PROBLEMS FOR SOLUTION

P. 175. Prove or disprove: Every *totally disconnected* (no connected subset contains more than one point) topological space is Hausdorff.

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SOLUTIONS

P. 163. (BULLETIN (6) **12** (1969), p. 873). Show that for any real f on the unit interval, the graphs of all its Bernstein polynomials $B_n f$ lie in the convex hull of the graph of f.

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Solution by O. P. Lossers, Technological University, Eindhoven, Netherlands.
First solution: We observe the following facts:

(i) If f is linear, i.e., if there exist constants a, b such that $f(x)=ax+b$ for all $x \in [0, 1]$, then $B_n f=f$;
(ii) if $f_1(x) \le f_2(x)$ for all $x \in [0, 1]$, then $(B_n f_1)(x) \le (B_n f_2)(x)$ for all $x \in [0, 1]$.

The assertion follows immediately from these observations and from the fact that the convex hull of the graph of f is the intersection of all supporting closed half-spaces.

Second solution: For each $x \in [0, 1]$ we have:

$$x = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} \left\{\frac{k}{n}\right\}, \qquad (B_n f)(x) = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} \left\{f\left(\frac{k}{n}\right)\right\},$$

so the point $(x, (B_n f)(x))$ is a convex combination of the points

$$\left(\frac{k}{n}, f\left(\frac{k}{n}\right)\right), \quad k = 0, 1, \ldots, n.$$

The assertion follows immediately from this observation.

Also solved by C. Davis, S. Spital and R. Roodrick, K. Salkauskas, and the proposer.