SMALL SCALE SOLAR MAGNETIC FIELDS: THEORY

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1. INTRODUCTION

One of the most exciting developments in solar physics over the past eight years has been the success of ground based observers in resolving features with a scale smaller than the solar granulation. In particular, they have demonstrated the existence of intense magnetic fields, with strengths of up to about 1600G. Harvey (1976) has just given an excellent summary of these results.

In solar physics, theory generally follows observations. Intergranular magnetic fields had indeed been expected but their magnitude came as a surprise. Some problems have been discussed in previous reviews (Schmidt, 1968, 1974; Weiss, 1969; Parker, 1976d; Stenflo, 1976) and the new observations have stimulated a flurry of theoretical papers. This review will be limited to the principal problems raised by these filamentary magnetic fields. I shall discuss the interaction of magnetic fields with convection in the sun and attempt to answer such questions as: what is the nature of the equilibrium in a flux tube? how are the fields contained? what determines their stability? how are such strong fields formed and maintained? and what limits the maximum field strength?

We also need to know what field strengths are possible beneath the surface of the sun, for magnetic fields are important probes for investigating its interior. In the photosphere the magnetic pressure in a flux tube is nearly equal to the gas pressure outside. If this balance persisted deep in the convective zone, fields of 10^7G might be formed - and dynamo theories of the solar cycle would have to be altered to accommodate them.

2. MAGNETIC FLUX TUBES AND CONVECTION

In a compressible fluid, convection is dominated by rising and expanding plumes. The numerical experiments of Graham (1975) show broad upwellings and narrow sinking regions. In a cell with fluid

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rising at its centre, the horizontal velocity is directed outwards over most of the depth. This picture is confirmed by observations: in a granule there is a broad central region with hot rising gas, surrounded by a narrower ring of rapidly sinking gas at the periphery (Kirk and Livingston, 1968; Deubner, 1976).

The kinematic transport of weak magnetic fields is well understood. Flux is rapidly swept aside and concentrated at the edges of convection cells and particularly at corners where several cells meet (Parker, 1963; Clark, 1965, 1966; Weiss, 1966; Clark and Johnson, 1967). The effect of radial inflow in three dimensional convection has been strikingly demonstrated by Galloway (1976). At the same time, the field within the cell is distorted by the motion and eventually expelled from a persistent eddy. In the sun, however, the lifetime of turbulent eddies is too short for this process to be completed (Weiss, 1966).

Flux concentration is limited by the Lorentz force. The magnetic field separates into ropes, where large scale convection is suppressed, while convection proceeds in the field-free region in between (Weiss, 1964). The formation of ropes is confirmed by dynamical calculations for two-dimensional (Peckover and Weiss, 1972, 1976; Weiss, 1975) and axisymmetric (Galloway, 1976) models. Pre-existing flux ropes must influence the pattern of motion itself so as to maintain the separation of vigorous convection from the almost stagnant flux tubes.

Opinions differ as to the structure of magnetic fields in the sun. On the one hand, turbulent dynamos seem to require strong small scale fields everywhere (e.g. Krause, 1976). On the other, Piddington (1974, 1975a,b,c, 1976a,b,c,d), in a spate of papers has castigated most other authors as supporters of "diffuse field theories". Those of us who have argued for flux ropes may not recognize his version of our views. Observations show that there is an intimate relationship between small scale magnetic fields and convection (e.g. Dunn and Zirker, 1973; Mehltretter, 1974). On a supergranular scale, the network fields display similar behaviour. Most of the flux seems to remain at cell boundaries, while the inner network fields (Harvey, 1976) involve only a relatively small flux. Apparently magnetic fields in the solar convective zone are normally concentrated into ropes. It does not follow, however, that these ropes are formed independently of convection, nor is it necessary to introduce twisted strands of a primeval field in order to provide models that are compatible with observations. Indeed, the only viable theories for explaining the structure of small scale magnetic fields rely on the interaction of those fields with convection.

Traditionally, it has been supposed that the field strength, B, in a flux rope cannot exceed the equipartition field, B_e. The equipartition field has an energy density equal to the kinetic energy density of the motion,

$$B_e^2/8\pi = \tfrac{1}{2}\rho U^2$$

where ρ is the density and U is the maximum speed of convection. This appealing, and uniquely simple, dimensional argument is incorrect. In the photosphere, B_e is less than 600G, yet fields of 1500G have been observed. The justification for the equipartition limit is that pressure fluctuations in a convecting fluid should be of order $\frac{1}{2}\rho U^2$ and, if the density and temperature in the flux rope are similar to those outside, the magnetic pressure cannot be greater than these pressure fluctuations. But if the flux tube is evacuated the internal pressure falls and the equipartition limit is irrelevant. A slight pressure excess is sufficient to squeeze the flux tube and to induce a downward flow of gas. Since the density in the sun increases rapidly with depth, the displaced matter can easily be accommodated.

3. STATIC EQUILIBRIUM IN A FLUX TUBE

Observations show that the magnetic field in a pore or sunspot drops to zero at the boundary in a distance too small to be resolved. The current sheet at the boundary has a very small but finite thickness owing to the finite electrical conductivity of the gas. It is reasonable, therefore, to adopt an equilibrium model in which the field is discontinuous at the boundary. Continuity of normal stress then requires that

$$P_i + B^2/8\pi = p$$

at the boundary, where P_i is the gas pressure within the flux tube and p is the pressure outside. Hence the magnetic field at the boundary cannot be greater than the value

$$B_p = (8\pi p)^{\frac{1}{2}}$$

that balances the external pressure when $P_i = 0$.

It has been suggested (e.g. Stenflo, 1975) that a twisted force-free field might have a mean longitudinal component $\langle B_z \rangle$ greater than B_p, contained by an aximuthal field at the boundary. The field on the axis may indeed be larger than B_p but Parker (1976a) has proved that $\langle B_z \rangle < B_p$ for cylindrical force-free fields. There are two effects that might allow the observed field to be greater. If the boundary of the flux tube is inclined at an angle θ to the vertical then the central field is greater than that at the boundary. With a monopole field, for instance,

$$\langle B_z \rangle = B_p \sec^2 \tfrac{1}{2}\theta \approx B_p(1+\tfrac{1}{8}\theta^2).$$

For small flux ropes this correction is negligibly small. Secondly, the Wilson depression in the flux rope makes it possible to observe fields at a greater geometrical depth (Spruit, 1976). The local pressure scale height in the external gas is about 200Km; if the

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level with optical depth unity is depressed by 100Km then the central field may rise by 30%. The influence of these effects on the average field is relatively slight and so $\langle B_z \rangle$ cannot appreciably exceed B_p. In the photosphere $B_p \approx 1600$G. Direct measurements of Zeeman splitting in an infrared line (Harvey, 1976) give values of $\langle B_z \rangle$ that are close to this. Reports of higher average fields are not likely to be verified.

Within the flux tube, large scale convection is suppressed by the magnetic field and the heat transport is reduced, so that the gas is cooled (e.g. Cowling, 1976a). Nonradiative transport in a strong magnetic field is highly anisotropic and the jump in temperature at the boundary of a flux tube can therefore be sustained. In small flux ropes the lateral radiative flux becomes significant (Zwaan, 1967; Spruit, 1976). Parker (1974c,d, 1975a, 1976a,d) has suggested that sunspots are refrigerated owing to the efficient transformation of energy into Alfvén waves. However, the amount of energy emerging from sunspots into the corona is comparatively small, and it is improbable that hydromagnetic waves can be generated so efficiently (Cowling, 1976a,b). Magnetic inhibition of convection remains the most likely explanation of cooling in sunspots and small flux ropes.

The simplest models of small flux tubes have assumed an axisymmetric vacuum field (Simon and Weiss, 1974; Meyer et al., 1976). More sophisticated magnetohydrostatic models, allowing for the internal pressure, have been put forward by Chapman (1974), Stenflo (1975), Spruit (1976) and Wilson (1976). Spruit has constructed a family of models with fluxes increasing from 3×10^{17}mx to 10^{19}mx, and surface radii from 80Km to 500Km. By a height of 400Km the radius has doubled: adjancent tubes must therefore merge and this effect may explain the more diffuse fields found by Simon and Zirker (1974). Spruit suggests that the appearance of bright faculae near the limb is caused by emission from the walls of flux tubes, seen in projection. Chapman, on the other hand, explained the disappearance of faculae at the centre of the disc by a temperature stratification with a cool layer overlaid by a hot region, heated presumably by hydromagnetic waves.

So far, only static models have been mentioned. The observations reported by Harvey (1976) indicate the presence of downward velocities in the photosphere, though it is not clear whether these are permanent or transient effects (see Durrant, 1976). Steady motion along the field lines has been discussed by Ribes and Unno (1976) and by Parker (1976b,c). To conserve mass there must also be upward motions, and the small scale fields are probably the sites of spicules with rapid upward surges (Parker 1974a,b; Unno et al., 1974).

4. PRODUCTION AND MAINTENANCE OF INTENSE MAGNETIC FIELDS

The manifest failure of the equipartition argument has stimulated various attempts to replace it. It has even been suggested

(Sreenivasan, 1973; Stenflo, 1975) that force-free fields can spontaneously be amplified by a Beltrami flow; of course, the energy must be supplied from somewhere and work is done by the external gas in compressing the flux tube. Provided that the total magnetic energy in the flux tube remains small compared with the kinetic energy of a granule, there is no difficulty in supplying enough energy to form a strong field within the lifetime of a granule. Parker (1974a,b) has considered mechanisms of hydraulic concentration by turbulent pumping, kneading and massaging of the flux tube (see the discussion by Durrant, 1976). The resulting increase in the field strength is only the appropriate equipartition field, which is still too low.

In sunspots, which are much larger than individual granules, inhibition of convection leads to cooling and a collapse to a final state with a strong magnetic field, as originally suggested by Biermann (see Cowling, 1976a). This thermal mechanism cannot be applied to flux ropes that are much smaller than the local scale of convection, for the field is swept aside before any thermal instability can grow (Galloway et al., 1976). So we have to discover what limits the concentration of flux by convection once the dynamical effects of the magnetic field have become important.

The idealized problem of laminar convection in a Boussinesq fluid with an imposed magnetic field has been fairly thoroughly investigated. Since pressure does not enter the equation of state, the pressure within the flux tube can always be reduced to ensure a magnetohydrostatic balance. In fact, pressure can be altogether eliminated from the governing equations, so the equipartition argument (which depends on balancing contributions to the total pressure) is obviously irrelevant. Busse (1975) pointed out that the peak field depended on the relative rates of viscous and ohmic dissipation, and could be made arbitrarily large by choosing a sufficiently high value for the ratio of the viscous to the magnetic diffusivity. This has been confirmed for fully nonlinear convection by several series of systematic numerical experiments (Peckover and Weiss 1971, 1976; Weiss 1975; Galloway, 1976) in which peak fields distinctly greater than the equipartition field have been produced. (In the most extreme case, with the diffusivity ratio equal to 10, the peak field $B^* \approx 6B_e$, though the particular numerical value is of no significance.)

In these computations runs were made with the thermal boundary conditions kept fixed while B_o, the average magnetic field over a whole convection cell, was varied. When B_o is small, flux concentration is purely kinematic and B^* is limited by diffusion. B^* remains proportional to B_o throughout the kinematic regime but, as B_o is increased, dynamical effects eventually become important. Thereafter, in the dynamic regime, flux concentration is limited by the Lorentz force and the rate of working of the buoyancy force is balanced by ohmic dissipation. The peak field B^* reaches its maximum value, B_m, at the transition from the kinematic to the dynamic regime, when ohmic dissipation in the flux rope becomes comparable with viscous dissipation in the convective cell.

For solar convection a similar criterion should apply (Galloway et al., 1976). Consider a flux rope of radius δ concentrated between granular convection cells with a radius d (where $d \gg \delta$). The rate of dissipation of kinetic energy in a granule is approximately $\frac{1}{2}\rho U^2 d^3/\tau$, where the lifetime τ of a convective eddy is approximately the turnover time d/U. The rate of ohmic dissipation in the flux rope is approximately $(\eta B^{*2}/\delta^2).\delta^2 d$, where η is the magnetic diffusivity, and is therefore independent of δ. The two rates are equal when

$$B^* \approx B_m \sim (Ud/\eta)^{\frac{1}{2}} B_e .$$

Hence fields much greater than B_e can be maintained if the magnetic Reynolds number $(Ud/\eta) \gg 1$. However, the value of η depends not on the laminar diffusivity but on the effective diffusivity provided by small scale oscillatory convection within the flux rope. This effective diffusivity can be estimated by taking a velocity of 1km s^{-1} and a horizontal scale of 100Km, corresponding to microturbulent velocities in sunspots (Beckers, 1976); then $\eta \approx 10^{11} cm^2 s^{-1}$. Hence, for intergranular fields $B_m \sim 15B_e \sim 5000G$. Thus it is possible to produce photospheric magnetic fields that are much greater than the equipartition limit. Since B_m exceeds the critical field B_p that balances the external pressure, the limit is in practice set by B_p. Depending on the flux contained, small ropes should have average fields of up to 1600G.

Thermal effects are only important near the photosphere. Deeper down, the magnetic pressure is much smaller than the gas pressure. Arguments similar to those above suggest that the peak field in flux ropes between supergranules should not be greater than about 10000G and even in the deep convective zone the field is not likely to reach significantly higher strengths (Galloway et al., 1976). Certainly the limit B_p cannot be approached except at the upper boundary of the convective zone.

5. STABILITY OF FLUX ROPES

In all simple magnetohydrostatic models the radius of the flux tube increases with height, owing to the vertical pressure gradient outside. Hence the field fans out at the boundary and is concave towards the external plasma. Parker (1975b) and Piddington (1975a) have argued that the configuration should therefore be intrinsically unstable, and liable to interchange (or flute) instabilities. A more careful treatment, based on the energy principle of Bernstein et al. (1958), shows that the field is stabilized locally by buoyancy effects (Meyer et al., 1976), provided that the flux is greater than about 10^{19}mx. Thus there is no need to invoke twisted magnetic fields (which have not been observed) to stabilize a sunspot. Small flux tubes are weakly unstable, though the instability probably grows too slowly for it to be significant. To hold the flux together, a deeper collar is still required and this may be provided by the interaction with external convection. Without such a collar, the total magnetic energy can be reduced by splitting up the flux tube (Wilson, 1976b).

Observations show that sunspots and pores have lifetimes much longer than the timescale for dynamical instability (the time taken by an Alfvén wave to travel across the flux rope). Obviously they are stable. Small scale fields also survive for longer than the dynamical timescale, but their behaviour is dominated by convection in the photospheric granules that surround them. Individual points and crinkles in the filigree are buffeted and jostled by granules, and their characteristic lifetime (about 10 minutes) is similar to the lifetime of a granule (Dunn and Zirker, 1973). Magnetic flux is shunted to and fro in the intergranular lanes and concentrated particularly at junctions. The field strength attained depends on the amount of flux brought together. Once the convection pattern alters, the flux disperses owing to the effective diffusion caused by small scale motion (cf. Meyer et al., 1974); for rope 100Km in radius, the diffusive timescale is about a minute.

6. CONCLUSION

The jump in gas pressure at the boundary of a flux rope, like the cooling of sunspots, is a shallow phenomenon, confined to the region where the magnetic pressure can balance the external pressure. The formation of a sunspot, which is much larger than individual granules, begins when a flux rope protrudes into the photosphere from below or when smaller flux tubes are assembled by supergranular convection. Magnetic inhibition of convection reduces the supply of energy to the photosphere and the temperature falls, so reducing the pressure. The spot then collapses until a stable equilibrium is reached.

Filamentary magnetic fields follow a different scenario. Their scale is determined locally by granular convection; in particular, the inner network fields can only be concentrated by small scale convection. Once the field is strong enough to impede heat transport, the temperature falls within the flux rope and the pressure difference $(p-p_i)$ is sufficient to squeeze the tube and to drive gas, mainly downwards, along the field lines. Deeper down, where the density is much greater and the magnetic pressure is much less than the external pressure, the density in the flux tube is actually increased and the cooling correspondingly enhanced in order to maintain a hydrostatic balance. Evacuation of the flux tube at the photosphere continues until the magnetic pressure there is strong enough to balance the external pressure. The resulting configuration is then maintained by a slow flow of gas across the field, driven by convection in adjacent granules.

A more detailed treatment of small scale magnetic fields is still needed. So far, we lack a decent theory of convection in a magnetic field, and the energy transport in a magnetic field can only be calculated by making rather arbitrary assumptions (cf. Deinzer, 1965; Spruit, 1976). With a better theory of convection, we would be able to construct improved equilibrium models and then to study their stability.

Nevertheless, we do now have a consistent qualitative understanding of photospheric magnetic fields. They are rapidly concentrated into ropes, whose appearance depends on a single parameter: the magnetic flux contained in them. For fluxes less than about 10^{17}mx the field is concentrated kinematically; the peak field strength is proportional to the flux and is limited by the effective (turbulent) diffusivity (Galloway et al., 1976). Thus small filamentary fields with strengths of several hundred to a thousand gauss must be common. For greater fluxes, the field reaches the limiting value B_p that balances the external pressure. So for fluxes above 10^{17}mx we expect to have fields of about 1500G. As the flux rises, the area of the tube increases with it until, for fluxes greater than about 10^{19}mx, the radius becomes comparable with that of a granule. Cooling is then more effective and a dark pore is formed. For yet higher fluxes, the field has to spread out in order to achieve a magnetostatic equilibrium. At the boundary the field becomes increasingly inclined, while the field strength at the centre becomes significantly greater than 1500G. Eventually, the field is almost horizontal at the edge of the pore. For fluxes greater than about 3 x 10^{20}mx, convection finally penetrates into the magnetic field, forming the filamentary penumbra characteristic of a sunspot.

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