## 7. COMMISSION DE LA MECANIQUE CELESTE

## Report of Meetings

President: D. Brouwer.
Secretary: M. S. Davis.

## First meeting, 16 August 196I

President Brouwer opened the meeting at $14^{\mathrm{h}} 0_{5}^{\mathrm{m}}$. Dr J. Kovalevsky was appointed official translator in the English, French and Russian languages and Dr M. S. Davis was appointed secretary.

President Brouwer regretfully announced the death of the members: Milankovitch, Vernic, and the former member von Zeipel.

After a brief discussion of typographical errors in the Draft Report, the President announced the following new Organizing Committee as well as recommended officers:

President: Y. Hagihara (Japan).
Vice-President: W. J. Eckert (U.S.A.).
Organizing Committee: G. A. Chebotarev (U.S.S.R.), J. Kovalevsky (France), G.
Lemaître (Belgium), P. J. Message (Great Britain), K. Stumpff (Germany), F. Zagar (Italy).
No opposition being expressed, the committee was taken as thus approved.

## COMMUNICATIONS

Professor Martin of Trieste proposed, in a letter to the President and with the support of Professor Heinrich of Prague, that a Sub-Commission on 'Evolution of Double Stars' be formed to deal with problems related to Celestial Mechanics. In the discussion that ensued, Dr Clemence agreed with the substance of the proposal but felt that there were probably not enough members to form a separate section. Professor Zagar thought that if this were done the door would be open to the formation of many Sub-Commissions. Professor Chebotarev pointed out that under the new rules new Sub-Commissions could not be formed-only working groups: The question as stated in the letter was then put to a vote and was defeated.

A communication from Professor Fabre was read by Dr Kovalevsky. It suggested that density models of the Earth be improved by analysis of artificial satellite observations to bring about better accord between the models used by geophysicists and those used by astronomers.

Another communication from Professor Belorizky stated that Commission 7 should concern itself more with problems in Celestial Mechanics rather than organizational matters. There was general agreement with his statement.

Professor Brouwer summarized significant progress in Celestial Mechanics since the last meeting of the Commission. In particular he indicated the following noteworthy mile-stones:
r. The new, more precise, determinations of the coefficients that appear in the expression for the Earth's potential.
2. Important progress in application of high-speed computers to problems in Celestial Mechanics.
3. Three-body Problem. Merman's new representation by convergent series similar to Sundman's methods.
4. Clemence's Theory of Mars. The error now does not exceed 0.0013 in the longitude and $0^{\prime \prime} \cdot 002$ in the latitude.
5. Satellite theory. Petrowskaja's proof that for $\mathrm{m} \leqslant 0.21$ Hill's series for the variation orbit are convergent.
6. Kovalevsky's use of a numerical integration to build a general theory of Jupiter VIII.
7. Makover and Bokhan's mass determination of Mercury from the motion of comet EnckeBacklund.
8. Duncombe's discussion of the observations of Venus (1750-1949).
9. Littlewood's treatment of the convergence and asymptotic properties of the solutions of the three-body problem.

Professor Brouwer invited discussion on these questions or any others of general interest to the Commission. It was decided that Dr Sarkian would present his paper at the next meeting of the Commission at which time Dr Eckert would present his report.

Professor Brouwer opened discussion on the form to recommend for the force function. Sir Harold Jeffreys suggested that the summation sign be preceded by a + sign instead of the - sign. He felt that this was called for since (i) some of the $f$ s are negative anyway and (ii) because comparisons with the value of $g$ would be simpler. Dr Herrick did not believe the minus sign was seriously needed. He suggested that longitude be uniformly measured eastwards. Professor Chebotarev pointed out that at the Institute of Theoretical Astronomy the plus sign is always used and that the notation for the coefficients is:

$$
\begin{aligned}
C_{n, m} & \text { when longitude-dependent } \\
-C_{m, 0} & \text { when longitude-independent }
\end{aligned}
$$

Professor Brouwer remarked that confusion could be removed by defining

$$
-\mathscr{f}_{m}=C_{m, 0}
$$

in the manner suggested by Chebotarev. Dr Kozai indicated that his use of $\mathcal{F}$ and $C$ as proposed would conflict with other established uses. Dr Garfinkel defended the use of the minus sign on historical grounds and also because the leading coefficient has a positive sign.

Professor Brouwer appointed the following committee to discuss this question and to bring in recommendations at the next meeting of the Commission:

Hagihara (chairman), Garfinkel, Sir Harold Jeffreys, Kozai, Chebotarev, Hori.
The Abbé Lemaitre presented a paper on the 3-body problem. By using the transformation

$$
\tau \doteq \int \frac{\mathrm{d} t}{r_{1} r_{2} r_{3}}
$$

the Levi-Cività regularization may be put in a symmetric form which permits the extension of the eccentricity into the 3 -body problem.

A second paper by Abbé Lemaitre described a method of numerical integration using a Lagrangean type formula and retaining one extra difference for reducing the size of the coefficients by a factor of about 4 , as well as producing a more random distribution of signs in the formula. This has the effect of reducing the accumulation of error due to rounding.

After some discussion of these papers the meeting was adjourned at $15^{\mathrm{h}} 35^{\mathrm{m}}$.

## Second meeting, 21 August ig6r

The meeting was opened by President Brouwer at $9^{h} 00^{m}$. Dr Eckert presented the following report of the committee dealing with material in machine-readable form. A number of suggestions for standardization was made. After some discussion, unanimous approval was given to Dr Eckert's report.

## REPORT BY DR W. J. ECKERT

In spite of the poor response to the notice in IAU Information Bulletin No. 5 of February 1961, there appears to be considerable desire on the part of astronomers for more information concerning the availability of machine-readable records.

The Committee proposes:

1. Members of the Committee will write to the observatories in their respective geographical areas requesting information concerning records that are available for exchange.
2. Information received will be consolidated and a list of the available records will be published in an IAU Information Bulletin.
3. Astronomers may obtain information concerning available files and advice or assistance in obtaining them by writing to the appropriate member of the Committee.
4. Most members of the Committee feel that comparatively few machine programs are suitable for exchange and in very few cases can they be exchanged without the direct co-operation of the writer of the program. The Committee is willing to list in its publication unusual programs suitable for exchange.

Letters will be written by the following Committee members:
G. M. Clemence or M. S. Davis: U.S.A., Canada, Latin America
D. H. Sadler: British Commonwealth (except Canada)
J. Kovalevsky: France, Belgium, Holland, Spain
P. Naur: Denmark, Sweden, Norway, Finland
W. Fricke: West Germany, Switzerland, Italy
M. F. Subbotin: U.S.S.R., Poland, East Germany, Bulgaria, Rumania
S. Hamid: Arab Countries

Material will be collected, edited and published by: W. J. Eckert and W. Bidelman

## REPORT OF COMMITTEE ON NOTATIONS

Professor Hagihara then reported that the committee on notations, which had been appointed at the first meeting of the Commission, had met twice with Sir Harold Jeffreys, Chebotarev, Garfinkel, Kozai and himself present. The committee found it desirable to unify notations for the Earth's gravitational potential and recommended the following:

1. Axial symmetry

$$
U=\frac{\mu}{r}\left[\mathrm{I}-\sum_{n=1}^{\infty} \mathfrak{y}_{\mathrm{n}}\left(\frac{R}{r}\right)^{n} P_{n}(\sin \beta)\right]
$$

## 2. General Case

$$
\begin{gathered}
U=\frac{\mu}{r}\left[\mathrm{I}+\sum_{n=1}^{\infty} \sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n} P_{n}^{m}(\sin \beta)\left\{C_{n, m} \cos m \lambda+S_{n, m} \sin m \lambda\right\}\right] \\
\text { where } P_{n}^{m}(x)=\left(\mathrm{I}-x^{2}\right)^{t m} \frac{\mathrm{~d}^{m}}{\mathrm{~d} x^{m}} P_{n}(x) \\
\text { and } \int_{-1}^{+1}\left[P_{n}^{m}(x)\right]^{2} \mathrm{~d} t=\frac{2}{2 n+\mathrm{I}} \frac{(n+m)!}{(n-m)!}
\end{gathered}
$$

## 3. Alternative Form

$$
\begin{gathered}
U=\frac{\mu}{r}\left[\mathrm{I}+\sum_{n=1}^{\infty} \sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n} p_{n, m}(\sin \beta)\left\{A_{n, m} \cos m \lambda+B_{n, m} \sin m \lambda\right\}\right] \\
\text { where } p_{n, m}(x)=\left(\frac{(n-m)!}{(n+m)!}\right)^{\frac{1}{2}} P_{n}^{m}(x)
\end{gathered}
$$

4. Longitude be measured positive to the East.
5. Recommends the computation and circulation of tables of the quantities

$$
P_{n}^{m}(x),\left(\frac{(n-m)!}{(n+m)!}\right)^{\frac{1}{n}} \quad n, m=0, \mathrm{r}, 2, \ldots 8
$$

by the Smithsonian Astrophysical Observatory.
In all the above formulae, $R$ represents the equatorial radius of the Earth.
The members of the Commission present agreed that the recommended notation did not pertain to the quantities $\mu, R, \beta$.

In the discussion which ensued, Dr Herget asked why the expression in 2 does not degenerate to that in I. Professor Hagihara replied that this form was retained to pay respect to former work in the field and Professor Brouwer added that the use of $\mathscr{f}$ has been standard for a long time. Dr Herget felt that by recommending this form, it would be perpetuated in the future. Dr Kovalevsky remarked that in this context longitude should be measured to the east only. Otherwise the possibilities of errors would be very large. Dr Herrick made the following suggestion:

$$
C_{n, 0}=-\mathfrak{f}_{n}=C_{n}
$$

His recommendation was that the zero subscript be dropped in the coefficient $C_{n, 0}$.
Herrick's suggestion was adopted followed by the adoption of the above recommendations of the committee.

Professor Sarkian then presented his paper on "Additional Accelerations in Celestial Mechanics".

Abbé Lemaître, at the suggestion of Professor Brouwer, elaborated on his method of numerical integration.

The proceedings of the Commission were adjourned at $11^{\mathrm{h}} 00^{\mathrm{m}}$.

