PART VI

MATTER-ANTIMATTER UNIVERSES AND PHYSICAL PROCESSES NEAR THE SINGULARITY

(Chairman: I. M. Khalatnikov)

QUANTUM DESCRIPTIONS OF SINGULARITIES LEADING TO PAIR CREATION*

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Abstract. Inhomogeneous generalizations of the Kasner cosmological models have been found by Gowdy, and can be used to exhibit simplified models of quantized gravitational fields. One finds that a quantum description can be given arbitrarily near the singularity. Graviton pair creation occurs, and can be seen to convert anisotropic expansion rates into the energy of graviton pairs.

1. Introduction

What I have to present are the results of studying a particular class of cosmological models which provide a mathematically convenient, but highly idealized, description of a cosmological singularity that develops into a pair creation epoch, and terminates in an adiabatic expansion with redshifting particle energies. This class of models was found by Gowdy (1971, 1974) as a set of exact solutions of the classical empty space Einstein equations describing inhomogeneous universes populated only by gravitational waves. Thus the pair creation we deal with is the creation of graviton pairs, but for the main qualitative features of the pair creation process that are the focus of interest here, it is not expected that gravitons are less representative than the photons, electron-positron pairs, or scalar quanta treated in previous work on pair creation in gravitational fields (De Witt, 1953; Parker, 1969, 1972; Zel'dovich, 1970, 1972; Zel'dovich and Starobinsky, 1971). There are two major differences from these groundbreaking treatments. One is that, by basing the work on exact classical solutions of Einstein's equations, the gravitational influences of the created pairs back upon the expanding universe are not ignored. The second is that, within some model of the quantum theory of gravity, a description of the Universe at times prior to $t_{\text{Planck}} = (\hbar G/c^5)^{1/2} \simeq 10^{-43}$ s can be analysed.

Berger (1972, 1973) was the first to apply the Gowdy cosmological models to questions of pair creation. She used the ADM (Arnowitt, Deser, Misner) quantization methods, and considered a problem of the sort previously posed, namely, assuming a 'no particle' state at some early time t_0, how many particles are there at much later times. I subsequently rewrote this work (Misner, 1973) using superspace quantization methods (see Misner, 1972 for an introduction), and began asking a somewhat different question: for a given quantum state of the Universe at the singularity, how many particles are there at the later times of classical adiabatic expansion?

Before going on to describe the results of these studies, I should give somewhat more detail about the model. Gowdy's T^3 metrics are Einstein-Rosen plane wave solutions with boundary conditions of spatial periodicity imposed to give space sec-

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tions a 3-torus topology. (Other Gowdy models have $S^2 \times S^1$ or S^3 topologies.) The T^3 metrics read

$$ds^2 = \exp(-\tau - \tfrac{1}{2}\lambda)\,(-e^{4\tau}\,dt^2 + d\theta^2) + e^{2\tau}(e^{\beta}\,d\sigma^2 + e^{-\beta}\,d\delta^2) \tag{1}$$

where the metric parameters τ, λ, and β are functions only of θ and t. Each of the three space coordinates $\theta\sigma\delta$ is treated as an independent angle to give the T^3 space topology. The Einstein equations include $\partial^2\tau/\partial t^2 = 0$, and one takes τ proportional to t, excluding any θ-dependence, as a coordinate condition. The parameter β satisfies a simple linear wave equation with a time-dependent phase velocity; explicit exact solutions can be written by a Fourier series for the θ-dependence leading to Bessel functions in the time dependence. There is a cosmological singularity at $\tau = -\infty$ which is of the Kasner-like type that is described in another paper here by Belinskii *et al.* (1974; see also Khalatnikov and Lifshitz 1963; Eardly *et al.*, 1972). In this model the Khalatnikov-Liftshitz parameter u governing the Kasner exponents is a function of θ only, $u = u(\theta)$, and gives the asymptotic values of $\partial\beta/\partial\tau$ at the singularity $\tau \to -\infty$. For late times $\tau \to +\infty$ a WKB solution to the wave equation for β is valid, and one can unambigously speak of gravitational waves or gravitons. In this limit the solution is the precise parallel of the DZN solution (Doroshkevich *et al.*, 1967) for unidirectional collisionless radiation in the Bianchi type I anisotropically expanding homogeneous universe, except that gravitons replace neutrinos or photons.

2. Dissipation of Anisotropy

One main result from studying these models is support for Zel'dovich's idea that pair creation will reduce expansion anisotropy. Although the solutions considered are not elaborate enough to evolve into Friedmann solutions, they do show energy in anisotropic motions near the singularity being converted into energy of pairs. However all created pairs have momenta along the single preferred axis, so high anisotropy remains in the particle (graviton) momentum distribution. More realistic models would have to include particle-particle collisions that would tend to isotropize the momentum distribution. In other studies of the dissipation of anisotropy (Matzner and Misner, 1972; Matzner, 1972) however, it was the conversion of anisotropy energy into particle energies that was most difficult to achieve, and Stewart (1969) had established limits on the rate at which such conversions could proceed in ideal gases of massless particles. (Dissipation via pair creation need not be subject to the limitations of Stewart's theorem.) Once the anisotropy is changed from that of cosmological expansion rates to anisotropy of particle momentum distributions as illustrated in these Gowdy models, it can then be dissipated completely within a few particle-particle collision times, so that Friedmann solutions would result. Because no graviton-graviton scattering occurs in the Gowdy models, this second step in the isotropization process does not occur in them, and they shed light only on the crucial first step.

3. Model Quantization

Let us now proceed to another question that these solutions help us with, namely, how can one describe the initial conditions of the Universe. The quantum models that can be based on the Gowdy cosmologies suggest that a quantum language could be developed in which 'the state of the Universe' a times prior to 10^{-43} s would have a formal significance corresponding to well defined elements of the mathematical structure. The model to be described here, and some I have given earlier (Misner, 1972), even suggest that early states of the Universe can be chosen so they have clear asymptotic forms near the singulalrity which might be called states for the Universe at the singularity. The fact that the singularity is quite evident in these model quantum theories poses the important question: can one formulate and prove *quantum singularity theorems* which require, even in a geometry with quantum limitations, a cosmological singularity of essentially the same inevitability and significance as the singularities in the classical Einstein theory? (For a viewpoint which can accept the singularity as part of physics rather than treating it as indicative of failures in physical theories, see the last section of Box 30.1 in MTW (Misner *et al.*, 1973).)

The model quantization to be described here allows one to begin to focus more concretely on some of these very abstract and speculative possibilities. Figure 1

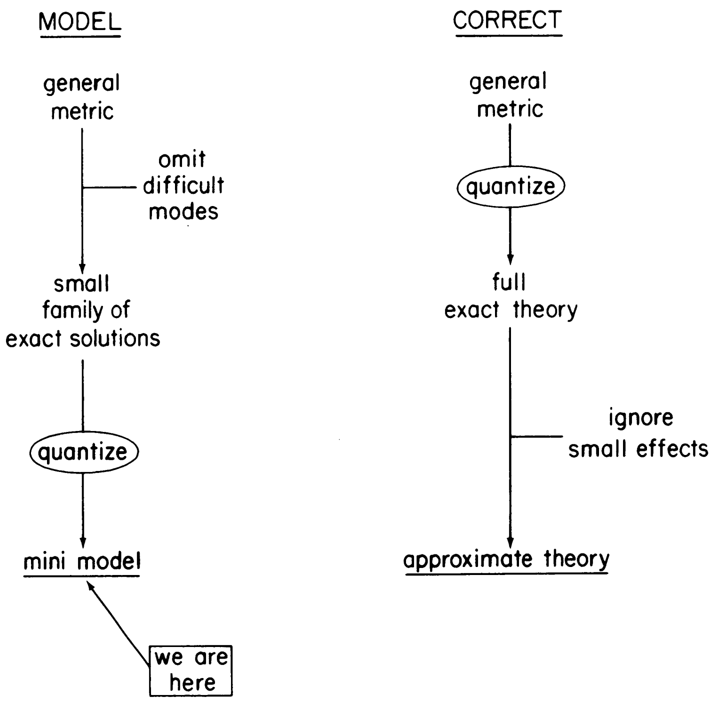

Fig. 1. Model quantum theories may show the format and some physical features of a quantum theory of gravity, but they ignore quantum fluctuations in many modes by setting both the coordinates and momenta of the omitted modes to zero in violation of the uncertainty principle.

warns, however, of the difference between a model quantization and an approximate calculation within a proper quantum theory of gravity. The model quantization shows the methodology being developed for quantizing the gravitational field, but it omits quantum fluctuations in infinitely many degrees of freedom and their possible interactions with the degrees of freedom which are retained. (In the Gowdy models the amplitudes for graviton propagation in all but one direction have been discarded.) The model quantization of the Gowdy metrics uses methods previously employed (Misner, 1972 and papers cited there) for a finite number of degrees of freedom, as generalized by Kuchař (1971, 1973) to models like the present one with infinitely many degrees of freedom. The format of the quantization actually follows most closely Moncrief's (1972) ideas leading to a theory with a *single residual constraint*, rather than the Dirac or ADM methods modelled in earlier work. In the Dirac approach one has infinitely many hamiltonian constraints, one per space point, while the ADM approach imagines that these have all been solved prior to quantization, with a resultant loss in formal covariance. Moncrief showed that, as a condition on the state functional Ψ, a single linear combination of the Dirac Hamiltonian constraints implies all of them. This format achieves much of the close ADM parallel to elementary quantum mechanics without a comparable loss in formal covariance.

In the model quantization of the Gowdy cosmologies, the single residual constraint that one finds is a Klein-Gordon type equation

$$(\underbrace{\square}_{-+++\cdots+\cdots} + \underbrace{\mathscr{R}}_{\int {}^3R\,{}^3g\,\mathrm{d}^3x.})\,\Psi = 0 \tag{2}$$

The wave operator here is defined in the infinite-dimensional minisuperspace in which the coordinates (supercoordinates, i.e. metric parameters) may be chosen as τ, $\lambda_0 \equiv (2\pi)^{-1} \oint \lambda\, \mathrm{d}\theta$, and q_n with $(q_n + iq_{-n})/\sqrt{2} = (2\pi)^{-1} \oint e^{in\theta}\beta\, \mathrm{d}\theta$, $n > 0$.

As indicated in Equation (2) the signature of the supermetric (metric in this minisuperspace) is Lorentz or hyperbolic with one negative sign and all others positive. The concept of a metric structure in superspace (De Witt, 1967) arises from this equation. That is, one first writes the constraint provided by Einstein's equations, which in this model is

$$\left[\frac{\partial^2}{\partial\tau\partial\lambda_0} + \sum_n \frac{\partial^2}{\partial q_n^2} + \mathscr{R}\right]\Psi = 0, \tag{3}$$

and then deduces from the form of this equation that a metric structure plays a significant rôle. In this model

$$\mathrm{d}s^2 = 2\delta\tau\delta\lambda_0 + \sum_{-\infty}^{\infty} (\delta q_n)^2 \tag{4}$$

defines the 'distance' in minisuperspace between two 3-space metrics g_{ij} and $g_{ij}+\delta g_{ij}$, each of the form specified by Equation (1). (The fourier components λ_n of λ, other than λ_0, do not appear in Equation (4) and are determined by explicitly solving the momentum constraints.) The second term $\mathscr{R}$ in Equation (2) arises from the curvature of the 3-dimensional $\tau=\text{const}$ space sections in a peculiar looking way,

$$8\pi\mathscr{R}=\int(\sqrt{{}^3g}\;{}^3R)\sqrt{{}^3g}\;\mathrm{d}^3x. \tag{5}$$

It is related to spatial coordinate invariance in a manner analyzed by Moncrief (1972), and is determined by the choice of the time coordinate condition. This term acts as a potential, or better as a variable mass term, in Equation (2). Because $\mathscr{R}$ can be both positive and negative in sufficiently general models, the wave propagation in this Klein-Gordon like equation is not normally restricted to the interior of the 'light-cones' of the supermetric, although this restriction does hold in the present models where $\mathscr{R}\leqslant 0$, and appears to be typical of the dominant behaviour quite generally near the cosmological singularity.

The residual constraint Equation (3) in this example can be solved by separation of variables. The wave functional Ψ factors, and the factor corresponding to a single Fourier component $q=q_n$ of β satisfies an equation of the form

$$i\frac{\partial\psi}{\partial t}=-\tfrac{1}{2}\frac{\partial^2\psi}{\partial q^2}+\tfrac{1}{2}e^{4t}q^2, \tag{6}$$

where some constant factors have been omitted. One sees that this equation presents the same mathematical problem as the Schroedinger equation of a simple harmonic oscillator whose spring constant is increasing exponentially in time. Two descriptions of states satisfying Equation (6) are then at hand. One uses the energy E and momentum p of the analogue oscillator mass, the other uses its excitation level quantum number N in the oscillator potential well. For the oscillator frequency $\omega=e^{2t}$ of Equation (6), the relationship between these two modes of description is given by

$$E=(N+\tfrac{1}{2})\,\hbar\omega=(N+\tfrac{1}{2})\,e^{2t}. \tag{7}$$

In general neither E nor N is constant, but there are limits in which one or the other is.

In the application which led us to Equation (6) q is a Fourier component q_n of the metric parameter β, so the statement 'ψ is a state of excitation level N' is read 'there are N gravitons present of wavelength mode n', and the case of constant N corresponds to this notion of a graviton being well-defined. The condition for constancy is that the oscillator frequency changes in Equation (6) be adiabatic:

$$\frac{t_{\text{osc}}}{t_{\text{change}}}=\frac{1}{\omega}\frac{\mathrm{d}\ln\omega}{\mathrm{d}t}=2e^{-2t}\ll 1. \tag{8}$$

Thus for $t\gg 1$ the adiabatic or WKB solution of Equation (6) is valid and gravitons propagate preserving their number N while undergoing the usual cosmological red-shifts. [Note: E is the energy of the analogue harmonic oscillator, but does not give the graviton energy.]

In the opposite limit, $t \to -\infty$, the potential term in the Schroedinger Equation (6) vanishes, and the analogue oscillator becomes a free particle. Then p and $E = \frac{1}{2}p^2$ become constants, so that from Equation (7) it is clear that N is rapidly varying. Figure 2 shows that the time evolution of ψ (idealized in Figure 2 as its classical limit, the position q of the analogue oscillator mass) can be divided into three eras, the 'free particle' motion with constant E and p near $t = -\infty$, a possible intermediate 'pair creation' stage where the potential is acting but does not change adiabatically, and the era $t \gg 1$ of constant graviton number N. The 'free particle' $t \to -\infty$ limit of the Schroedinger Equation (6) translates into a Kasner-like singularity for the metric of Equation (1). The model quantum theory finds no obstacles to the analysis of this singularity regime, and wave packet solutions of Equation (6) [with $e^{4t} = 0$] are familiar. What is not appropriate is any attempt to describe this singularity era in terms of graviton numbers N. From Equation (7) with constant E, one sees that N must begin (at $t = -\infty$) arbitrarily large and decrease rapidly, but from Figures 2 and 3

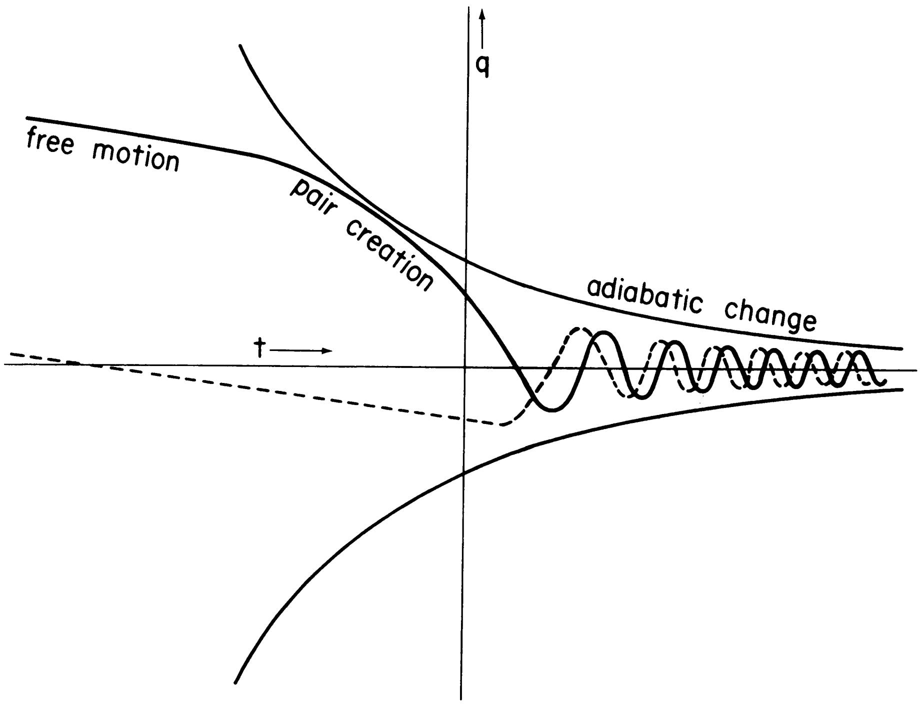

Fig. 2. The Newtonian mechanical motion of a mass point under the influence of a time dependent potential $V = \frac{1}{2}e^{4t}q^2$ presents the same mathematical problem as the evolution of one Fourier component of the gravitational waves in the Gowdy universes. The figure shows schematically two possible classical motions confined within the potential walls $\frac{1}{2}e^{4t}q^2_{\text{wall}} = E_{+\infty}$. In the typical case (full curve) the straight line free motion is terminated by interaction with the potential wall ('pair creation' era) at some time $t < 0$ when an adiabatic approximation is not yet valid. Classically this is an era of parametric amplifications of the wave amplitude q, while quantum mechanically the amplification of zero-point fluctuations (creation of quanta) also occurs. For $t > 0$ the time dependence of the potential is adiabatic, and the potential influences the motion for all initial conditions (i.e., for the broken curve also), but the excitation state $N = E/\omega$ (number of quanta) is an adiabatic invariant and remains constant.

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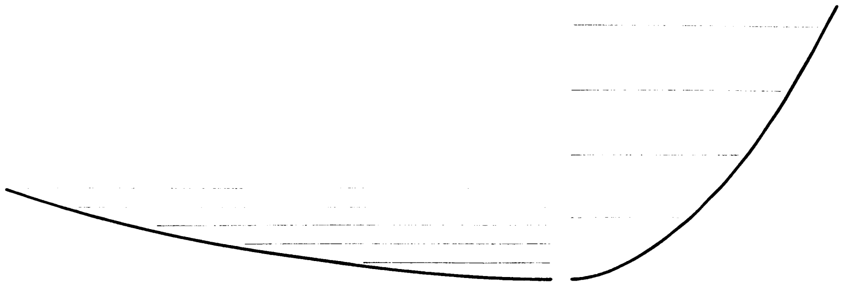

Fig. 3. The oscillator potential $V=\frac{1}{2}e^{4t}q^2$ is sketched at two different times. When the potential is stronger, the quantum excitation states, shown as horizontal lines, are more widely spaced. When the potential is too weak to have any influence, the system (gravitational wave amplitude or analogue mechanical oscillator) will remain at fixed energy while the potential and the quantum levels within it change. In this case the evolution from one time (one side of the diagram) to another could correspond to a large change in N without any change in the wave function ψ, as occurs in the quantized Gowdy models near the cosmological singularity. Under conditions of adiabatic change in the potential, as for $t\rightarrow+\infty$ in these model cosmologies, the wave function changes under the influence of the potential to maintain its excitation level N constant.

one sees that N is irrelevant during this 'free particle' portion of the analogue oscillators motion. The oscillator potential is too weak at these times to influence the wave function ψ, but the level spacing of the harmonic oscillator states is rapidly changing, and with it the excitation level N assigned to a fixed wave packet. Only when the potential begins to influence the evolution of the wave function ψ, as in the 'pair creation' era in Figure 2, do the excitation levels N in this potential become physically meaningful.

Unfortunately, although this model theory can be solved exactly both classically and quantum mechanically a good summary has not yet been formulated of the graviton pair creation which the model embodies. What seems to be lacking is a properly insightful measure of pair creation – a measure which would ignore the changing N values when momenta p are constant near the singularity, and ignore the changing p and E values in the late stages when N is constant, and then summarize the extent of the non-adiabatic work done in creating pairs in the intermediate stage.

Another approach to the description of the quantum evolution of this model universe is to ask not 'how much graviton creation has occurred?' but instead 'how does the graviton population at late times depend on the conditions at the initial singularity?' This is an S-matrix approach – relate the final state to the initial state – but one in which different modes of description are applicable to the two limits. Classically the initial states near the singularity in these cosmological models can be characterized by the constants $p_{-\infty}$ and q_0 in the linear ('free particle') solution

$$q=q_0+p_{-\infty}\tau \tag{9}$$

of the Einstein equations near $\tau=-\infty$. The resultant final state for $\tau\rightarrow+\infty$ may then be found to correspond to a fixed number N of gravitons. The relationship between the initial state and final state is the cosmological S-matrix. In this example one finds

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from semi-classical arguments a relationship of the form (Misner, 1973)

$$N \propto [p_{-\infty}(u)]^2 + (q_0)^2, \tag{10}$$

and Berger (1974) has given a complete and exact solution for the S-matrix in the quantum model.

In Equation (10), which refers to some fixed Fourier component q_n of the gravitational field amplitude β, one of the initial conditions $(q_0, p_{-\infty})$ is familar in the classical description of the Kasner-like singularity. This is the momentum $p_{-\infty} = p_n$ conjugate to q_n, which is found to be just a Fourier component of the Khalatnikov-Lifshitz parameter $u = u(\theta)$ describing the expansion rate anisotropy near the singularity. The other initial condition parameter q_0 in the initial conditions for q_n (Fourier component of β) reflects inhomogeneities not in the expansion rates, but in the shape of the model universe near the singularity. Thus Equation (10) describes how inhomogenieties in the initial singularity appear as gravitational waves (gravitons) after the expansion slows and graviton production stops. But it does not suggest that any choice for the initial conditions $(q_0, p_{-\infty})$ is more natural or appealing than any other. One lacks on the one hand a 'ground state' as a uniquely unexcited state among the many possible initial 'free particle like' wave packets, and on the other hand one lacks as well an energy-like controlling variable to allow statistical states of many degrees of freedom to be limited by a single parameter. It is possible that one or both of these simplifications in discussing the initial state of the Universe could occur in more general models. As Belinski *et al.* (1974) have described, the more typical singularity behaviour is closer to the mixmaster model than to the Kasner one. But Jacobs, Zapolsky and I have found (reported in Misner, 1972) that quantum models of the mixmaster cosmology do have something that could be called a ground state near the singularity, and they also provide an energy-like variable which is asymptotically constant near the singularity and limits all the q_0 and $p_{-\infty}$ type initial condition variables in that problem. Thus if inhomogeneous universes could be studied which had mixmaster-like singularities, one might hope to find simpler answers than in the present model. One could then perhaps ask what spectrum and number of quanta result if the Universe begins in a 'ground state singularity'; or one could ask the same questions for an initially stationary statistical state with a single excitation parameter governing all modes and wavelengths simultaneously. Although suitable techniques for approaching these questions are not known now, the recent progress in advancing from the homogeneous Kasner models first quantized only a few years ago, to their inhomogeneous analogues in the Gowdy models now, suggests that some effort in this direction is justified.

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DISCUSSION

Icke: Are the predictions of your model only to be tested in inaccessible places like the early universe, or is there some hope that laboratory tests may be used?

Misner: I am not aware that quantum gravity can be implicated in observations in any domain except, possibly, the early universe. Thus Novikov's report this morning is very exciting since it holds out the hope that some second set of measurements, not equivalent *a priori* to G, h and c, could point to a time of 10^{-43} s or smaller. Observations bearing on this prospect then have a most fundamental significance.

Starobinsky: In the model considered by Prof. Misner, only gravitons moving in one direction are taken into account, so the time of isotropization of such a model need not be equal to the Planck time but depends upon initial conditions just as in classical solutions. Only when the excitation of all quantum modes in three dimensions is taken into consideration may the isotropization time be of the order of the Planck time for any initial conditions.