2

Advanced pinch technique: Still one loop

In this chapter, we study several more advanced aspects of the pinch technique (PT), still sticking to one-loop processes, mostly in perturbation theory but with some discussion of one-dressed-loop effects related to gluon mass generation. One of these applications of the pinch technique also has nonperturbative consequences, coming from the invocation of a gauge-field condensate; it allows us to conclude, as we show in this chapter, that the dynamical gauge-boson mass in QCD vanishes like q^{-2}, modulo logarithms, at large momentum. Finally, we introduce one of the main themes of the rest of the book: the pinch technique is realized to all orders by calculating conventional Feynman graphs in the background-field Feynman gauge. The subjects covered include the following:

1. The pinch technique and the operator product expansion (OPE) at one loop, where we see how only *gauge-invariant* condensates such as $\langle \mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}\rangle$ arise in PT Green's functions and how this condensate governs the vanishing at large momentum of dynamically generated gauge-boson mass in QCD.
2. Uses of the pinch technique in studying gauge-boson mass generation, both dynamic in QCD (no symmetry breaking, whether by Higgs–Kibble fields or other mechanisms) and with spontaneous symmetry breaking.
3. The background field method and the effective action.
4. The one-loop equivalence between the pinch technique and the background field method in the Feynman gauge.

2.1 The pinch technique and the operator product expansion: Running mass and condensates

As mentioned more than once, the pinch technique is essential to unveiling the nonperturbative effects that are vital in understanding confinement in QCD. One of the oldest and most familiar nonperturbative phenomena of QCD is the gauge-field

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condensate $\langle \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} \rangle$ appearing in the OPE of hadronic or leptonic currents. This condensate is explicitly gauge invariant, so it has physical significance.[1]

If there is to be dynamical mass generation (by which we mean generation of mass where gauge invariance of the usual classical action forbids such a mass), the dynamical mass must be a function of the momentum and must decrease at large momentum. If it did not vanish at infinite momentum, there would be a corresponding bare mass, not allowable in cases of interest to us. From the viewpoint of Schwinger–Dyson equations, there simply would be no massive solution for the gauge propagator unless the mass vanished at infinite momentum. This situation is already familiar for the constituent mass of the light quarks in QCD, which for all practical purposes have zero bare mass protected by a chiral symmetry forbidding a quark mass at any finite order of perturbation theory. Nevertheless, there is a large constituent mass that must also decrease with momentum. The mass is a sign of spontaneous chiral symmetry breaking, and there is another characteristic sign of chiral symmetry breaking: a nonzero value of the quark condensate $\langle \bar{q}(x) q(x) \rangle$. One cannot exist without the other. The OPE tells us how these are related: the running quark mass $M(q)$ decreases at large momentum as

$$M(q) \to \text{const.} \frac{-\langle \bar{q} q \rangle}{q^2}. \tag{2.1}$$

What is the corresponding relation between gluon mass and gauge-field condensate? As in every use of the OPE, the first step is to pick a matrix element with the right quantum numbers and bring together the space-time arguments of the condensate fields, thereby picking up the appropriate c-number multiplier of the condensate for that particular matrix element.

The OPE was used to find the contribution of the $\langle \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} \rangle$ condensate to the conventional gluon propagator, with disappointing but not unexpected results at one-loop order. Not only did this condensate appear, but gauge-dependent condensates involving the ghost fields c and $\bar{c}$ also appeared. It seemed that no physical results could be obtained from the OPE for a gauge-dependent quantity such as the usual gluon propagator. Then Lavelle [1] did the same calculation for the PT propagator in $d = 3, 4$, with very different results. *Only* the gauge-invariant condensate appeared and in just such a way that it could be interpreted as contributing to a running mass. Lavelle's results are equivalent to saying that the scalar

[1] The condensate $\langle \mathrm{Tr}\, A^{\mu} A_{\mu} \rangle$, which is explicitly not gauge invariant, can be made gauge invariant by turning it into the gauged nonlinear sigma model, as we indicate in Section 2.2.4. This is equivalent to minimizing the space-time integral of $\mathrm{Tr}\, A_{\mu} A^{\mu}$ over all local gauge transformations.

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inverse of the PT propagator, in Euclidean space, behaves in $d = 3, 4$, and at large momentum as

$$\widehat{d}^{-1}(q) \to q^2 + c_d \frac{g^2 \langle \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} \rangle}{q^2}, \tag{2.2}$$

where

$$c_3 = \frac{29N}{30(N^2 - 1)} \qquad c_4 = \frac{17N}{18(N^2 - 1)}. \tag{2.3}$$

(Actually, powers of logarithms of q can also occur, but we ignore them here.) The constants are positive, so this OPE correction has the right sign to represent a running mass because the condensate is also positive. In both cases, quark constituent mass and dynamical gluon mass, the running mass decreases like q^{-2} times a vacuum expectation value (VEV) of a gauge-invariant condensate. The difference is, of course, that there is no symmetry breaking for gluon mass generation, and indeed the gauge-field condensate is in no sense an order parameter for any kind of symmetry breaking. The simple physical reasoning for the connection of the condensate and the gluon mass is that a gluon mass allows for the construction of many different quantum solitons that cannot exist at the classical (zero-mass) level, including center vortices and nexuses. Condensates of these solitons are favored because of their large entropy (large number of possible space-time configurations) relative to their (finite) action and so lead to a gauge-field condensate. Lavelle's finding is the converse: a condensate allows for gluonic mass generation. Ultimately, this connection exists only because the gauge theories of interest show *infrared slavery* – the infrared manifestation of the ultraviolet phenomenon of asymptotic freedom in $d = 4$. Infrared slavery means that the perturbative PT propagator has a one-loop proper self-energy of the wrong sign (opposite to that of QED), with consequent intolerable infrared diseases such as tachyons. These theories must find a cure for infrared slavery, and that cure is dynamical gluon mass generation.

2.2 The pinch technique and gauge-boson mass generation

2.2.1 General remarks

As in Chapter 1, we consider only the one-loop case, postponing the all-orders generalization to later chapters.

There are several closely related ways of endowing a gauge boson with mass. The most straightforward way to generate gauge-boson masses is through

Higgs–Kibble–Goldstone symmetry breaking[2] with elementary scalar fields, as in the Georgi–Glashow model [2] or the electroweak (EW) sector of the standard model. A second way [3, 4] of generating gauge-boson mass is through dynamical effects associated with taming the infrared singularities of strongly coupled gauge theories such as QCD, which has no elementary scalars. Some gauge theories with no scalar fields can show dynamical symmetry breaking, in which the elementary scalar fields are replaced by composites arising from homogeneous solutions of the Schwinger–Dyson equations. A variant of these cases has elementary Higgs–Kibble–Goldstone fields and possibly symmetry breaking, but the VEVs in the scalar sector are too small (perhaps even zero, as in the EW sector at high temperature) to remove the infrared singularities of the underlying NAGT [5]. A third, rather specialized way, is through a Chern–Simons term in three dimensions. It can happen that the perturbative gauge-boson mass coming from the Chern–Simons term is too small to overcome infrared slavery, and dynamical mass generation comes into play. The pinch technique is important in estimating the critical Chern–Simons coupling (which is quantized) separating perturbative behavior of the theory from the need for nonperturbative dynamical mass generation [6].

All these cases have two vital ingredients in common. First, they require massless longitudinally coupled scalars, one for each gauge boson that gets mass. This is a subtle matter because the massless scalars do not appear (at least directly) in the S-matrix, yet they can appear in the pinch technique proper Green's functions. Every Goldstone-like scalar, whether elementary or composite, that is eaten by a gauge boson to give it mass is canceled out of the S-matrix by other massless poles or current conservation.

If the massless scalars are not elementary Goldstone fields, then they arise as composite excitations in a strongly coupled gauge theory.[3] By a composite excitation, we mean a pole in an off-shell Green's function representing a field that does not exist in the classical action but that occurs in the solution of the Schwinger–Dyson equation for that Green's function, as a sort of bound state. Therefore, the second vital ingredient is strong coupling, which, as far as we know, can only come from the infrared instabilities of a NAGT.

The residue, at zero momentum, of these Goldstone-like poles is essentially the square m^2 of a gauge-boson mass. The classical action does not have the

[2] If the gauge theory has local gauge symmetry at the classical level, so-called spontaneous symmetry breaking is not actually breaking this local gauge symmetry but simply realizing it in a different way. Without explicit symmetry breaking, such as fermion masses for local axial symmetries, no gauge-dependent object can have a nonzero VEV, as Elitzur's theorem tells us. Gauge-fixing terms break a gauge symmetry explicitly, but the pinch technique effectively removes such breaking.

[3] Other massive composite excitations may have to arise as well to save unitarity at high energies; see Lee et al. [7].

corresponding elementary field because if it did, there would be a symmetry violation, or a violation of perturbative renormalizability, or both. If renormalizability is at issue, the squared mass runs with momentum q and vanishes at large q; when mass generation is an infrared effect, as it will always be for us, a typical decrease is $m^2(q) \sim q^{-2}$ (modulo logarithms), as Lavelle [1] found. Without some such large-momentum falloff, the Schwinger–Dyson equation would have no solutions without extra infinities not corresponding to perturbative renormalization principles.

Here we will introduce two cases of the dynamical gauge-boson mass generation, saving other examples for later chapters. The first case is dynamical mass generation in QCD; the second is the well-known case of elementary Higgs–Kibble–Goldstone scalars. In both cases, the pinch technique is an essential tool for ensuring gauge invariance of the results.

2.2.2 Dynamical gauge-boson mass generation in QCD

In $d = 4$, the necessary strong coupling comes from asymptotic freedom as expressed in the wrong sign of the beta function (see Eq. (1.69)). Unfortunately, it is not helpful to associate the infrared phenomenon of mass generation with the ultraviolet phenomenon of asymptotic freedom. This is only a question of terminology because (perturbative) asymptotic freedom necessarily implies infrared singularities that are more virulent than in QED. The situation is actually more clearly seen in $d = 3$, where NAGTs are superrenormalizable, there is no renormalization group, and the ideas of asymptotic freedom are not relevant. Yet $d = 3$ NAGTs have – in even worse form than in $d = 4$ – serious infrared singularities.[4] We prefer, then, to suggest these low-momentum singularities with the term *infrared slavery* because, ultimately, they lead to confinement. The one-loop PT propagator in $d = 3$ clearly shows infrared slavery through the negative (and, as with asymptotic freedom, wrong) sign of a certain gauge-invariant constant [3, 4]. This sign is absolutely critical because if it were positive, the infrared behavior of radiatively corrected gluon propagators would be less singular than at tree level. But in the physical case of a negative sign, the infrared singularities show up as potential tachyons or ghost particles, that is, unphysical objects with imaginary mass or couplings. No other solution for the wrong sign is known aside from dynamical gauge-boson mass generation, which generates positive terms in the PT proper self-energy that overcomes the negative and singular behavior.

[4] This was not fully appreciated until the pinch technique came along because, before that, people had only investigated the standard Feynman propagator, which is gauge dependent. For the PT results, see [3, 4, 8].

We will briefly explore here, at the one-loop level, the nonperturbative dynamics leading to gauge-boson mass generation, deferring a detailed treatment of the PT Schwinger–Dyson equations to Chapter 6. Dressed propagators and vertices come, at least formally, from resumming Feynman graphs and have the power to inform us about phenomena that do not occur even when perturbation theory is summed to all orders in the coupling g. This kind of resummation is familiar from the skeleton graph expansion of Schwinger–Dyson equations and the resummation of the effective potential [9]. To truncate the otherwise infinite series of Schwinger–Dyson equations requires us to understand how to construct approximate forms for three- and higher-point Green's functions that, in spite of their approximate nature, *exactly* satisfy the PT Ward identities and that are expressed in terms of lower-point functions such as the PT propagator itself. If the vertex functions used do not obey exactly the Ward identities, gauge invariance is lost. The method of vertex constructions that satisfies these Ward identities is called the *gauge technique* and is discussed in Chapter 5. Because the $d = 3$ case is so instructive, we begin with it.

2.2.3 The need for dynamical mass in $d = 2 + 1$ QCD

We will easily see the problems of infrared slavery in $d = 2 + 1$ by calculating the one-loop perturbative PT proper self-energy. This goes exactly as in the $d = 3 + 1$ case of Section 1.3.3 except for the values of the integrals. The result [3, 4] for the scalar part of the one-loop PT inverse propagator is

$$\widehat{d}^{-1}(q) = q^2 + \pi b_3 g_3^2(-q^2)^{1/2} + \mathcal{O}(g_3^4), \tag{2.4}$$

where

$$b_3 = \frac{15N}{32\pi} \tag{2.5}$$

and g_3 is the $d = 3$ coupling with dimensions of $(\text{mass})^{1/2}$. Infrared slavery is simply the fact that b_3 is positive, which has the implication that there is a pole in the propagator for a spacelike momentum ($q^2 < 0$). This indicates a *tachyonic* pole – a pole corresponding to an imaginary mass.

There is also a tachyonic pole in $d = 4$, as one can see from the renormalized version of Eq. (1.68): the propagator has a pole at

$$-q^2 = \text{const.} \times \mu^2 \mathrm{e}^{-1/bg(\mu)^2}, \tag{2.6}$$

again satisfied with tachyonic q^2.

What could be the cure for this unphysical behavior? At first glance, it could be easy: Because the coupling g_3^2 has dimensions of mass, the omitted g_3^4 term might

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well provide a sufficiently positive term to overcome the negative one-loop term. This is indeed what happens nonperturbatively but not to any order of perturbation theory, where the coefficient of g_3^4 is identically zero to all orders. (If it were not zero, we could add a bare mass term to the action, which is not perturbatively renormalizable.)

This is only the beginning of the bad perturbative behavior. At $\mathcal{O}(g_3^{2N})$, each perturbative integral, by simple dimensional reasoning, has the infrared behavior $g_3^4(g_3^2/q)^{N-2}$, with poles of infinitely high order in the inverse propagator. But with nonperturbative generation of a (nontachyonic) mass m, the infrared behavior of every propagator in a loop is $\sim 1/m^2$, and an easy power counting shows that $|q|$ in the perturbative ordering expression is replaced by the dynamical mass $m \sim g_3^2$ so that all terms are of $\mathcal{O}(m^2)$ for order $N \geq 2$.

Of course, a one-loop pinch technique calculation only clearly shows us (i.e., gauge invariantly) the disease, not the cure. We take up the cure in Chapter 9, but for now, it is important to know something about what this cure looks like.

2.2.4 What do vertices and propagators look like when dynamical mass is generated?

The question is how to write PT Schwinger–Dyson equations with the right structure to represent composite massless scalars in both $d = 3$ and $d = 4$. There is a quite simple answer [10]: abstract this structure from an infrared-effective action in which we add to the usual NAGT action a mass term that is a gauged nonlinear sigma model. We add to the classical action,[5] the integral of Eq. (1.5), the term

$$S_m = \int \mathrm{d}^d x\, m^2 \,\mathrm{Tr}\left(U^{-1}\mathcal{D}_\mu U U^{-1}\mathcal{D}^\mu U\right) \tag{2.7}$$

to arrive at a total infrared-effective action (without fermions):

$$S\{A,m\} = \int \mathrm{d}^d x \left[-\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + m^2 \mathrm{Tr}\left(U^{-1}\mathcal{D}_\mu U U^{-1}\mathcal{D}^\mu U\right)\right]. \tag{2.8}$$

Here U depends on $\mathcal{R}$, the $N \times N$ unitary matrix representative of the group element $\mathcal{R}$. We also add the prescription that the exponential of this effective action is to be integrated over the Haar measure of the local gauge group.

We emphasize that we are not proposing to take S_m seriously as an addition to the true NAGT action, which always consists only of the usual Yang–Mills term. It is only part of an effective action, whose consequences should be studied only at the

[5] This action is to be used in $d = 3, 4$; for simplicity, we do not explicitly indicate the dimension as we did in the previous section.

classical level. However, it is legitimate for us to guess, from this classical study, what kind of structure the ingredients of the (PT) Schwinger-Dyson equation must have and then show that the Schwinger-Dyson equation with the assumed structure is actually self-consistent and physical. What we will find is what we spoke of earlier: massless longitudinally coupled Goldstone-like scalars, with couplings proportional to squared gauge-boson masses.

Because S_m is an effective action, we interpret integration over the group as finding the extrema over $\mathcal{R}$ of S_m. The matrix U undergoes gauge transformations along with the potential

$$U \to VU, \quad A_\mu \to VA_\mu V^{-1} + V\frac{\mathrm{i}}{g}\partial_\mu V^{-1}. \tag{2.9}$$

It is then elementary that the modified potential

$$C_\mu \equiv U^{-1}\mathcal{D}_\mu U, \tag{2.10}$$

is formally gauge invariant. (Furthermore, it is a gauge transformation by U^{-1} of the gauge potential A_μ.) We define a potential that is gauge covariant by multiplication from the left by U and from the right by U^{-1}:

$$\widetilde{A}_\mu \equiv UC_\mu U^{-1} = A_\mu + \frac{\mathrm{i}}{g}(\partial_\mu U)U^{-1} \qquad \widetilde{A}_\mu \to V\widetilde{A}_\mu V^{-1}. \tag{2.11}$$

It appears that we have added new degrees of freedom to the NAGT action by introducing U, but we have not, at least perturbatively. The reason is that the classical equations of motion for U are not independent of those for A_μ but follow from them, and U can be solved as a (nonlocal) functional of A_μ. The equations of motion for A_μ are as follows:

$$\left[\mathcal{D}^\mu, G_{\mu\nu}\right] + m^2\widetilde{A}_\nu = 0. \tag{2.12}$$

Because of the identity

$$\left[\mathcal{D}^\nu, \left[\mathcal{D}^\mu, G_{\mu\nu}\right]\right] \equiv 0, \tag{2.13}$$

it must be that

$$[\mathcal{D}^\nu, \widetilde{A}_\nu] = 0. \tag{2.14}$$

But this equation is precisely the equation of motion found by varying U. After a certain amount of algebra, one can show that this U equation of motion is equivalent to

$$\partial^\nu C_\nu = 0. \tag{2.15}$$

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In other chapters, we will find nonperturbative features of these equations and connect them to confinement via center vortices and to the Gribov ambiguity.[6] In particular, there are massless scalar excitations longitudinally coupled to the gauge potential. These massless scalars must be present if a gauge boson is to have mass in a way that preserves local gauge symmetry. We will see that such scalar fields actually represent long-range pure-gauge parts of the gauge potential that carry critical topological information about confinement and topological charge.

For now, we pursue the perturbative solution of the equation of motion for U and find massless scalars there. Write $U = \exp[\mathrm{i}\omega]$ and find [10]:

$$\mathrm{i}\omega = -\frac{1}{\Box}\partial \cdot A - \frac{1}{2}\left[\frac{1}{\Box}\partial \cdot A, \partial \cdot A\right] + \frac{1}{\Box}\left[A_\mu, \partial^\mu \frac{1}{\Box}\partial \cdot A\right] + \mathcal{O}(A^3). \quad (2.16)$$

Substitution of Eq. (2.16) in the gauged nonlinear sigma model action reveals an infinite set of vertices for the potential A_μ, longitudinally coupled to the massless scalars. This massless scalar is completely analogous to the Goldstone scalar of spontaneous symmetry breaking even though, in the present case, there is no symmetry breaking (and no elementary Higgs field). The lowest-order vertex is quadratic and yields a transverse mass term of the form

$$\int \mathrm{d}^d x\, m^2 \operatorname{Tr}\left[\left(A_\mu - \partial_\mu \frac{1}{\Box}\partial \cdot A\right)\left(A^\mu - \partial^\mu \frac{1}{\Box}\partial \cdot A\right)\right] \quad (2.17)$$

and a free gauge-boson propagator that has the structure expected from a pinch technique propagator:

$$\mathrm{i}\Delta^{(0)}_{\alpha\beta}(q) = P_{\alpha\beta}(q)d(q) + \xi\frac{q_\alpha q_\beta}{q^4}, \quad (2.18)$$

where

$$d(q) = \frac{1}{q^2 - m^2}. \quad (2.19)$$

The next vertex is a three-gluon vertex, to be added to the conventional free vertex. We convert the result to PT form by choosing the free vertex to be Γ^ξ of Eq. (1.37) and find [4, 11] that

$$\widehat{\Gamma}^{m,\xi}_{\mu\alpha\lambda}(q,k,-q-k) = \Gamma^\xi_{\mu\alpha\lambda}(q,k,-q-k) - \left[\frac{m^2}{2}\frac{q_\mu k_\alpha (q-k)_\lambda}{q^2k^2} + \text{c.p.}\right], \quad (2.20)$$

where c.p. stands for *cyclic permutations*. This vertex obeys the PT Ward identity of Eq. (1.40) with the propagator of Eqs. (2.18) and (2.19). The new vertex $\widehat{\Gamma}^{m,\xi}$ has,

[6] The Gribov ambiguity is that setting up a covariant gauge fixing in the usual way does not completely fix the gauge, so the gauge potential is ambiguous for a given field strength. For example, suppose that A_μ is in the Landau gauge; then, by Eqs. (2.10) and (2.15), so is C_μ, which is a gauge transformation by U^{-1} of A_μ.

as we mentioned earlier, terms with longitudinally coupled massless poles whose residue is m^2. This is the only way that the Ward identity can be satisfied if the propagator is transverse and has mass, just as the only way a massive gauge-boson propagator can be transverse is if it has similar poles in the transverse projector $P_{\mu\nu}$. There are infinitely many other vertices coming from the gauged nonlinear sigma model term, all with these poles, but we will go no further in exhibiting any of them explicitly.

Now apply the usual pinch technique to the mass-modified action of Eq. (2.8), in which we substitute the solution for U, as in Eq. (2.16). After much calculation [11] (originally done in the light-cone gauge), we find the one-loop pinch technique proper self-energy. In $d = 4$, it is as follows:

$$\widehat{\Pi}(q^2) = \frac{bg^2}{\pi^2} \int \mathrm{d}^4k \left[-q^2 d(k)d(k+q) + \frac{4}{11} d(k) + \frac{m^2}{11} d(k)d(k+q) \right]. \quad (2.21)$$

(We will take up the $d = 3$ analog later.) This reduces, as it must, to the formerly calculated one-loop proper self-energy of Eq. (1.68) in the limit of zero mass. Note that there are no massless poles in $\widehat{\Pi}$. In fact, the only trace of the massless poles comes from the second and third terms on the right-hand side (rhs), which do not vanish at $q = 0$. There are then massless poles in the tensorial proper self-energy coming from the longitudinal term in the transverse projector $P_{\mu\nu}$ that multiplies $\widehat{\Pi}$. Ultimately, these terms cannot appear in the S-matrix because of gauge current conservation.

With Eq. (2.21), we are actually only a few steps away from being able to study nonperturbative dressed-loop effects and analyze gauge-boson mass generation. The next steps are to integrate the rhs of this equation over m^2, with a spectral weight that is used in the Källen–Lehmann[7] representation of the pinch technique propagator; by this means, we construct a simple example of a gauge technique vertex and a PT Schwinger–Dyson equation for the proper self-energy. But we will postpone this study until later and use our work here to draw a few lessons that apply to such a nonperturbative study:

1. However gauge-boson mass is generated, it is accompanied by Goldstone-like longitudinally coupled scalars.
2. These scalar poles appear in vertices of all order, as, for example, in the transverse mass term of Eq. (2.17) and the three-vertex of Eq. (2.20), with couplings proportional to squared gauge-boson masses.

[7] The spectral weight is not positive definite, but this is not required for the existence of a spectral representation. See the discussion of Section 1.7.

3. These scalars do not appear in the S-matrix and (in our gauged nonlinear sigma model approach) even cancel out in the final expression for the pinch technique proper self-energy, as in Eq. (2.21).
4. (This lesson will be learned in part through a one-loop calculation in the next chapter.) A gauge-boson mass violating a symmetry or perturbative renormalizability must be a running mass, vanishing at large q, or the Schwinger–Dyson equation has no finite solution. Furthermore, a nonrunning mass by itself leads to violations of unitarity at high energy.
5. Dynamical generation of mass does not interfere with satisfaction of ghost-free Ward identities.

Next, we see if these lessons apply to Higgs–Kibble symmetry breaking.

2.2.5 Mass generation through Higgs–Kibble–Goldstone fields

There are two ways of describing mass generation in this case, depending on the description of gauge fixing in the theory (which, of course, is ultimately irrelevant). The method we mostly use in this book for the pinch technique was first developed in [12] for the Georgi–Glashow model and was later generalized to all orders of electroweak (EW) theory in [13]. Numerous other authors have used a similar form of the pinch technique for EW processes; Degrassi and Sirlin [14] have pointed out the relation of the pinch process to equal-time commutators of symmetry currents and given explicit expressions for the one-loop PT proper self-energies. All these authors use a *modified* R_ξ gauge analyzed by Fujikawa et al. [15] and originally from 't Hooft [16], which we will term the *FLS gauge*. In the FLS gauge, the Goldstone bosons decouple from the gauge bosons that eat them, at least at tree level (but not beyond). The value of this gauge, as we will see shortly, is that the tree-level gauge propagators have no longitudinal parts in the Feynman version of the FLS gauge (or 't Hooft–Feynman gauge), just as there are none for the free gauge propagator in the Feynman R_ξ gauge with $\xi = 1$ (see Eq. (1.31)). But this is not an unalloyed virtue because the PT calculations in the FLS gauge seem to differ from what we have said so far: the Goldstone bosons in general have a gauge-dependent mass that is the same as that of the ghosts.[8] This is a sign that the Goldstone bosons as well as the ghosts do not appear in the S-matrix. Moreover, the proper self-energies and vertices are independent of ξ but satisfy Ward identities ostensibly different from those we have already used, such as transversality of the gauge boson proper self-energy. However, these pieces can be reshuffled [12] to

[8] In the FLS–Feynman gauge, $\xi = 1$, the ghosts and Goldstone bosons, and the gauge bosons that eat them, all have the same mass.

yield transverse proper self-energies and vertices that do have the properties listed in our lessons from dynamical mass generation in QCD.

There is another approach that is more similar in spirit to what we have done so far for QCD-like theories; it uses the standard sort of R_ξ gauge that we have used for QCD. In this gauge, the Goldstone bosons and ghosts are massless scalar fields longitudinally coupled to gauge bosons. The massless poles do not appear in the S-matrix but are important in enforcing current conservation (Ward identities). Demanding that these Ward identities be satisfied leads uniquely to a set of PT Green's functions that are independent of ξ and obey other physical requirements. It is worthwhile to understand this approach because similar massless scalar excitations must and do occur in the vertices of QCD-like gauge theories if there is to be dynamical gluon mass generation. Although these Goldstone-like excitations do not contribute to the perturbative S-matrix, they are the carriers of long-range topological information that is responsible for nonperturbative phenomena, including confinement and chiral symmetry breakdown. We will begin with a brief sketch of what happens in this QCD-like description. Needless to say, either description results in the same physics, as described by the pinch technique.

Symmetry breaking in a standard R_ξ gauge The main new feature with symmetry breaking is that Ward identities used for pinching, though unchanged in basic structure (see Eq. (1.62)), have new vertex terms involving the massless Goldstone bosons even at tree level, similar in structure to those found for dynamical mass generation in QCD. These new terms are essential for current conservation or, more generally, for satisfaction of the Ward identities.

To be explicit, consider the Georgi–Glashow model [2] in $d = 4$, containing an $SU(2)$ gauge field, a Fermion doublet ψ, and a triplet ϕ_a of scalar bosons. With these scalars and fermions, the Georgi–Glashow model is asymptotically free, with the beta function coefficient b of the pure gauge-boson theory (see Eq. (1.69)) changed to $19/(48\pi^2)$. The action is as follows:

$$S_{GG} = \int \mathrm{d}^4x \left\{ -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \mathrm{i}\bar{\psi}\,[\not{\mathcal{D}} - M_0]\,\psi + \frac{1}{2}\left[\left(\mathcal{D}_\mu\phi_a\right)(\mathcal{D}^\mu\phi_a\right) \right.$$
$$\left. - V(\phi_a^2) - h\bar{\psi}\tau_a\psi\phi_a \right\}, \tag{2.22}$$

to which we will add a gauge-fixing term. We will also use the convenient matrix notation

$$\phi = \frac{1}{2}\tau_a\phi_a, \tag{2.23}$$

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where the τ_a are the Pauli matrices. The potential V has the form

$$V(\phi_a^2) = \frac{\lambda}{2}(\phi_a^2 - v^2)^2, \tag{2.24}$$

with the symmetry-breaking minimum taken by convention to be in the 3 direction so that $\phi_a = v\delta_{a3}$ at the classical level. This gives a Higgs mass to the gauge bosons with group indices 1,2, with value $M_g = vg$, and the third gauge boson remains massless. There is also a symmetry-breaking term in the fermion mass matrix $\mathcal{M}$, which becomes

$$\mathcal{M} = M_0 + hv\tau_3 \equiv M_0 + m\tau_3. \tag{2.25}$$

In the presence of symmetry breaking, we write the scalar fields in matrix form:

$$\phi = \frac{1}{2}[v\tau_3 + \chi_a\tau_a], \tag{2.26}$$

where, by definition, the VEV of χ_a vanishes.

In a conventional R_ξ gauge, there is a quadratic coupling term between the original gauge potential and the χ_a for $a = 1, 2$. This is just what tells us that the gauge bosons with $a = 1, 2$ swallow the corresponding Goldstone bosons and become massive (the gauge potential $A_{3\mu}$ remains massless, and χ_3 describes a massive Higgs–Kibble field). From now on, we use the notation W_μ for the massive gauge bosons.

This quadratic coupling is no particular problem. Because the Goldstone fields are eaten by the W-bosons, they become – at least in perturbation theory – dependent fields, expressible entirely in terms of the gauge potential and possibly other fields. Save only the W-bosons and the ϕ fields with indices $a = 1, 2$ and write the quadratic part of the Lagrangian, including the gauge-fixing term:

$$\begin{aligned} S_2 = \int \mathrm{d}^4x \, \Bigg\{ &-\frac{1}{2}\widetilde{\mathrm{Tr}}\left(\partial_\mu W_\nu - \partial_\nu W_\mu\right)(\partial^\mu W^\nu - \partial^\nu W^\mu) \\ &+ \widetilde{\mathrm{Tr}}\left(\partial_\mu \chi + v\left[W_\mu, \frac{1}{2}\tau_3\right]\right)^2 - \frac{1}{\xi}\widetilde{\mathrm{Tr}}\left(\partial_\mu W_\mu\right)^2 \Bigg\}, \end{aligned} \tag{2.27}$$

where $\widetilde{\mathrm{Tr}}$ means taking the trace only over terms involving $\tau_{1,2}$. We define an anti-Hermitean Goldstone matrix $G = \frac{1}{2\mathrm{i}}\tau_a G_a$, with $a = 1, 2$, by a re-ordering of $\chi_{1,2}$:

$$\chi = \left[G, \frac{1}{2}\tau_3\right] \qquad \text{or} \qquad \chi_1 = -G_2, \quad \chi_2 = G_1. \tag{2.28}$$

Now couple the fields W_μ and G to currents V_μ and T, respectively. A short calculation using the action S_2, in terms of G rather than χ, yields (in momentum space) the two equations

$$\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right) W^\nu + \frac{1}{\xi} q_\mu q_\nu W^\nu - M_g^2 \left(W_\mu - \frac{q_\mu G}{M_g}\right) = V_\mu \qquad (2.29)$$

$$q^2 G - M_g q_\nu W^\nu = T.$$

The solution for the G equation is

$$G = \frac{M_g q_\mu W^\mu + T}{q^2}, \qquad (2.30)$$

and this, substituted in the equation for W_μ, yields an equation for this potential with a modified source term:

$$\left(q^2 - M_g^2\right)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W^\nu + \frac{1}{\xi} q_\mu q_\nu W^\nu = V_\mu - \frac{q_\mu M_g T}{q^2}. \qquad (2.31)$$

Note that the inverse propagator (coefficient of W^ν on the left-hand side) has just the form of Eq. (1.30) that we expect of a PT propagator, with the proper self-energy given by M_g^2.

We are concerned, as usual, with the gauge dependence of amplitudes, so we note that the ξ-dependent term in the solution of Eq. (2.31) is

$$W^\nu = \frac{\xi q^\nu q^\mu}{q^4}\left(V_\mu - \frac{q_\mu M_g T}{q^2}\right) + \cdots. \qquad (2.32)$$

Without gluon mass generation by symmetry breaking (i.e., if M_g and m were zero), the current V_μ would have to be conserved on shell,[9] or otherwise there would be ξ dependence even in the tree-level S-matrix. The situation would then be just the same as for QCD-like theories. But with symmetry breaking, a different current is conserved on shell; the massless Goldstone pole has modified the massive gauge-boson source and is essential for current conservation and Ward identities, as we now show. This modification of the source term is essential because it must happen that the combined source in Eq. (2.31) must be conserved on shell:

$$q_\mu V^\mu - M_g T = 0 \;\text{ on-shell.} \qquad (2.33)$$

It is this quantity multiplied by ξ that would appear in the tree-level S-matrix and so must vanish.

The Goldstone contribution is necessary because with symmetry breaking, the divergence $q_\mu V^\mu$ is generally not zero, even at tree level. Consider our earlier example $V_\mu = \bar{u}_1(p+q)\gamma_\mu u_2(p)$, where the fermion momenta $p, p+q$ are on

[9] For example, $V_\mu \sim \bar{u}_1(p+q)\gamma_\mu u_2(p)$, where the labels 1,2 distinguish different $SU(2)$ fermion eigenstates.

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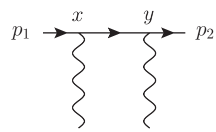

Figure 2.1. Part of a one-loop Feynman graph for on-shell quark-quark scattering. Gauge propagators end at space-time points x, y on the quark line.

shell and the mass of the fermion labeled 1 is $M_0 + m$ and of the fermion labeled 2 is $M_0 - m$. Then $q_\mu V^\mu$ is

$$\begin{aligned} q_\mu \bar{u}_1(p+q)\gamma^\mu u_2(p) &= \bar{u}_1(p+q)\left[S^{-1}(p+q) - S^{-1}(p) - 2m\right] u_2(p) \\ &= -2m\bar{u}_1 u_2. \end{aligned} \tag{2.34}$$

The last equation follows because the inverse propagators annihilate the on-shell spinors. The Goldstone source term of Eq. (2.31), proportional to $M_g T$, turns out to be $2m\bar{u}_1(p+q)u_2(p)q_\mu/q^2$ when one recognizes that $m = (h/g)M_g$. Now we see that the vertex $V_\mu - M_g T q_\mu/q^2$ does obey the expected on-shell Ward identity. One reads off from Eq. (2.34) the off-shell Ward identity of our tree-level example:

$$q_\mu \left[V^\mu - M_g T \frac{q^\mu}{q^2}\right] = S^{-1}(p+q) - S^{-1}(p). \tag{2.35}$$

This is the same as the Ward identity used earlier for QCD-like theories, and the pinch technique proceeds from it as before.

We digress to give the Degrassi–Sirlin explanation of how the symmetry currents coupled to the gauge bosons are related to the pinching out of various propagators. Figure 2.1 shows a part of a Feynman graph that occurs in the S-matrix. This could be a part of several of the complete S-matrix graphs shown in Figure 1.1. This figure by itself is the tree-level version of the matrix element:

$$\langle p_1 | T\left[J^a_\mu(x) J^b_\nu(y)\right] | p_2 \rangle, \tag{2.36}$$

aside from the two gauge propagators, which we do not exhibit explicitly. The T operation is covariant time ordering, and the currents $J^{a,b}_\alpha$ are symmetry currents coupled to the gauge bosons. A gauge propagator with a longitudinal momentum acts to take the divergence of this time-ordered product, say, with respect to x. The

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currents themselves are supposed to be conserved, so the only effect is the matrix element of the equal-time commutator:

$$\langle p_1|\delta(x_4 - y_4)\left[J_0^a(x), J_\nu^b(y)\right]|p_2\rangle. \tag{2.37}$$

The equal-time commutator here is

$$\mathrm{i} f_{abc}\delta(x - y)\langle p_1|J_\nu^c(y)|p_2\rangle, \tag{2.38}$$

in which the points x and y are made to coincide and a fermion propagator is missing, just as would happen with our standard PT formalism.

If just the currents V_μ that we introduced in Eq. (2.29) were used for the symmetry currents, our pinch arguments would fail because these are not conserved. We must add the T currents, sources of the Goldstone bosons, to get conserved currents, and conservation arises through the coupling of the massless Goldstone field. It is not hard to check, by resumming graphs, that these Goldstone particles and currents come from the tree-level mixing of Goldstone fields and gauge fields that is no longer eliminated when we use a standard R_ξ gauge.

The Goldstone poles cannot appear in the S-matrix because they are not independent elementary fields; their appearance at one place must be canceled by another appearance elsewhere. The pinch technique, of course, shows this cancellation. The essence of it is that in the pinch technique, the inverse propagator has the form

$$-\mathrm{i}\widehat{\Delta}^{-1}_{\alpha\beta}(q) = P_{\mu\nu}(q)\left[q^2 + \mathrm{i}\widehat{\Pi}(q)\right] + \frac{1}{\xi}q_\alpha q_\beta. \tag{2.39}$$

For the charged bosons, $\widehat{\Pi}(q = 0) \neq 0$, so the Goldstone bosons appear as the longitudinal massless poles of the transverse projector. In the S-matrix, these terms in the propagator itself strike currents that are conserved and cannot contribute to the S-matrix. On the other hand, the Goldstone poles that allow these currents to be conserved annihilate the physical part of the propagator or inverse propagator, leaving only gauge-dependent kinematic terms. We know that these must cancel in the pinch technique. Now we go on to the more widely used, and equivalent, formulation in the FLS gauge.

Symmetry breaking in the FLS gauge In the FLS gauge, the gauge-fixing term is chosen to cancel the quadratic coupling of W_μ and $\partial_\mu S$:

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{\xi}\widetilde{\mathrm{Tr}}\left(\partial_\mu W_\mu - M_W \xi S\right)^2. \tag{2.40}$$

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The quadratic cross-term between W_μ and $\partial_\mu S$ now cancels. However, the S^2 term in the gauge-fixing Lagrangian now makes these Goldstone fields, and also the ghosts, massive. A short calculation gives the following:

$$\mathrm{i}\Delta_{\mu\nu} = \frac{g_{\mu\nu}}{q^2 - M_g^2} + \frac{q_\mu q_\nu(\xi - 1)}{(q^2 - M_g^2)(q^2 - \xi M_g^2)} \tag{2.41}$$

$$\mathrm{i}\Delta_{\mathrm{S}} = \mathrm{i}\Delta_{\mathrm{gh}} = \frac{1}{q^2 - \xi M_g^2},$$

where $\Delta_{\mu\nu}$ is the W propagator, Δ_{S} is the Goldstone propagator, and Δ_{gh} is the ghost propagator. Clearly, all the ξ dependence in these propagators must cancel in the S-matrix and therefore in the pinch technique, and they do, after some very lengthy calculations [12]. At $\xi = 1$, the tree-level gauge propagator has no longitudinal terms that can pinch, which considerably simplifies the calculations. Moreover, in this Feynman gauge, the gauge bosons, ghosts, and Goldstone particles all have the same mass M_W, and no unphysical masses can appear.

For general ξ, one often decomposes the W propagator as follows [12, 15]:

$$\mathrm{i}\Delta_{\mu\nu} = \mathrm{i}\Delta^1_{\mu\nu} + \mathrm{i}\Delta^2_{\mu\nu}, \tag{2.42}$$

$$\mathrm{i}\Delta^1_{\mu\nu} = \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_g^2} \right] \frac{1}{q^2 - M_g^2},$$

$$\mathrm{i}\Delta^2_{\mu\nu} = \frac{q_\mu q_\nu}{M_g^2(q^2 - \xi M_g^2)}.$$

This is one way of isolating the gauge dependence of the free propagator into Δ^2. The first term, Δ^1, is the propagator in the so-called unitary gauge $\xi = \infty$. The one-loop pinch technique decomposition has been worked out [12] with this separation of the W-propagator, followed by a demonstration of how to recover the usual results with massless Goldstone particles that we discussed earlier. The idea is simply to recompose the W-propagator by writing the $1/M_g^2$ term in Δ^1 with the identity

$$\frac{1}{M_g^2} = \frac{q^2 - M_g^2}{q^2 M_g^2} + \frac{1}{q^2}. \tag{2.43}$$

This shows not only the massless Goldstone poles but one of the ways in which they cancel out in the S-matrix.

The pinch technique has been worked out to all orders for the standard electroweak model [13]. We will discuss it in Chapter 10.

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2.3 The pinch technique today: Background-field Feynman gauge

It would be very awkward to carry on with the pinch technique in the light-cone gauge (see Section 2.2.4). Even two-loop calculations would be exceptionally difficult, not only because of the intrinsic difficulties of working in a noncovariant gauge but also because what we have done so far at the one-loop level does not really suggest how to generalize the pinch technique. Fortunately, there is a simple way to generalize the pinch technique to all orders of perturbation theory (and to nonperturbative applications, with the help of the gauge technique). It consists of calculating ordinary Feynman graphs in the Feynman gauge of the background-field method. Just as with ordinary Feynman graphs, the sum of such graphs can be reorganized into a dressed-loop expansion from which nonperturbative effects can arise. Much of the rest of this book is devoted to demonstrating these points, which are not at all evident.

The background-field method [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27] goes back decades in the study of general relativity and NAGTs. It gives an effective action as a functional of specified background fields (gauge potentials) – an effective gauge action guaranteed to depend on the background fields only through gauge-invariant constructs such as $\mathrm{Tr}\, G^2_{\mu\nu}$. Unfortunately, just as in any other covariant formulation of NAGTs, there has to be gauge fixing and ghosts, and the coefficient functions of the background field constructs depend on the specific gauge chosen (so they depend on ξ in an R_ξ gauge). If these coefficient functions were gauge independent, there would be no need for an independent pinch technique. It is essential that the gauge-fixing term can be chosen to have full local non-Abelian gauge invariance for the background fields in order that this effective potential be gauge invariant for these fields. In other words, gauge fixing is used only for the quantum fields – those integrated out in the functional integral defining the effective potential.

As we said earlier, the first connection was made at the one-loop level [28, 29, 30], where it was shown that the one-loop pinch technique and the one-loop background Feynman gauge method gave precisely the same results. This raises the question of why these two seemingly disparate approaches should give the same answer but by no means answers it. It does not seem plausible that calculations in a specified gauge should actually give gauge-invariant results for Green's functions, as the pinch technique does. In fact, if one were to calculate Green's functions in some other version of the background-field method, for example, the Landau gauge, the results would not be the same as the pinch technique gives. The pinch technique can be used to combine pieces of Feynman graphs in the background Landau gauge, just as in any other gauge, and the usual pinch technique results emerge. Ultimately, as the rest of this book shows, the background Feynman gauge is singled out because of the absence, in this particular gauge, of certain longitudinal numerator parts that give pinches.

We cannot emphasize too strongly that the pinch technique is a way of enforcing gauge invariance and several other physical properties for off-shell Green's functions. The background-field method, in a general gauge, is not. It is a remarkable and extraordinarily useful result that PT Green's functions can also be calculated in the background Feynman gauge, but this needs a very extensive demonstration.

Before presenting the background-field method, we first quickly review the well-known construction of the effective action and the problems encountered with it in gauge theories.

2.3.1 The effective action

A brief review of the scalar field case Consider first a Euclidean $d = 4$ ϕ^4 field theory. The generating functional is written

$$Z[J] = \int [\mathrm{d}\phi] \exp\left\{\mathrm{i}S[\phi] + \mathrm{i}\int \mathrm{d}^4x\, J(x)\phi(x)\right\} \equiv \mathrm{e}^{\mathrm{i}W[J]}, \tag{2.44}$$

where $S[\phi]$ is the scalar field action. Functional derivatives of W with respect to the source J yield connected quantum Green's functions. In particular,

$$\frac{\delta W}{\delta J(x)} = \langle \phi(x) \rangle \equiv \Phi(x), \tag{2.45}$$

the expectation value of the quantum field in the presence of the source J.

The Legendre transform of W is more useful because it generates the 1PI graphs:

$$\Gamma[\Phi] = W[J] - \int \mathrm{d}^4x\, J(x)\Phi(x); \qquad \frac{\delta\Gamma}{\delta\Phi(x)} = -J(x). \tag{2.46}$$

The functional $\Gamma[\Phi]$ is the effective action, and its functional derivatives with respect to Φ yield 1PI Green's functions. Usually, we are interested in setting the source J to zero at the end of the calculation, and so the preceding equation shows that Γ is stationary in Φ. This condition of stationarity is the Schwinger–Dyson equation for Φ.

The effective action can be found directly by introducing a background field Φ and shifting the argument ϕ of the action $S[\phi]$ by this amount. At the outset, this shift field Φ is arbitrary and independent of J. The shift yields a new generating functional $\tilde{Z}$:

$$Z \to \widetilde{Z} = \int [\mathrm{d}\phi] \exp\left\{\mathrm{i}S[\phi + \Phi] + \mathrm{i}\int \mathrm{d}^4x\, J(x)\phi(x)\right\} \equiv \mathrm{e}^{\mathrm{i}\widetilde{W}[J,\Phi]}. \tag{2.47}$$

We might as well write this by shifting variables back, through $\phi \to \phi - \Phi$, so that

$$\widetilde{W}[J, \Phi] = W[J] - \int d^4x\, J(x)\Phi(x). \tag{2.48}$$

This is precisely the effective action $\Gamma[\Phi]$, provided that J and Φ are related by Eq. (2.45). This equation amounts to $\delta\widetilde{W}/\delta J = 0$.

Let us calculate the effective action $\widetilde{\Gamma}$ corresponding to $\widetilde{Z}$. Do the Legendre transform:

$$\widetilde{\Gamma}[\bar{\phi} + \Phi] = \widetilde{W}[J, \Phi] - \int d^4x\, J(x)\bar{\phi}(x), \tag{2.49}$$

where the Legendre-transform variable $\bar{\phi}$ is defined by

$$\bar{\phi}(x) = \frac{\delta\widetilde{W}}{\delta J(x)}. \tag{2.50}$$

When the equations of motion (2.45) are satisfied, $\bar{\phi}$ vanishes.

We have written $\widetilde{\Gamma}$ as a functional of only one variable. To show this, use

$$\frac{\delta\widetilde{W}}{\delta\Phi(x)} = -J(x), \tag{2.51}$$

to find the total variation of $\widetilde{\Gamma}$:

$$\delta\widetilde{\Gamma} = -J\delta(\bar{\phi} + \Phi). \tag{2.52}$$

The final conclusion is that when the equations of motion in Eq. (2.45) hold,

$$\widetilde{\Gamma}[\bar{\phi} = 0, \Phi] = \Gamma[\Phi]. \tag{2.53}$$

A possible construction of Γ comes from summing all connected 1PI Feynman graphs, using the field-shifted action of Eq. (2.47) to find the 1PI graphs. Or, one can simply integrate the Schwinger–Dyson equation for Φ, written in terms of 1PI skeleton graphs.

All the scalar-field results have analogs for NAGTs but with certain complications from gauge fixing and ghost terms. There is a further generalization [9] of effective-action methods that allows us to construct an effective action that is 2PI, so that no connected graph in Γ can be separated by cutting only two (distinct) lines. We discuss this generalization briefly in the following section.

The two-particle-irreducible effective action By introducing a two-point source $K(x, y)$ and corresponding Legendre transform, we can [9] define a generating

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functional and its Legendre transform, the effective action $\Gamma[\Phi, G]$ that depends on a propagator function Δ as well as on Φ:

$$\begin{aligned} Z[J, K] &= \int [\mathrm{d}\phi] \exp \left\{ \mathrm{i}S[\phi] + \mathrm{i} \int \mathrm{d}^4 x\, J(x)\phi(x) \right. \\ &\qquad \left. - \frac{\mathrm{i}}{2} \int \mathrm{d}^4 x \int \mathrm{d}^4 y\, \phi(x) K(x, y) \phi(y) \right\} \\ &= \mathrm{e}^{\mathrm{i}W[J,K]} \\ \Gamma[\Phi, \Delta] &= W[J, K] - \int \mathrm{d}^4 x\, J(x)\Phi(x) \\ &\qquad - \frac{1}{2} \int \mathrm{d}^4 x \int \mathrm{d}^4 y\, [\Phi(x) K(x, y) \Phi(y) + \Delta(x, y) K(x, y)]. \end{aligned} \tag{2.54}$$

The functional derivatives

$$\frac{\delta W}{\delta J(x)} = \langle \phi(x) \rangle \equiv \Phi(x) \tag{2.55}$$

$$\frac{\delta W}{\delta K(x, y)} = \frac{1}{2} [\Phi(x)\Phi(y) + \Delta(x, y)] ,$$

where Δ is the connected two-point function, lead to the functional derivatives for the effective action:

$$\frac{\delta \Gamma}{\delta \Phi(x)} = -J(x) - \int \mathrm{d}^4 y\, K(x, y)\Phi(y)$$

$$\frac{\delta \Gamma}{\delta \Delta(x, y)} = -\frac{1}{2} K(x, y). \tag{2.56}$$

As before, physical processes correspond to vanishing sources, so now Γ is stationary with respect to both the one- and two-point functions. The vanishing variation of $\delta\Gamma/\delta\Delta$ yields the Schwinger–Dyson equation for Δ. The graphical construction for Γ now involves the sum of connected 2PI graphs, those that cannot be separated by cutting only two (distinct) lines. These graphs necessarily have dressed propagators for their lines. There are, in addition, some one-dressed-loop terms. Even if Γ is approximated by saving only a few terms in the dressed-loop expansion, the equations resulting from requiring stationarity may well reveal nonperturbative effects not visible even in resummed perturbation theory.

One can go further and introduce sources for three- and four-point functions, along with Legendre transforms analogous to those of Eq. (2.54). The resulting Γ is now stationary (at vanishing sources) with respect to N-point functions with $N = 1, 2, 3, 4$, and the stationarity requirements are the corresponding Schwinger–Dyson equations. Or [31], one can simply look at the sum of connected graphs (with correct combinatoric factors!) for W in the absence of sources and resolve these into skeleton graphs for these N-point functions. The derivatives of W with respect to these N-point functions come out to be the Schwinger–Dyson equations.

2.3.2 The background-field method for gauge fields

The goal is to produce an effective action that is gauge invariant in terms of the classical potentials $\widehat{A}_\mu$ that appear in it so that it is a functional of the classical field strengths such as $\mathrm{Tr}\, \widehat{A}_{\mu\nu}\widehat{A}^{\mu\nu}$. We would like to imitate the principle of shifting the variable of integration (which we also call the quantum potential) by a classical potential and then shift back to produce the effective action, as we did for scalar fields. So we want to split the original quantum variable A_μ in the functional integrals into a classical part $\widehat{A}_\mu$ and a quantum part Q_μ:

$$A_\mu = \widehat{A}_\mu + Q_\mu. \tag{2.57}$$

The original action of Eq. (1.5) is invariant under the inhomogeneous gauge transformation of Eq. (1.10), which is just a change of variable of integration. How do we apportion this gauge transformation,

$$A_\mu \to V\frac{\mathrm{i}}{g}A_\mu V^{-1} + V\partial_\mu V^{-1}, \tag{2.58}$$

between $\widehat{A}_\mu$ and Q_μ? Furthermore, the NAGT generating functional with no background field given in Eq. (1.5), which we repeat here:

$$\begin{aligned} Z[J_\mu] = &\int [\mathrm{d}Q_\mu][\mathrm{d}\bar{c}][\mathrm{d}c]\exp\Big\{\mathrm{i}S[Q] \\ &\quad + \mathrm{i}\int \mathrm{d}^4x\left[\frac{1}{2\xi}\mathrm{Tr}\left(\partial_\mu Q^\mu\right)^2 + (\bar{c}\partial_\nu\mathcal{D}^\nu c) + J_\mu(x)Q^\mu(x)\right]\Big\} \\ \equiv\ & \mathrm{e}^{\mathrm{i}W[J_\mu]}, \end{aligned} \tag{2.59}$$

is not gauge invariant because of the ghost-antighost and gauge-fixing terms, as well as the term involving $J \cdot A$. So how do we get overall gauge invariance of some sort in $W[J]$ and its Legendre transformation, the effective action?

Gauge invariance of the effective action as a functional of the classical potential $\widehat{A}_\mu$ means that it is invariant under a standard gauge transformation of this potential in which the full inhomogeneous term goes with $\widehat{A}_\mu$ (and it would seem that no inhomogeneous term goes with the quantum potential Q_μ):

$$\widehat{A}_\mu \to V\frac{\mathrm{i}}{g}\widehat{A}_\mu V^{-1} + V\partial_\mu V^{-1}; \qquad Q_\mu \to V\frac{\mathrm{i}}{g}Q_\mu V^{-1}. \tag{2.60}$$

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The combined transformations preserve gauge invariance of the NAGT action $S[\widehat{A}_\mu + Q_\mu]$. Following the scalar field construction, we proceed by coupling only the quantum potential to the current so that the term $J \cdot Q$ remains unchanged.

Next is the gauge-fixing term. If it involves derivatives (and therefore has ghosts), we can replace the ordinary derivatives with the covariant derivative with respect to the classical potential. For example, in R_ξ gauges,

$$\frac{1}{2\xi}\mathrm{Tr}\left(\partial_\mu Q_\mu\right)^2 \to \frac{1}{2\xi}\mathrm{Tr}\left(\left[\mathcal{D}_\mu(\widehat{A}), Q_\mu\right]^2\right), \tag{2.61}$$

where $\mathcal{D}_\mu(\widehat{A}) = \partial_\mu - \mathrm{i}g\widehat{A}_\mu$ is the covariant derivative with respect to the classical potential.[10] The Faddeev–Popov determinant undergoes a corresponding change.

At this point, the new generating functional $\widetilde{Z}$, given by

$$\begin{aligned}\widetilde{Z}[J_\mu, \widehat{A}_\nu] = &\int[\mathrm{d}Q_\mu][\mathrm{d}\bar{c}][\mathrm{d}c]\exp\Bigg\{\mathrm{i}S[C+Q]\\ &+\int \mathrm{d}^4x\,\mathrm{i}\left[\frac{1}{2\xi}\mathrm{Tr}\left(\left[\mathcal{D}_\mu(\widehat{A}), Q_\mu\right]\right)^2 + \mathcal{L}_{\bar{c}c}(x) + J_\mu(x)Q_\mu(x)\right]\Bigg\}\\ \equiv\ & \mathrm{e}^{\mathrm{i}\widetilde{W}[J_\mu, C_\nu]},\end{aligned} \tag{2.62}$$

(where $\mathcal{L}_{\bar{c}c}$ is the Lagrangian expressing the Faddeev–Popov determinant), is invariant under the combined transformations of Eq. (2.60) plus a homogeneous rotation of the current under which

$$J_\mu \to V J_\mu V^{-1}. \tag{2.63}$$

The corresponding Legendre transform

$$\begin{aligned}\widetilde{\Gamma}[\mathcal{A}_\mu, \widehat{A}_\mu] &= \widetilde{W}[J_\mu, \widehat{A}_\mu] - \int \mathrm{d}^4x\, J^\mu(x)\mathcal{A}_\mu(x)\\ \frac{\delta\widetilde{W}}{\delta J_\mu(x)} &= \langle Q_\mu(x)\rangle \equiv \mathcal{A}_\mu(x),\end{aligned} \tag{2.64}$$

is also invariant.

Just as with the scalar field, the next step is to change the variable of integration back so that in Eq. (2.62), $Q_\mu \to Q_\mu - \widehat{A}_\mu$. This changes the argument of the action back to a conventional form. Then the generating functional becomes

$$\mathrm{e}^{\mathrm{i}\widetilde{W}[J_\mu, \widehat{A}_\mu]} = \mathrm{e}^{\mathrm{i}W[J_\mu] + \mathrm{i}\int \mathrm{d}^4x\, J_\mu(x)\mathcal{A}_\mu(x)}. \tag{2.65}$$

Here W is the conventional exponent, except that it is calculated in a special gauge. Making the shift $Q_\mu \to Q_\mu - \widehat{A}_\mu$ in the gauge-fixing term of Eq. (2.61) yields the

[10] The unadorned covariant derivative $\mathcal{D}_\mu$ is always with respect to the quantum potential.

gauge-fixing term

$$\frac{1}{2\xi}\mathrm{Tr}\, G^2; \qquad G = \left[\mathcal{D}^{\mu}(C), Q_{\mu} - C_{\mu}\right]. \tag{2.66}$$

This is somewhat unconventional because this term depends on the classical potential. One must also calculate the Faddeev–Popov determinant,

$$\det \frac{\delta G}{\delta \theta}, \tag{2.67}$$

for an infinitesimal gauge transformation $V \approx \mathbb{I} - \mathrm{i}\theta$, under which the new variable of integration Q_{μ} transforms as

$$\delta Q_{\mu} = -\frac{1}{g}[\mathcal{D}_{\mu}, \theta]. \tag{2.68}$$

Note that this corresponds to transforming Q_{μ} inhomogeneously, as in Eq. (2.58), which is necessary because the action is only invariant under such a gauge transformation.

For the scalar field case, we were finished at this point because it was trivial to show that $\widetilde{W} = \Gamma$. It is only slightly more elaborate [23] to show that

$$\widetilde{W}[\mathcal{A}_{\mu} = 0, \widehat{A}_{\mu}] = \Gamma_C[\widehat{A}_{\mu}], \tag{2.69}$$

where Γ_C is the conventional effective action with the gauge-fixing term of Eq. (2.66). The complication is that the gauge-fixing term in the conventional effective action also depends on the external potential.

The Feynman rules for the effective action, consisting, as usual, of the sum of 1PI connected graphs in the presence of the external potential C_{μ}, are given by Abbott [23] and in the appendix. They are different from the usual rules because this external potential appears in the gauge-fixing term and in the ghost-antighost action. As far as we are concerned right now, two of the main differences are that the three-gluon vertex with one external potential leg is precisely the same as the vertex Γ^{ξ} of Eq. (1.38) and that the ghost-external potential vertex, unlike the conventional (asymmetric) ghost-gluon vertex, is conserved.

One can, of course, use any value for ξ in the gauge choice for the background-field method without affecting the fact that the effective action is a gauge-invariant functional of C_{μ}. Unfortunately, the coefficient functions found from the functional integrals over Q_{μ} still lead to ξ-dependent quantities, and so we are only part way along the path to true quantum gauge invariance. It is nevertheless true that simply by setting $\xi = 1$, we do get the gauge-invariant Green's functions of the pinch technique. In fact, Chapter 1 and our subsequent remarks already give us what amounts to a proof of this for one-loop quantities. The integrands in Eq. (1.61)

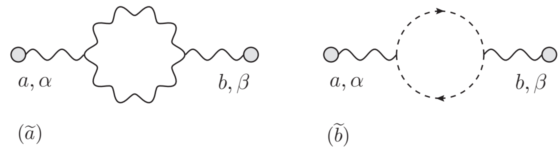

Figure 2.2. Feynman diagrams contributing to the one-loop background gluon self-energy. Shaded circles on external lines represent background fields.

for the PT proper self-energy and Eq. (1.84) for the three-gluon vertex are precisely those that would be found using the background-field method at $\xi = 1$. The background-field method at $\xi \neq 1$ does not give the PT results, but these can be recovered by applying the pinch technique as usual, thereby coming back to the background-field method results at $\xi = 1$.

2.3.3 Pinch technique and background Feynman gauge correspondence

Let us have a closer look at the announced connection between the pinch technique and the background Feynman gauge. The key observation [28, 29] is that at $\xi_Q = 1$, the tree-level vertex that occurs in the action multiplying $\widehat{A}_\alpha(q)A_\mu(k_1)A_\nu(k_2)$, to be denoted by $\widetilde{\Gamma}^{\xi_Q}_{\alpha\mu\nu}(q, k_1, k_2)$ (see Feynman rules of the appendix), collapses to the expression for $\Gamma^{\mathrm{F}}_{\alpha\mu\nu}(q, k_1, k_2)$, given in Eq. (1.42). Because in addition, at $\xi_Q = 1$, the longitudinal parts of the gluon propagator vanish, one realizes that at this point, there is nothing there that could pinch. Thus, ultimately, the background Feynman gauge is singled out because of the total absence, in this particular gauge, of any pinching momenta.

It is relatively straightforward to verify at the one-loop level the correspondence between the PT two- and three-point functions and those of the background Feynman gauge [28, 29]. For example, the two Feynman diagrams contributing to the background Feynman gauge gluon self-energy are shown in Figure 2.2. Using the background Feynman gauge Feynman rules, we obtain

$$
\begin{aligned}
(\widetilde{a})_{\alpha\beta} &= \frac{1}{2} g^2 C_A \int_k \frac{1}{k^2(k+q)^2} \widetilde{\Gamma}_{\alpha\mu\nu}(q, -k-q, k) \widetilde{\Gamma}_{\beta}^{\mu\nu}(q, -k-q, k) \\
(\widetilde{b})_{\alpha\beta} &= -g^2 C_A \int_k \frac{1}{k^2(k+q)^2} (2k+q)_\alpha (2k+q)_\beta .
\end{aligned}
\tag{2.70}
$$

We can simply compare the two terms on the rhs of Eq. (1.63) with the two terms given in Eqs. (2.70). Evidently, the PT and background Feynman gauge gluon self-energies are identical at one loop.

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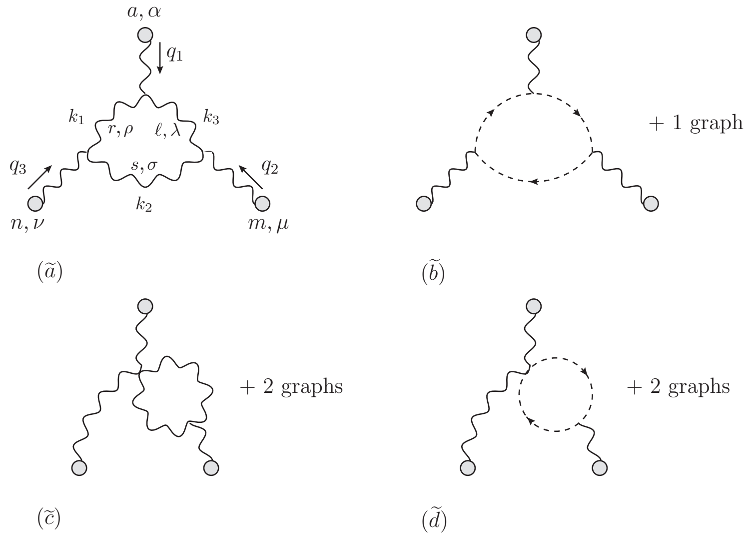

Figure 2.3. One-loop diagrams contributing to the three-gluon vertex in the BFM. Diagrams $(\widetilde{c})$ carries a $1/2$ symmetry factor.

Similarly, the one-loop diagrams contributing to the background Feynman gauge three-gluon vertex are shown in Figure 2.3; it is easy to see that the sum $(\widetilde{a}) + (\widetilde{b})$ coincides with the term $\widehat{N}_{\alpha\mu\nu}$ of Eq. (1.85), while diagrams $(\widetilde{c})$ give exactly the term $\widehat{B}_{\alpha\mu\nu}$ of Eq. (1.86). Finally, diagrams $(\widetilde{d})$ vanish by virtue of elementary group-theoretical identities.

Although it is a remarkable and extremely useful fact that the one-loop PT Green's functions can be calculated in the background Feynman gauge, particular care is needed for the correct interpretation of this correspondence. First, the pinch technique enforces gauge independence (and several other physical properties, such as unitarity and analyticity) on off-shell Green's functions, whereas the BFM, in a general gauge, does not. This is reflected in the gauge invariance of the BFM n-point functions in the sense that they satisfy (by construction) QED-like Ward identities, but are not gauge independent, i.e., they depend explicitly on ξ_Q. For example, the BFM gluon self-energy at one loop is given by [30]

$$\widetilde{\Pi}^{(\xi_Q)}_{\alpha\beta}(q) = \widetilde{\Pi}^{(\xi_Q=1)}_{\alpha\beta}(q) + \frac{\mathrm{i}}{4(4\pi)^2} g^2 C_A (1-\xi_Q)(7+\xi_Q) q^2 P^{\alpha\beta}(q). \quad (2.71)$$

Had the BFM n-point functions been ξ_Q independent, in addition to being gauge invariant, there would be no need to introduce the pinch technique independently.

We emphasize that the objective of the PT construction is not to derive diagrammatically the background Feynman gauge but rather to exploit the underlying BRST

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symmetry to expose a large number of cancellations and eventually define gauge-independent Green's functions satisfying Abelian Ward identities. Thus, that the PT Green's functions can also be calculated in the background Feynman gauge always needs a very extensive demonstration. Therefore, the correspondence must be verified at the end of the PT construction and should not be assumed beforehand. Moreover, the ξ_Q-dependent BFM Green's functions are not physically equivalent. This is best seen in theories with spontaneous symmetry breaking: the dependence of the BFM Green's functions on ξ_Q gives rise to *unphysical* thresholds inside these Green's functions for $\xi_Q \neq 1$, which limits their usefulness for resummation purposes (this point will be studied in detail in Chapter 11). Only the case of the background Feynman gauge is free from unphysical poles because then (and only then) do the BFM results collapse to the physical PT Green's functions.

It is also important to realize that the PT construction goes through unaltered under circumstances in which the BFM Feynman rules cannot even be applied. Specifically, if instead of an S-matrix element, one were to consider a different observable, such as a current correlation function or a Wilson loop (as was in fact done in the original formulation [4]), one could not start out using the background Feynman rules because *all* fields appearing inside the first nontrivial loop are quantum ones. Instead, by following the PT rearrangement inside these physical amplitudes, the unique PT answer emerges again.

Perhaps the most compelling fact that demonstrates that the PT and BFM are intrinsically two completely disparate methods is that one can apply the PT within the BFM. Operationally, this is easy to understand: away from $\xi_Q = 1$, even in the BFM, there are longitudinal (pinching) momenta that will initiate the pinching procedure. Thus, one starts out with the S-matrix written with the BFM Feynman rules using a general ξ_Q and applies the PT algorithm as in any other gauge-fixing scheme; one will recover again the unique PT answer for all Green's functions involved (i.e., the Green's functions will be projected to $\xi_Q = 1$).

2.3.4 The generalized pinch technique

As we have seen in detail, the PT projects us dynamically to the background Feynman gauge, regardless of the gauge-fixing scheme from which we may start. A question that arises naturally at this point is the following: could we devise a PT-like procedure that would project us to some other value of the background gauge-fixing parameter ξ_Q? As was shown by Pilaftsis [32], such a construction is indeed possible; the systematic algorithm that accomplishes this is known as the *generalized pinch technique*.

The starting point of the generalized PT is precisely the decomposition given in Eqs. (1.37), (1.38), and (1.39). However, unlike the pinch technique where *all*

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longitudinal momenta are allowed to pinch, in the generalized pinch technique, the Γ^{ξ} of Eq. (1.38) does not trigger any pinching (even though it contains longitudinal momenta), playing essentially the role of Γ^{F}; the pinching momenta of the generalized pinch technique come from the $\Gamma^{\mathrm{P}\xi}$ of Eq. (1.39) and, of course, the tree-level gluon propagators. At the end of this procedure, one recovers diagrammatically the background Green's functions calculated at the desired value of $\xi \to \xi_Q$.

To be sure, the generalized pinch technique represents a fundamental departure from the primary aim of the pinch technique, which is to construct gauge-fixing, parameter–independent, off-shell Green's functions. The generalized pinch technique, instead, deals exclusively with gauge-fixing, parameter–dependent Green's functions, with all the pathologies that this dependence entails. Nonetheless, it is certainly useful to have a method that allows us to move systematically from one gauge-fixing scheme to another at the level of individual Green's functions. In addition to the possible applications mentioned by Pilaftsis [32], we would like to emphasize the usefulness of the generalized pinch technique in truncating gauge-invariant (i.e., maintaining transversality) sets of Schwinger–Dyson equations written in gauges other than the Feynman gauge (see Chapter 6). This possibility becomes particularly relevant, for example, in attempts to compare SDE predictions with lattice simulations, which are carried out usually in the Landau gauge.

The method can be systematically generalized to more complicated situations [32]. For instance, a method may be projected from the R_ξ gauges to one of the generalized BFM gauges, such as the BFM axial gauge. This, of course, leads to a proliferation of pinching momenta; the resulting construction is therefore more cumbersome but remains conceptually rather straightforward.

2.4 What to expect beyond one loop

Everything in Chapters 1 and 2 illustrates the pinch technique at the one-loop level. The pinch technique would be of little interest unless everything in these chapters had an all-order generalization. A good part of the rest of the book is devoted to showing that the PT propagator has the following indispensable properties to all orders, and even nonperturbatively:

1. It truly is gauge independent.
2. It is independent of what group representation or spin the external particles used to construct the S-matrix have. (The reader should check this for the one-loop pinch technique.)
3. It has only physical threshholds even if (or especially if) the gluons get a mass through the strong interactions. There are no unphysical ghost contributions.

4. Its Green's functions have conventional analytic properties and spectral representations, except that in certain cases, conventional positivity requirements do not hold.
5. It, along with similarly defined pinch technique vertices, participates in ghost-free Ward identities that are analogous to those of QED and with similar consequences such as generalizations of the familiar QED identity $Z_1 = Z_2$.
6. The PT propagator defines a running charge that is gauge invariant and scheme independent.
7. Once the ghost-free Ward identities are imposed, it is unique, which can be understood because (as we will show) PT propagators and vertices are simply those of the background Feynman gauge, which is a uniquely defined graphical prescription with the same Ward identities.

As a result of the preceding requirements – and we emphasize once again not the other way around – we show that the PT Green's functions to all orders are identical to those of the background-field method in the Feynman gauge.

After the technical developments that establish these points come the applications. They range from perturbative effects, such as a physical and gauge-invariant definition of the neutrino charge radius, to nonperturbative effects, such as the all-order resummation needed to define the widths of unstable gauge bosons beyond tree level, to setting up the tools necessary for calculating the dynamical mass of gauge bosons in the magnetic sector of QCD or of high-temperature electroweak theory.

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