SELF-CONSISTENT OBLATE-SPHEROID MODELS

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ABSTRACT. A method for the construction of models of axisymmetric galaxies is presented. In this formulation we determine the distribution function corresponding to a given gravitational potential and the associated mass density distribution. Although the realization of the model is numerical, the underlying theory is analytic and exact. This method allows us to construct a wide range of models without having to use linear programming and a large amount of computer time. Here we present the results from the application of this method to the "perfect" oblate-spheroid mass model. A large class of valid self-consistent distribution functions which depend on three isolating integrals of the motion is found. The kinematics of many models are consistent with those observed for elliptical galaxies. In particular, models generated by this formulation are in agreement with the observed values of the ratio of the maximum projected rotational velocity to the velocity dispersion along the line of sight versus ellipticity.

FORMULATION OF THE METHOD: We consider a model with a given mass density and gravitational potential which satisfies Poisson's equation and we seek solutions for the distribution function (d.f.) which satisfies Liouville's equation (Bishop 1986a). Self-consistency requires that the matter described by the d.f. be the source of the potential. We choose potentials which admit of three isolating integrals of the motion (I_1, I_2, I_3) and according to Jeans' theorem, the d.f. will depend on them. We make the ansatz: $f(I_1, I_2, I_3) = g(I_1, I_2, I_3)h(I_1, I_2)$. If $g(I_1, I_2, I_3)$ is an assigned function we seek solutions for $h(I_1, I_2)$, the derived function. Thus if B is equal to

$$B(I_1, I_2, \mathbf{x}) = \int g(I_1, I_2, I_3)J(I_1, I_2, I_3, \mathbf{x})dI_3,$$

where $J(I_1, I_2, I_3, \mathbf{x})$ is the Jacobian determinant of the transformation, then the fundamental integral equation is

$$\rho(\mathbf{x}) = \int B(I_1, I_2, \mathbf{x})h(I_1, I_2)dI_1dI_2. \tag{1}$$

Equation (1) relates a function of two variables in configuration space to a function of two variables in integral space. Since $\rho(\mathbf{x})$ and $B(I_1, I_2, \mathbf{x})$ are known the integral

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equation can be solved for $h(I_1, I_2)$ and the d.f., and its solution, is unique. In order to solve equation (1) numerically, we approximate the continuous stellar system as a discrete system. We divide the configuration and the integral space into a set of finite cells. We choose a set of integrals such that the division of the integral space and the configuration space is accomplished with the same cell structure. All continuous equations are made into discrete equations and equation (1) becomes a set of linear equations which is composed of a lower triangular matrix.

MASS MODEL AND RESULTS: We demonstrate this method with the "perfect" oblate spheroid mass model (de Zeeuw 1985). The corresponding potential admits of three isolating integrals and the motion separates in prolate spheroidal coordinates (λ, μ, ϕ). All orbits are short axis tubes and the motion consists of two librations and one rotation. The double turning points of the orbit are isolating integrals, so $(\lambda^+, \mu^+, \lambda^-) \longleftrightarrow (E, L_z, I_3)$. A natural choice of cell shape has boundaries which are spheroids of constant λ and hyperboloids of constant μ. In each cell there is one double turning point (λ^+, μ^+).

We used $\sim$ 400 cells and $\sim$ 4000 orbits and tested $\sim$ 100 assigned functions. We found a broad range of self-consistent models ($f \geq 0$) (Bishop 1986a,b). We found that rotation curves first rise linearly and then drop off slowly. The maximum values occur near the half-mass radius. The observed values of (V_m/σ_c) versus the ellipticity (ϵ) fall within the allowed region defined by the models (Fig. 1). The lower boundary has approximately equal numbers of stars rotating in each direction. The upper boundary has a range of (N_+/N_-) from 1.9 to 1.002.

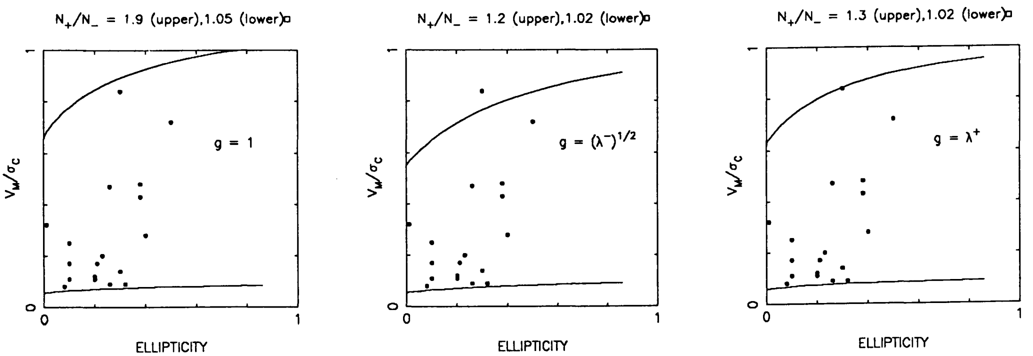

Figure 1. Plot of the ratio of the maximum projected rotational velocity, V_M, to the central line of sight velocity dispersion projected on the sky, σ_c, versus the ellipticity, for the given models. The two lines for each model represent two different rotational states of the same model. N_+/N_- is the ratio of the number of stars moving counter-clockwise to clockwise. Plotted points are values of observed galaxies from Illingworth (1977. *Ap.J.Letters*, **218**, L43.), Schechter and Gunn (1979. *Ap.J*, 229,472.), Davies (1981. *M.N.R.A.S.*, 194, 879.).

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