the two cycle $(1\ k)$, then $\tau(e(\sigma_2, \cdots, \sigma_m)) = e(\sigma_k^{-1}\sigma_2, \cdots, \sigma_k^{-1}, \cdots, \sigma_k^{-1}\sigma_m)$.

Define $e[\sigma_2, \cdots, \sigma_m] \in T_m(L/F)$ to be

$$\sum_{\tau \in S_{m-1}} e(\sigma_{\tau(2)}, \cdots, \sigma_{\tau(m)}) .$$

It is now not hard to derive the following result.

THEOREM 2.6. *The basic idempotents of $E_m(L/F)$ are exactly the idempotents $e = e[\sigma_2, \cdots, \sigma_m]$ where $\{1, \sigma_2, \cdots, \sigma_m\} \subset G$ is a subgroup of order m. $\Psi(e)$ is the fixed field of this subgroup.*

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