ON CALCULATION OF RELATIVISTIC EFFECTS IN NUMERICAL PREDICTION OF THE ARTIFICIAL SATELLITE MOTION

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ABSTRACT. This paper discusses the algorithm of the calculation of relativistic effects in numerical prediction of the artificial satellites motion. An attempt to estimate the values of relativistic effects in the motion of two artificial satellites Navstar and Lageos is made. These values are compared with the quantities of other weak perturbations and the errors of the calculated position of the satellite due to the inaccurate approximation of perturbations.

1. INTRODUCTION

The application of the laser technique in the practice of astronomical observations requires predicting the artificial satellites motion with permanently increasing accuracy.

Within the frame of Newtonian celestial mechanics one can't construct a model of artificial satellite motion representing observations with the precision of 1-5 cm on large intervals of time because the quantities of relativistic perturbations are compared with the precision of observations (Brumberg, 1984).

2. ALGORITHM

A sufficiently simple algorithm of the calculation of relativistic perturbations in the numerical prediction of artificial satellite motion can be constructed by using relativistic equations of the satellite motion suggested by Brumberg (1984)

$$\ddot{\underline{r}} = - \frac{M_1}{r^3} \underline{r} + \frac{M_2}{R^3} \left[-\underline{r} + \frac{3}{R^2} (\underline{R}\ \underline{r})\ \underline{R} \right] +$$

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$$+ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad +$$

$$\begin{aligned} &+ \frac{m_1}{r^3}\Big\{\Big[-(\alpha+\gamma)\,\dot{\underline{r}}^2 + \frac{3\alpha}{r^2}(\underline{r}\,\dot{\underline{r}})^2 + 2(\beta+\gamma-\alpha)\cdot \\ &\cdot \frac{M_1}{r}\Big]\underline{r} + 2(\gamma-\alpha+1)(\underline{r}\,\dot{\underline{r}})\dot{\underline{r}}\Big\} + \frac{2m_2}{R^3}\Big\{\Big[-(\alpha+\gamma)\cdot \\ &\cdot(\dot{\underline{R}}\,\dot{\underline{r}} + \frac{1}{2}\dot{\underline{r}}^2) + \frac{3\alpha}{R^2}\big((\underline{R}\,\dot{\underline{R}})(\underline{R}\,\dot{\underline{r}}) + \frac{1}{2}(\underline{R}\,\dot{\underline{r}})^2\big)\Big]\underline{R} + \\ &+ (\gamma-\alpha+1)(\underline{R}\,\dot{\underline{r}})\dot{\underline{R}} + (\gamma-\alpha+1)\big[(\underline{R}\,\dot{\underline{R}}) + (\underline{R}\,\dot{\underline{r}})\big]\dot{\underline{r}}\Big\} \end{aligned} \tag{1}$$

Here $\underline{r}$ is the geocentric position vector of the satellite, $\underline{R}$ is the heliocentric position vector of the Earth determined by the differential equations

$$\begin{aligned} \ddot{\underline{R}} = &- \frac{M_2}{R^3}\underline{R} + \frac{m_2}{R^3}\Big\{\Big[-(\alpha+\gamma)\,\dot{\underline{R}}^2 + \frac{3\alpha}{R^2}(\underline{R}\,\dot{\underline{R}})^2 + \\ &+ 2(\beta+\gamma-\alpha)\frac{M_2}{R}\Big]\underline{R} + 2(\gamma-\alpha+1)(\underline{R}\,\dot{\underline{R}})\,\dot{\underline{R}}\Big\}, \end{aligned} \tag{2}$$

M_1 and M_2 are masses of the Earth and the Sun multiplied by the constant of gravitation and $m_i = M_i/c^2$ $(i=1,2)$ are gravitational radii of the Earth and the Sun, β and γ are parameters of gravitational theory (for the general relativity $\beta = \gamma = 1$). Coordinate parameter α determines the choice of the systems of the quasi-Galilean coordinates. The cases of $\alpha=1$, $\alpha=0$, $\alpha=2$ correspond to standard coordinates, harmonic coordinates, Painlevé coordinates respectively.

The equations (1) have been derived by the assumption that gravitational field in which the satellite moves is described by the linear combination of the two static Schwarzschild metrics related to the Earth (M_1) and the Sun (M_2). The geocentric metric is introduced by the assumption that the ratio M_1/M_2 may be neglected.

Equations (1) and (2) completed by other perturbations are integrated simultaneously in the process of the numerical prediction. The results of the numerical or analytical integration of equation (2) may be obtained independently of the integration of the equation (1) as well. In this case tabulated quantities $\underline{R}$ and $\dot{\underline{R}}$ are used in the numerical integration.

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3. RESULTS

Let us give the estimation of quantities of relativistic perturbations and compare them with the effect of other weak perturbations and errors which the unprecise knowledge of coefficients of the earth gravitational field and coordinates of the Moon and the Sun introduces in the satellite position. As an example we consider the motion of two earth artificial satellites Navstar and Lageos with the following orbital parameters

Navstar	Lageos
T = 43085 s	T = 13528 s
e = 0.01	e = 0.04
$i = 63^{\circ}.4$	$i = 109^{\circ}.8$

The estimations will be made by means of our numerical model of the earth artificial satellite motion.

The numerical model of the earth artificial satellite motion is a program for the precise calculation of space positions of artificial satellites. The equations of motion have been written by using regular elements introduced by E.Stiefel and G.Scheifele (1971).

The approximation of perturbing forces has been made in rectangular coordinates (Bordovitsyna, 1984), known correlations (Stiefel, Scheifele, 1971) have been used for transformation into regular elements. The equations of the motion are integrated by Everhart's (1974) method.

Peculiarities of the realisation of numerical algorithms in Kustaanheimo-Stiefel space were discussed by us in (Bordovitsyna, 1984).

Accuracy estimations of the numerical integration of equations of the motion of artificial satellites Navstar and Lageos are shown in tables 1 and 2 (Bordovitsyna, Sharkovsky, 1983). A comparison of the estimation obtained by regular elements (α) with estimations obtained by rectangular coordinates (x) and parametric variables (u) of Kustaanheimo-Stiefel is made. The integration is performed by Everhart's method of the 11^{th} order, $\Delta r=(\Delta x^2+\Delta y^2+\Delta z^2)^{1/2}$ is a modulus of the difference of position vectors, obtained by direct and indirect integrations, The present estimations indicate that the application of regular elements with the numerical method of the high order limits the error by 1 cm for both objects.

Let us get to estimating the effect of the relativistic perturbations on the motion of the considered objects. The estimation will be made by the numerical integration of the equation (1) with and without the calculation of the relativistic terms of these equations. In tables 3 and 4 there were data on relativistic effects due to the Earth (r_{Earth}) and the Sun (r_{Sun}) on the motion of artificial satellites Navstar and Lageos.

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Table 1
Accuracy estimations of numerical integration of Navstar equations of motion, Δr, m

Type of variable	The number of revolutions					
	10	20	30	40	50	60
x	$1\cdot10^{-3}$	$1\cdot10^{-2}$	$6\cdot10^{-2}$	$1\cdot10^{-1}$	$1.5\cdot10^{-1}$	$2\cdot10^{-1}$
u	$3\cdot10^{-4}$	$5\cdot10^{-4}$	$7\cdot10^{-4}$	$9\cdot10^{-4}$	$1\cdot10^{-3}$	$1\cdot10^{-3}$
α	$1\cdot10^{-3}$	$1\cdot10^{-3}$	$1\cdot10^{-3}$	$1\cdot10^{-3}$	$1\cdot10^{-3}$	$1\cdot10^{-3}$

Table 2
Accuracy estimations of numerical integration of Lageos equations of motion, Δr, m

Type of variable	The number of revolutions				
	40	80	120	160	200
x	$5\cdot10^{-2}$	$1\cdot10^{-1}$	$3\cdot10^{-1}$	$5\cdot10^{-1}$	1
u	$5\cdot10^{-4}$	$1\cdot10^{-3}$	$3\cdot10^{-3}$	$5\cdot10^{-3}$	$1\cdot10^{-2}$
α	$1\cdot10^{-3}$	$1\cdot10^{-3}$	$5\cdot10^{-3}$	$1\cdot10^{-2}$	$1\cdot10^{-2}$

Table 3
Relativistic effects in the Navstar's motion

Estimated values	The number of revolutions					
	10	20	30	40	50	60
Δr_{Earth}, m	3.2	6.2	9.5	12.5	15.4	17.3
Δr_{Sun}, m	2.3	6.6	21.2	41.8	62.5	113.0

Table 4
Relativistic effects in the Lageos's motion

Estimated values	The number of revolutions			
	10	20	30	40
Δr_{Earth}, m	2.8	5.6	7.9	11.0
Δr_{Sun}, m	0.2	0.6	1.2	2.0

These estimations were obtained by using standard coordinates, when α =1 in (1). The differences of estimation

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owing to the use of harmonic, standard and Painlevé coordinates amount to 3 m for Lageos and 9 m for Navstar at the end of the interval of the prediction.

For comparison let us consider the effect of the high harmonics of the geopotential V_{nm} on the motion of satellites during the same intervals of time. The necessary data are given in table 5.

Table 5

The effects of the high harmonics of the geopotential on the motion of Navstar and Lageos

The name of the satellite			
Lageos		Navstar	
Disturbing force	Δr, m	Disturbing force	r, m
$V_{10,10} - V_{7,0}$	53.71	$V_{9,9} - V_{9,0}$	3.0
$V_{15,15} - V_{11,0}$	0.35	$V_{10,10} - V_{10,0}$	0.5
$V_{20,20} - V_{16,0}$	0.79	$V_{11,11} - V_{11,0}$	0.04
$V_{24,24} - V_{23,0}$	0.01	$V_{12,12} - V_{12,0}$	0.01

Here Δ r are given for such intervals of time: 7 days for Lageos and 30 days for Navstar.

The present estimations (tables 3-5) show that the effects of relativistic perturbations are comparable and even surpass in value the effect of high harmonics of the geopotential for artificial satellites under consideration.

Further let us compare the quantity of the effect of relativistic perturbations with the errors, which the unprecise knowledge of coefficients of the gravitational field of the Earth and errors of the calculation of coordinates of the Moon and the Sun bring in calculated positions of satellites.

The results given in table 6 show the effects of errors of harmonic coefficients on the accuracy of the prediction of the satellite motion.

Table 6

The effects of errors of harmonic coefficients

The satellite	The number of revolutions					
	10	20	30	40	50	60
Navstar	12.5	22.5	45.0	62.0	77.5	95.0
Lageos	4.8	6.7	16.0	22.2		

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These estimations have been obtained by us for the GEM-9, being a sum of modulus of errors due to inaccuracies of gravitational coefficients.

The variation of coefficients has been done according to the rule of 3σ, where σ is a square error of the coefficient.

Let us consider the influence of errors of coordinates of the Moon and the Sun on the accuracy of the prediction of the artificial satellites motion. The dependence of the accuracy of the calculation of coordinates of satellite Navstar on the accuracy of representation of coordinates of the Moon and the Sun may be seen from the results given in table 7. These results show that the modern theories of the motion of the Moon and the Sun make it possible to calculate the effect of lunar-solar perturbations on the artificial satellite motion with sufficient accuracy.

Table 7

The effects of errors of coordinates of the Moon and the Sun on calculated positions of Navstar

r_{Sun}, km	r_{Sat}, m	r_{Moon}, km	r_{Sat}, m
85700	2.54	2100	320.00
32000	0.37	140	9.44
9300	0.34	2	0.13
2600	0.04	0.2	0.004

On the contrary, the data given in table 6 show that the errors of the determination of general gravitational coefficients of modern standard approximations of geopotential don't allow to reveal both the relativistic effects and the influence of perturbations of high geopotential harmonics in the real motion of the artificial satellite.

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