Near Earth Objects, our Celestial Neighbors: Opportunity and Risk
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Abstract. Some of the problems related to Near Earth Objects (NEOs), like orbit determination and ephemeris computation, are not new, and had to be dealt with since the beginning of NEO astronomy. The latter practically started with the discovery of Comet D/1770 L1 Lexell, that passed very close to the Earth in 1770; studies of the chaotic dynamics of this exceptional object continued well into the XIXth century. At the end of the XXth century there has been a renewal of interest in NEOs, as attested by IAU Symposium 236.

Keywords. orbit determination; multiple solutions; close approaches

1. A close passage

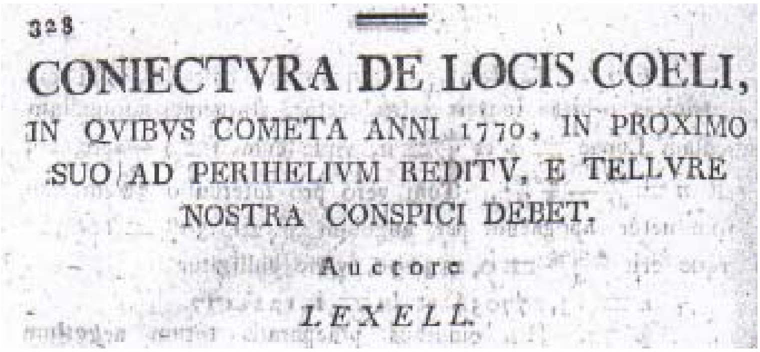
Astronomers started to deal with objects coming very close to the Earth in 1770, when Charles Messier discovered, on the night between 14 and 15 June, the comet that is now known under the name of comet D/1770 L1 Lexell. The comet was heading towards the Earth; within a few days from the discovery it became visible to the naked eye, reaching the second magnitude. The minimum distance from the Earth was reached on 1 July, at about six times the lunar distance, and in a few more days the comet disappeared due to its proximity to the Sun; Pingré computed an ephemeris for its recovery based on a parabolic orbit, as it was then customary, and Messier was able to see the comet again starting from the beginning of August. The comet then remained under observation until the beginning of October.

It soon became clear that, although the ephemeris by Pingré had allowed the recovery of the comet in August, a parabolic orbit could not account for the entire set of observations. Prosperin tried to use three parabolas to fit separately the observations of June, August, and September, but this was evidently unsatisfactory, and unjustifiable from the point of view of Celestial Mechanics.

2. Lexell's work

The solution to this problem was found by Lexell, who showed that the comet was on an elliptical orbit with a period of five and a half years, far shorter than the 76 years of the other case then known, that of comet Halley; his solution accounted well for the entire set of observations. Messier was puzzled by this finding, and asked why the comet had not been observed before, given its short orbital period and its small perihelion distance; to this, Lexell answered that in May 1767 the comet and Jupiter had been very close to each other, and that the action of the gravity of the giant planet had greatly transformed the orbit of the comet. In fact, before 1767 the comet had a much larger perihelion distance, implying that it could not become very bright, thus explaining the fact that it had not been observed before.

Lexell also found that the orbit of the comet in 1770 was nearly resonant with that of Jupiter; in fact, in the time it took the comet to make two revolutions about the Sun, the giant planet would make one revolution. As a consequence, in 1779 the comet would encounter Jupiter again, at a distance even closer than in 1767, and would be expelled from the inner solar system into an orbit of large perihelion distance and period, that would make it invisible again for the telescopes of the time. The comet, in fact, was not observed again in 1782, as it should have been if it had remained in its 1770 orbit. Lexell's reconstruction of what had happened in 1767, as well as his prediction of what would happen in 1779, became generally accepted and the comet, although discovered by Messier, now brings Lexell's name. It can be said that the work of Lexell started the modern understanding of the dynamics of small solar system bodies.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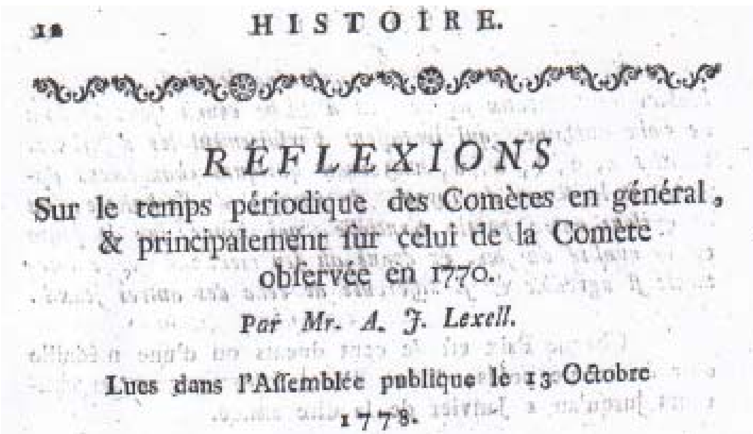
CONIECTVRA DE LOCIS COELI,
IN QVIBVS COMETA ANNI 1770, IN PROXIMO SVO AD PERIHELIVM REDITV, E TELLVRE NOSTRA CONSPICI DEBET.

Auctore

LEXELL.

Figure 1. The Latin title of Lexell (1777b); in English it may be translated as "Conjecture about where in the sky the comet of 1770, in its next return to perihelion, has to be looked for from our Earth".

It is interesting to note that one of the current problems of NEO astronomy, that of predicting the future position of an Earth-approaching body from a short observed arc, was a major concern also for Lexell, as testified by the title of Lexell (1777b), reproduced in Figure 1.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REFLEXIONS
Sur le temps périodique des Comètes en général, & principalement sur celui de la Comète observée en 1770.

Par Mr. A. J. Lexell.

Lues dans l'Assemblée publique le 13 Octobre 1778.

Figure 2. The French title of Lexell (1778b); in English it may be translated as "Considerations about the periods of comets in general, and principally on that of the comet observed in 1770".

The title of another of Lexell's papers, that of Lexell (1778b), reproduced in Figure 2, reflects one of the major worries of current NEO astronomy, i.e. that of communicating to fellow scientists and to the interested public about NEO matters, given the potential implications in terms of hazard (the idea that Earth approaching comets could be on orbits of period much shorter than that of P/Halley could have been rather hard to accept at the end of the XVIIIth century).

3. Enter LeVerrier: Line of Variations, chaos and all that...

In mid XIXth century, at about the same time of the celebrated work that led to the discovery of Neptune, LeVerrier reconsidered the orbital problems posed by Lexell's comet. LeVerrier critically examined the available observations, and among them identified a subset that he trusted; he then tried to compute an accurate orbit for the comet, taking also into account the gravitational action of the Earth. After many computations, described in detail in LeVerrier (1844), LeVerrier (1848) and LeVerrier (1857), he concluded that the best-fit orbit of the comet was poorly constrained and that all what could be done was to identify a line in the space of orbital elements —what we would nowadays call the Line of Variations (LoV), see Milani *et al.* (2005a)— in which the point corresponding to the true orbit most probably lies. As described by Milani *et al.* (2005b), current impact monitoring software robots exploit the same concept.

Thus, LeVerrier expressed the six orbital elements of comet Lexell as functions of a single unknown parameter, that he called μ; he also showed that the observations could be used to find the possible range of variation of μ, since outside a certain range, the path of the comet on the sky would have been measurably different from the observed one. Figure 3 shows an excerpt from LeVerrier (1857), in which the LoV is given as a straight line, parametrized by μ, in the space of orbital elements.

$$
\begin{aligned}
a &= 3{,}147\,86 + 0{,}01\mu,\\
e &= 0{,}785\,7161 + 0{,}000\,7260\mu,\\
\varepsilon &= 356^{\circ}\,15'\,55'',51 - 12'',67\mu,\\
\varpi &= 356.16.26{,}03 - 27{,}16\mu,\\
\varphi &= 1.34.19{,}53 + 4{,}17\mu,\\
\theta &= 131.53.56{,}00 + 83{,}00\mu.
\end{aligned}
$$

Figure 3. The Line of Variations introduced by LeVerrier for comet Lexell: from top to bottom, semimajor axis, eccentricity, mean longitude at epoch, longitude of perihelion, inclination and longitude of node.

LeVerrier computed the post-1770 time evolution of orbits lying on the LoV, so as to obtain a global view of all the possible outcomes, as done much later by Carusi *et al.* (1982); a comparison of the orbits examined in the latter paper with those given in Figure 3 reveals that, in fact, the uncertainty affecting LeVerrier's orbit solution was mostly in the magnitude of the heliocentric velocity vector, since the orbits computed by Carusi *et al.* only differed from each other for that quantity, and they appear to lie, in orbital elements space, along a line almost coincident with LeVerrier's LoV.

The computations showed, among other things, that the comet could approach Jupiter extremely closely in 1779, as close as less than three and a half radii of the planet from its centre; nevertheless, the comet could not become a satellite of Jupiter, not even temporarily, for any allowed value of μ. The range of post-1779 orbits included the possibility, for the comet, to leave the solar system on a hyperbolic orbit. The reason for this wide range of possible outcomes was the extreme sensitivity of the subsequent evolution to the precise value adopted for μ; this sensitivity is a crucial part of the modern concept of chaos, and in fact Le Verrier's computations probably represent the first instance of this concept in scientific literature.

Figure 4 shows the largest perturbations computed by LeVerrier, that are very close to his assumed nominal orbit .

− 0,3	3,784	0,6632
− 0,2	4,909	0,5251
− 0,1	8,923	0,5349
0,0	+ 60,097	0,9127
+ 0,1	− 58,617	1,0926
0,2	+ 49,751	0,8987
0,3	12,444	0,6532

Figure 4. A small excerpt from LeVerrier's computations: for the values of μ in the left column, the corresponding post-1779 values of semimajor axis and eccentricity of the orbit of comet Lexell.

4. NEO studies have a long tradition

After a long hiatus in the second half of the XIXth century and in the first three quarters of the XXth, the subject is again attracting the attention of astronomers. Some of the basic ideas have not changed; however, new observational and computational techniques have allowed us to make significant progress in NEO studies. We have learnt many things in the process, and these Proceedings aim at presenting a comprehensive view of the current status and achievements of NEO studies.

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