MASS TRANSFER IN SEMI-DETACHED BINARIES

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Abstract. The dynamics of gas streams in semi-detached binaries is reviewed from the standpoint of the theory of matched asymptotic expansions. Physical concepts are emphasized, and some astronomical applications are considered. The model of impact-driven accretion disks is also discussed briefly.

1. Introduction

There have been many theoretical studies of gas streams in close binaries since the classical discussion given by Kuiper (1941). Apart from the process of mass ejection from the contact component, the dynamics of gas streams is, in broad outline, a relatively uncontroversial subject. A less happy state exists for the theory of accretion disks. Rees (1976) and Lin and Pringle (1976) have summarised the view that turbulent or magnetic viscosity plays a crucial role in the accretion process. As part of this article, I would like to suggest an alternative picture – the mechanism of impact-driven accretion disks. Much of my discussion follows the development given by Lubow and Shu (1975). The mathematical derivations and tabular results can be found in our published work; I review here mainly the physical ideas underlying our calculations, and I comment on some astronomical applications.

Firstly, however, I indicate the advantages and the limitations of my analysis. Our calculations consider the steady-state inviscid (i.e., frictionless) flow. They require only a fraction of a second of CPU time on a computer, in contrast to the several hours required by conventional techniques of numerical hydrodynamics (Biermann, 1971; Prendergast and Taam, 1974; Flannery, 1976), but they have the disadvantage that we are not able to follow the development of the flow in time.

We are able to make progress analytically because the dynamical problem for short-period binaries contains a small parameter whose existence has long been recognized, but which has remained not fully exploited until our work. This parameter ϵ is the ratio of the thermal speed, $(kT/m)^{1/2}$, to the relative orbital speed, Ωd, of the two stars. Since ϵ typically has values on the order of a few percent, we are able to develop a perturbational technique on the basis of the smallness of ϵ. Carrying out the calculations to two orders in ϵ gives all important properties of the stream. Simple scaling laws exist for a given mass ratio of the two stellar components to accommodate all mass transfer rates and all values of ϵ, as long as it is small.

The primary assumption of the analysis is the adoption of the Roche model with the implicit assumption of synchronous spin of the contact component. This assumption is probably justified in short-period systems where the contact component is of late enough spectral type to have a convective envelope (Zahn, 1966). The latter restriction excludes the direct applicability of our analysis to massive binary X-ray sources, but for these systems good arguments exist, in any case, that the mass transfer is driven by a stellar wind and not by Roche lobe overflow (Ostriker and Davidson, 1973; van den Heuvel, 1975).

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A secondary assumption that we made for the sake of simplicity is that the small parameter ϵ remains constant throughout the flow. The assumption of isothermal flow is justified if the temperature of the gas is governed by the ambient radiation field and not by internal sources of heat. It can be relaxed at the expense of considerably greater computational effort.

2. Mechanism of Mass Loss from the Contact Component

According to generally accepted ideas, a star fills its Roche lobe when it expands because of core evolution or when the Roche lobe descends onto the surface of the star because of the decrease in orbital separation, d, brought on by the effects of gravitational radiation (Plavec, 1968; Paczynski, 1971; Faulkner, 1971). From the point of view of stellar interiors, the surface of a star is defined by the condition that the pressure has dropped to such low values that it can be essentially taken to be zero. The gradient of the pressure is, however, not zero. Indeed, in hydrostatic equilibrium it provides the force which balances gravity and holds the surface at its given location. But the effective gravity is zero at the inner Lagrangian point, L_1, and the unbalanced pressure gradient there must lead to a flow of material away from the L_1 region toward the detached component of the binary system. The contact component ruptures in the neighborhood of the L_1 region, and the situation is somewhat similar to an inner tube with a hole wherein air is pushed out through the hole by the high pressure inside the inner tube.

From the point of view of stellar atmospheres, however, the foregoing description lacks precision. On length scales characterized by the pressure scale height, the atmosphere of a star extends to infinity; hence, a relevant question to ask is: how much of the mass of the atmosphere is above the Roche lobe?

The answer to this question requires an understanding of the mass loss flow on the scale characterizing the structure of the surface layers. Clearly, in a frame which corotates with the orbital motion of the two stars, the gas must start nearly at rest in the surface layers of the contact component and eventually be accelerated by the reversal of gravity near L_1 to hypersonic speeds. We showed that the transition from subsonic flow to supersonic flow on matter-carrying streamlines must be made in a small neighborhood of L_1 which is of fractional size ϵ in comparison with the orbital separation, d. Figure 1 shows a magnified view of this region schematically.

Since the flow in the surface layers is subsonic, the hole at L_1 makes its presence felt on the entire star, and material from the entire surface is eventually pushed toward the hole at L_1. The horizontal flow is not straightforwardly from high pressures to low, except on the equator, because of the strong deflections provided by the Coriolis forces. The flow in the orbit plane of the L_1 region develops as indicated in the caption to Figure 1 and results eventually in a narrow jet of gas which leaves the L_1 region at an angle θ_s with respect to the line joining the two stellar centers. In brief, the gas is pushed by pressure to where it can be pulled by gravity. For isothermal flow conditions, the gravity field dominates the far downstream behavior, and the gas stream develops gaussian density profiles in the two directions normal to the length of the stream.

In a steady state, the density of the material at L_1 must be high enough to accommodate the mass loss rate $\dot{M}$ required by stellar structure considerations. Since the mass loss occurs within a length ϵd of the L_1 point and since the matter reaches velocities

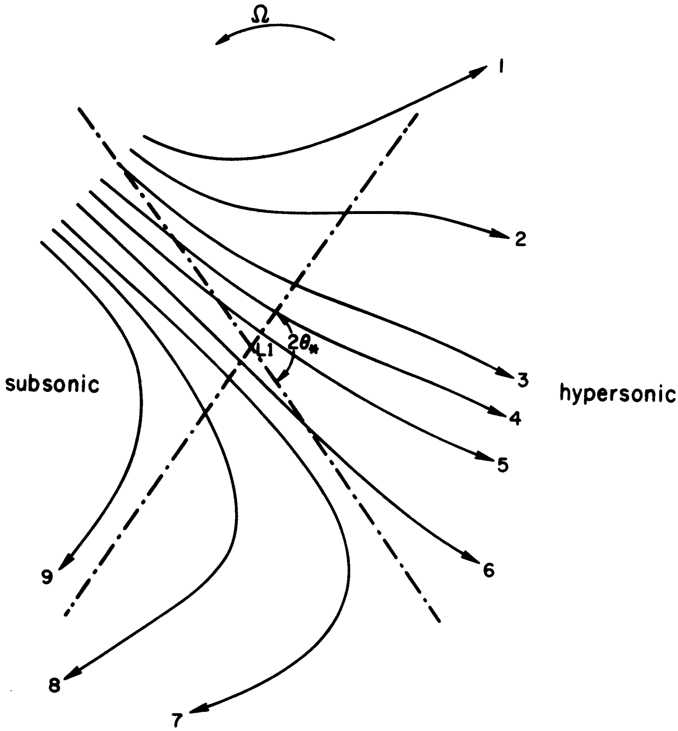

Fig. 1. Schematic diagram of the flow in the L_1 region when plotted in the orbit plane. The dashed-dotted lines show the equipotential curves which intersect at the point L_1 at an angle $2\theta_*$. The streamline 9 lies deep in the surface layer of the contact component and corresponds to a very slow flow which is counter to the sense of orbit rotation. The gradient of the pressure is unbalanced by gravity near L_1, and this leads to a rightwardly directed flow for streamlines 1–8. As the fluid elements on streamlines 7 and 8 cross the Roche surface of the contact component, they must climb uphill against the effective potential in the bottom sector, leading to deceleration and eventual deflection by the Coriolis force back toward high pressures in the surface layers of the contact component. On streamlines 1–5, the fluid elements also encounter a hill in the top sector, but they are deflected by the Coriolis force into the downhill portion of the saddle-potential in the right sector. There is little gas in this sector, and this leads to rapid acceleration by the gravity away from the L_1 region. On streamline 6, only a slight hill is encountered, and the behavior is more similar to that of streamlines 1–5 than streamlines 7 and 8. The gas between streamlines 6 and 8 suffers great rarefaction, whereas the material on streamline 1 is rarefied to begin with, since it originates high in the atmosphere of the contact component. The material between streamlines 2 and 6 results far down stream in a narrow jet of gas which exits the L_1 region with hypersonic velocities and gaussian density profiles in the two directions perpendicular to the jet. (After Lubow and Shu, 1975)

of the order of the sound speed, $\epsilon\Omega d$, at L_1, the density at L_1, ρ_{L_1}, must satisfy the following order of magnitude relation:

$$\rho_{L_1}\,(\epsilon\Omega \mathrm{d})\,(\epsilon d)^2 = \dot{M}, \tag{1}$$

i.e., ρ_{L_1} must be of order ϵ^{-3} in units of $\dot{M}/\Omega d^3$. If the actual density ρ at L_1 is less than this value, the instantaneous mass loss rate would be less than the required rate $\dot{M}$, and the star will continue to grow relative to the Roche surface until ρ equals ρ_{L_1}, and vice-versa.

If the surface layers were in perfect hydrostatic equilibrium, the density on the Roche surface would equal the density ρ_{L_1} since the Roche lobe is an equipotential surface. Actually, we have estimated that a correction factor of order ϵ^{-1} may be required because the general surface layers are more dense than the dynamical regions near L_1. If we now compute the optical depth to the layer where the density is ϵ^{-4} $(\dot{M}/\Omega d^3)$, we will have given an approximate answer to our original question of how much of the atmosphere sits above the Roche lobe.

3. Properties of the Stream in the Orbit Region

The continuation of the stream motion from the L_1 region into the orbit region is shown in Figure 2. Note that the stream trajectory is confined within the Roche lobe of the detached component. The center of the gas stream follows a ballistic trajectory to lowest order in ϵ; this fact provides a formal justification for the procedure carried out by some earlier workers (cf. Kruszewski, 1966). The stream width, height, density, and differential velocity field are, however, not amenable to a ballistic analysis and must be computed as a higher order effect using singular perturbation theory. The results have been tabulated as Table 3 of Lubow and Shu (1975) for a variety of mass ratios. The stream maintains gaussian density profiles in the two directions perpendicular to the length of the stream; the Gaussian width and height generally decrease as the detached component is approached. The stream width and height scale as ϵd, and the central stream density scales as ϵ^{-2} $(\dot{M}/\Omega d^3)$. The differential velocity field turns out to have zero absolute vorticity, a result which is readily understandable in terms of Kelvin's circulation theorem (Lubow and Shu, 1975, §IVd).

In Algol systems and cataclysmic variables, the quantity ϵ^{-2} $(\dot{M}/\Omega d^3)$ can be estimated to have values typically in the range of roughly 10^{-8} to 10^{-10} gm cm^{-3}. Such densities tend to be somewhat higher than the corresponding electron densities estimated for the stream by Batten (1970, 1973); if this difference is real, it may provide useful information concerning the ionization equilibria.

In novae and dwarf novae, simple observational checks exist also for the velocity field – the differential velocities as well as the mean velocities. In these systems, Warner and Nather (1971) and Krzeminski and Smak (1971) have interpreted various features in the light curves in terms of the impact of the stream with the edge of an accretion disk. Such a disk always forms in these systems because the detached component, presumably a white dwarf, is too small to intercept the stream on any part of its direct trajectory. The resulting collisions of the stream with itself creates a directly rotating disk.

We have argued that the equilibrium size of this disk is determined by the condition that the outer edge of the disk nearly constitutes a simple periodic orbit and that the tangential component of the stream speed at stream center before impact must equal the speed of the disk edge. This condition of zero slip is enforced by stability considerations of the Kelvin-Helmholtz type.

In this model, the inward motion of the stream is arrested upon impact by a radiating shock, and the luminosity of the hot spot so formed is equal to $\frac{1}{2}u^2$ $(\dot{M}\Omega^2 d^2)$ where u is the dimensionless inward component of the stream's mean velocity just before impact. For mass transfer rates $\dot{M}$ between 10^{-9} and 10^{-7} $M_\odot$ yr^{-1}, the quantity $\dot{M}\Omega^2 d^2$ is typically $10^{32}-10^{34}$ erg s^{-1} in these systems, whereas $\frac{1}{2}u^2$ can be seen from Table 2 of

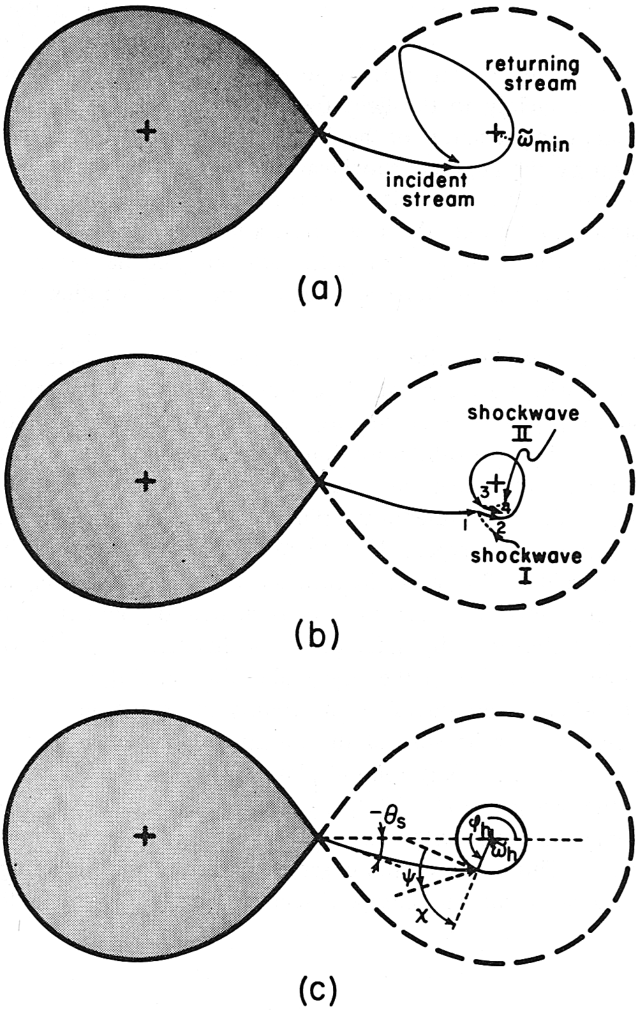

Fig. 2. Stream trajectory and disk formation in the orbit plane graphed for the case of equal masses for the two stellar components. (a) The stream which leaves L_1 returns to strike itself after having reached a minimum distance, $\tilde{\omega}_{min}$, without having been intercepted by the surface of the detached component. (b) Schematic interaction of the stream with the disk edge in the steady-state flow. In the interior of the disk, the material spirals slowly in toward the detached component. (c) Geometry for stream-disk interaction for the steady-state configuration. The disk edge corresponds nearly to a simple periodic orbit. At the point of impact of the stream center with the disk edge, $(\tilde{\omega}_h, \varphi_h)$, a condition of zero slip applies: $|v_{stream}| \cos \psi = v$ *disk edge*|. (After Lubow and Shu, 1975)

Lubow and Shu (1975) to have values typically between 0.5 and 1.5. Thus, the luminosity of the hot spot can contribute a significant fraction of the light seen in such systems. The greatest uncertainty in computing the exact fraction from the observational data would seem to be the bolometric corrections to be applied to the hot spot and the disk.

A striking feature of the light curves in cataclysmic variables is the rapid irregular flickering that occurs with moderate amplitudes. Careful time-resolved observations by Warner and his colleagues of this activity in eclipsing systems have revealed that this flickering is confined mainly to the light from the hot spot. Our interpretation of this phenomenon depends on the nature of the differential velocity field in the stream.

After deflection by the radiating shockwave, the stream becomes the new disk edge. The absolute vorticity in the stream is zero upstream from the shock. In general, it will conflict with orbital motion in the disk, even after modification by passage through a curved shockfront. The resolution of this difficulty must be the formation of a turbulent boundary layer. I believe this turbulence to be the source of the flickering activity in the hot spot.

To check this assertion, note that we would expect the time scale of variation of the largest eddies (those which are comparable in size to the hot spot itself and which have the most turbulent energy) to be on the order of $(\Delta\omega)^{-1}$ where $\Delta\omega$ is the difference in absolute vorticity required for orbital motion in the disk and that possessed by the post-shock stream. For mass ratios near unity, $(\Delta\omega)^{-1}$ works out to be about 1% of the orbital period, on the order of a few minutes in dwarf novae. Moreover, since the driven turbulence is only mildly supersonic, we expect there to be the usual cascade of turbulent energy to smaller and smaller eddies. These are in turn associated with shorter and shorter time scales. The shortest time scale is set by viscous dissipation, and it is smaller than the longest time scale of flickering by a factor of roughly $(Re/Re_{cr})^{-1/2}$ where $Re_{cr} \sim 10^3$ and Re is the Reynolds number of the large-scale differential flow (Landau and Lifshitz, 1959, Chap. III). There is considerable uncertainty in estimating Re for the radiative portion of the shock layer because of the sensitive dependence of the coefficient of viscosity on temperature in a fully ionized plasma. We can roughly take Re to be 10^{11} or more. Hence the factor $(Re/Re_{cr})^{-1/2}$ is $\lesssim 10^{-4}$, and we can expect the flickering in cataclysmic variables to occur with time scales ranging from several minutes to a small fraction of a second. This is indeed the observed nature of the flickering deduced from power spectral analysis of the light curves of such systems (for example, Robinson, 1973, Figure 6).

4. Sizes of Disks

We come now to the comparison of the sizes of the theoretical equilibrium disks and the observed disks. For eclipsing cataclysmic variables, a common method of determination of the disk size is to estimate the phase angle between when the hot spot is seen face-on and when it is in mid-eclipse by the detached component (see, for example, Smak, 1971; Krzeminski and Smak, 1971; Warner and Nather, 1971; Warner and Peters, 1972). There is, however, an error in the usual assumption that the face of the hot spot is parallel to the disk edge because it will be inclined to it by an amount appropriate for an oblique radiating shock. The amount of inclination will be of the order of 10° for a strong shock in an optically thick medium. If this effect is left uncorrected, there will be a tendency to infer disk sizes that are systematically larger than the actual sizes.

A related comment is that since the hot spot is highly turbulent, its orientation in space may not be fixed as a function of time even for constant mass transfer rates. Thus, observed time variations in the eclipse data do not necessary imply changes in the size of the disk. We hope to perform some theoretical calculations to estimate the magnitude of such effects. In any case, our view is that the disk probably has too much inertia for its size to change abruptly except possibly during outbursts.

Given these cautionary remarks, the available data on the light curves of cataclysmic variables seem to be reasonably consistent with the theoretical determinations. This provides a welcome check to the contention that the Kelvin-Helmholtz instability does indeed regulate the disk size to a hydrodynamic equilibrium value.

If the massive part of the disk is indeed as small as indicated by our calculations and those of Flannery (1976), then we can be confident that the stream does impact hypersonically and inelastically on the disk edge and that the mass transfer process must be essentially 100% efficient as long as the energy released from the accretion process does not energize a strong wind from the disk (cf. Shakura and Sunyaev, 1973). For Algol-type systems, this picture would then be consistent with the assumption made in evolutionary calculations that there is no mass loss from semi-detached systems, only mass transfer (Plavec 1968; Paczynski 1971). This assertion would not apply to contact systems.

5. Impact-driven Accretion Disks

Let us now consider the mechanism of accretion of mass by the detached component when a gaseous disk is present. The disk has spin angular momentum as well as orbital angular momentum and the former provides a formidable barrier to accretion. Rees (1976) and Lin and Pringle (1976) have already alluded to the usual proposal for the resolution of this difficulty which invokes turbulent or magnetic viscosity (Prendergast and Burbidge, 1968; Pringle and Rees, 1972; Novokov and Thorne, 1973; Shakura and Sunyaev, 1973; Lightman, 1974; Lynden-Bell and Pringle, 1974). Unfortunately, this proposal leaves unexplained the origin of the turbulence in a disk which satisfies Rayleigh's criterion for stability. It also has the problem of disposing of the excess angular momentum transported to the disk edge by the action of friction in the inner parts.

In view of these difficulties, I prefer to pursue what seems to me to be a more natural approach. I believe that if the material in the disk spirals slowly inward to accrete onto the detached component, it does so because it is constantly being slammed into by a jet stream. The stream is, after all, the reason that the disk exists at all, and it is only natural to investigate the possibility that the stream may provide the motive force which prevents the disk material from simply circulating around the detached component forever as Saturn's rings do around Saturn.

The specific mechanism I have in mind is the internal shockwave driven into the disk by the impact of the stream on the disk edge. As the shockwave propagates radially inward, it is swept along by the unperturbed disk flow in the azimuthal direction, and the wavefront forms a trailing spiral (see Figure 3). The presence of this wave prevents the gas in the disk from executing exact simple periodic orbits. This leads to the possibility, at least in principle, that the wave will remove spin angular momentum from the unperturbed disk either by convective transport or by gravitational back reaction on the two stellar components. In a steady state, accretion of material from the disk occurs with the removal from the disk of spin angular momentum which is deposited either as spin angular momentum of the detached component or as orbital angular momentum of the two stars. Indeed, if we can solve the problem of wave transport of mass correctly,

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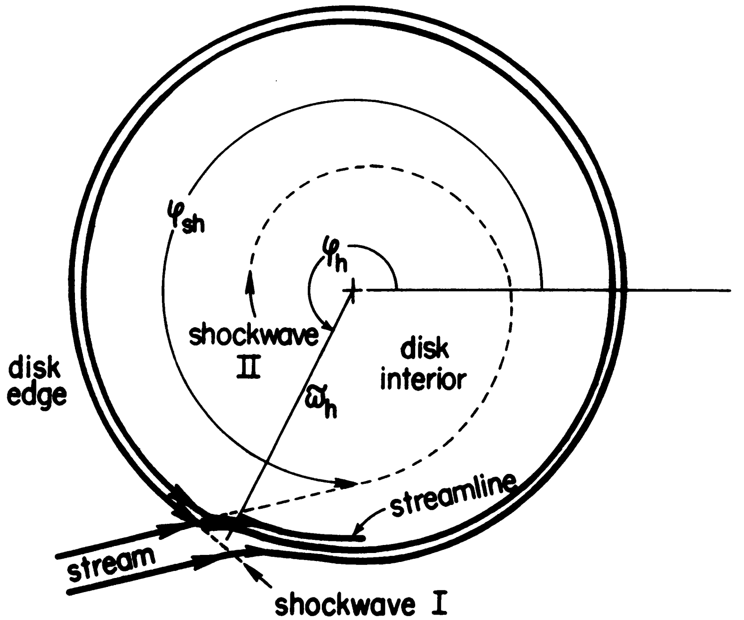

Fig. 3. Schematic diagram of the accretion flow driven by the stream impact. The impact of the stream on the existing disk edge is imagined to produce an inward displacement of the edge, whereas the stream itself is turned by the strong radiating shockwave I to become the new disk edge (cf. Figure 2). The subsequent bumping of gas elements by neighboring elements in the disk interior corresponds to a weak inwardly-propagating shockwave whose front describes a tightly-wound trailing spiral. The presence of shock II prevents the gas elements in the disk interior from moving in exact simple periodic orbits. Thus, the change in the spin angular momentum of the gas element induced by the tidal gravitational field of the contact component will not exactly balance the change in the other half of the orbit. Each element of gas spirals inward toward the central star with the spin angular momentum lost per cycle either being absorbed by the wave (and eventually deposited as spin angular momentum of the detached component) or being absorbed by the orbital motion of the two stars. The latter is accomplished via the gravitational torques which act on the asymmetric excess of matter produced by the presence of shockwave II (cf. Figure 4). In a steady state, the material in the disk is distributed in a manner which allows, per unit time, the stream to deposit a strip of gas on the outside edge of the rotating disk while the central object removes a strip containing the same amount of gas from the inside.

Newton's third law will automatically guarantee the simultaneous resolution of the angular momentum difficulty.

The correct solution of the wave transport problem is, however, no easy feat because the unperturbed distribution of density with position in the disk is not given *a priori. Instead, it* is to be found as part of the solution to the self-consistent problem.

The only safe assumption is that the density of the equilibrium disk must be much greater than that of the stream because of the physical difficulty in overcoming the angular momentum barrier. This assumption suffices for us to treat the disturbance produced in the disk by the stream impact as a small perturbation. Despite this simplification, the resulting problem is still quite complex, and I have not found a completely satisfactory solution; let me outline the basic ideas.

Consider the case when the accretion luminosity in the disk is small in comparison with the luminosity of the two stellar components. We can then avoid the complications involved in discussing the heat transfer problem by postulating the disk flow to occur isothermally.

To describe the compressional wave generated by the impact of the stream, consider the radial component, u, of the fluid velocity at a given radius $\tilde{\omega}$ in the disk, as a function of the azimuthal angle, φ. The stream at impact has a width which is an order ϵ smaller than the radius of the disk edge, and the azimuthal extent of the induced compressional wave in the outer portions of the disk will have dimensions comparable to the stream width. Let us begin, then, by ignoring the curvature of the flow. In this case, the induced compressional wave has a triangular waveform well downstream from the center of the impact point $(\tilde{\omega}_h, \varphi_h)$ (see Figure 4a).

As the wave propagates inwards, the azimuthal extent, $\tilde{\omega}(\varphi_{sh} - \varphi_{tail})$, of the triangular waveform increases as the square-root of the distance propagated whereas the velocity discontinuity at the shockfront decreases by the inverse of the same factor. Thus, the triangular wave changes shape as it dissipates, but it preserves its area. The value of this area can be related to the conditions of the impact of the stream with the disk edge through momentum considerations. Note that the area is negative. This means that if we

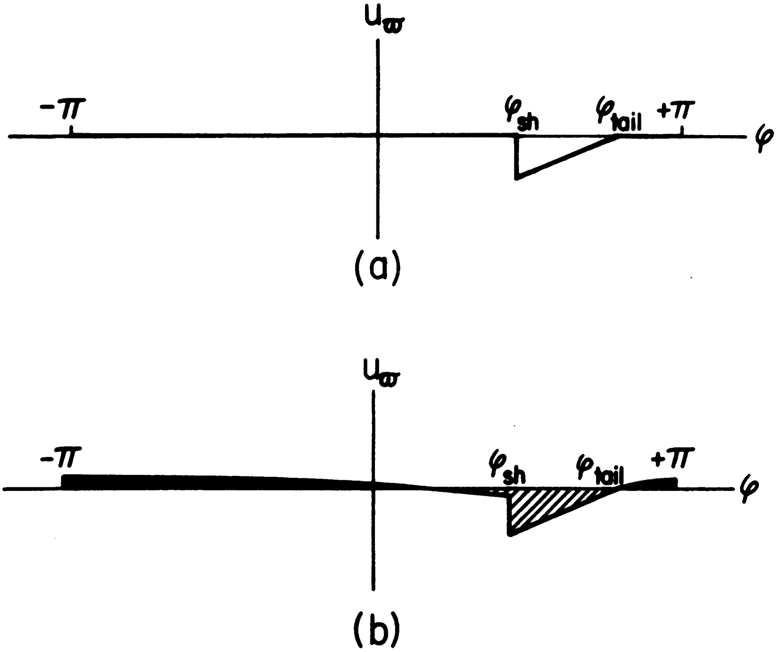

Fig. 4. Schematic diagram of waveform produced in the disk interior by the impact of the stream with the disk edge. (a) The case when the curvature of the disk flow is ignored. A shock is located at $\varphi = \varphi_{sh}$ where there is a sudden decrease in u as the compressive effect of the stream's original impact converges to make itself felt. For $\varphi_{tail} > \varphi > \varphi_{sh}$, the inward motion is reduced as the gas reexpands to the lower pressure of the surrounding medium. (b) The case when the curvature of the disk is not ignored. The flow is periodic in φ. The positive phase arises because Lindblad oscillations are set up by the tendency to conserve angular momentum. The grey area under the positive phase may nearly cancel the negative contribution of the triangular pulse.

were to follow the Lagrangian motion of a fluid element instead of adopting the present Eulerian description, we would find the fluid element to be displaced a net amount inwards as it passes through the wave.

The curvature of the flow in the actual disk introduces two complications. The first complication is purely geometrical and can be handled easily along with slow variations of the unperturbed surface density of the disk, σ_0, with position in the disk. The area of the triangular pulse, $\int u\tilde{\omega}\mathrm{d}\varphi$, no longer remains constant as the wave propagates inward, but varies as $(\tilde{\omega}\sigma_0)^{-1/2}$. Note that the shock strengthens if $\tilde{\omega}\sigma_0$ decreases as we go into the disk. To complete the self-consistent solution, we wish to determine σ_0 so that the mass transfer integral

$$\oint \sigma u \tilde{\omega} \mathrm{d}\varphi \qquad (2)$$

equals $-\dot{M}$ for each circle of radius $\tilde{\omega}$. In expression (2), $\sigma = \sigma_0 + \sigma_1$ is the total surface density, unperturbed plus perturbation.

The second complication introduced by the curvature of the flow is dynamical in origin. Although I have not yet resolved this complication mathematically, I can see that it has the following physical consequences. We can no longer be sure that the integral (2) has only negative contributions. Although $|u|$ achieves its largest values in the triangular pulse, the range of azimuth covered by this negative phase is small; thus its contribution to the integral (2) might be cancelled by a positive phase where the integrand is small relative to the integrand in the negative phase but where the range of integration in φ is large (see Figure 4b). Such a positive phase does not occur for the rectilinear problem, but it is unavoidable in rotational flow because of the tendency to conserve angular momentum. Returning to a Lagrangian description, we can say that a radial displacement of a rotating element of gas generally excites Lindblad oscillations which tend to exhibit equal positive and negative phases.

I would guess that the positive phase in our problem cannot exactly cancel the negative phase since the kick provided by the stream impact, unlike that of a vibrating piston, is always directed inward. In the presence of the shockwave, the gas element must lose a net amount of angular momentum in a complete circuit around the disk (Roberts and Shu, 1972), and the net amount will have to be absorbed by the wave unless the tidal gravitational field of the contact component plays a role. If the angular momentum is absorbed by the wave, a spatial attenuation of the wave additional to the effects discussed earlier will occur. This attenuation may not be critical for disks in Algol-type systems because the wave does not have to propagate very far before reaching the surface of the detached component; however, it is potentially very serious for systems where the detached component is a compact object. The interaction of the wave with the tidal gravitational field will be crucial in the latter case. Fortunately, the issue is a complex one only in a technical sense; there are no fundamental obstacles of unknown physics which would block the ultimate resolution of this problem.

Let me finish this review with the following comment. The current interest in accretion disks concentrates on those cases where the accretion luminosity is large. Thus, the primary emphasis has naturally been on the problem of the radiative transfer. However, the basic energy source which powers the accretion luminosity is the same in all models (except for a factor of 3, see Lynden-Bell and Pringle, 1974, p. 611), and Rees (1976) has already commented that comparisons of the spectrum of radiation emitted from the disks will not readily discriminate between competing models for the accretion process. A better test is the prediction of the masses of accretion disks. These can be checked, for example, against our growing knowledge concerning the dynamics of nova explosions (Sparks and Starrfield, 1973; Weaver, 1974). The simplest interpretation for the asphericity observed for nova shells would yield masses for the disks in novae on the order of $10^{-4}-10^{-6}\,M_\odot$. Such a range of masses may plausibly arise if the impact-driven accretion process is dominant over other effects.

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DISCUSSION

Carter: You have predicted that the disc will be smaller than in the models described by Pringle, in consequence of the no-slip requirement where the stream meets the disc. I understand that Pringle and Lin do not believe that this condition will be satisfied. Could you enlarge on the instability argument by which you would justify your no-slip assumption, and describe where you think their argument goes wrong?

Shu: For the small disks, the no-slip requirement is almost the same as the requirement that the disk edge, which constitutes (nearly) a simple periodic orbit, have the same specific angular momentum as the stream of the impact point. I prefer to state the requirement in the form of the hydrodynamic process which tends to maintain the disk size at its equilibrium value, namely, the spin-up or spin-down of the disk by the Kelvin-Helmholtz instability if the disk is not at its equilibrium value. If the disk has a little internal viscosity which tends to transfer angular momentum outwards, the disk edge

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will try to move outward past the equilibrium value given by the inviscid solution; however, the addition of fresh stream material will continue to spin down the disk edge toward the equilibrium value. If the mass transfer rate inwards driven by the impact is much larger than the mass outflow produced by the viscous tendencies, the disk edge will be close to, but perhaps slightly larger than, its inviscid equilibrium value. If the reverse is true, Lin and Pringle will be correct in asserting that the disk will be considerably larger. My personal guess is that the break even point occurs at an effective turbulent or magnetic Reynolds number on the order of 10^5 to 10^7. If the actual Reynolds number is much larger, I suspect the impact driven accretion is dominant over the viscious mechanism. On the other hand, Lin and Pringle require an effective Reynolds number of about 10^2 to maintain the disk edge at a value close to the Roche lobe. Thus, two crucial dynamical questions which need to be resolved are (a) what is a reasonable value for the effective Reynolds number? (b) what is the exact efficiency of the impact-driven accretion?