ENTREPRENEURS, FINANCIERS, AND BOOM–BUST CYCLES

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In this paper, a dynamic general equilibrium model with infinitely lived entrepreneurs and financiers is developed to investigate a possible mechanism that explains business cycles and financial crises. The highest growth rate is achievable only if financiers coexist with entrepreneurs, given a certain extent of financial market imperfections. However, if financiers coexist with entrepreneurs, the economy is highly likely to face a financial crisis at certain parameter values. These two-sided implications of the coexistence of entrepreneurs and financiers explain why both instability and high growth are frequently observed in modern economies. Furthermore, our model can obtain countercyclical movements in total factor productivity growth that cannot be explained by the standard real business cycle theory but were observed in the Great Recession of 2007–2008.

Keywords: Boom–Bust Cycles, Entrepreneurs, Financiers, Financial Market Imperfections

1. INTRODUCTION

A remarkable characteristic of modern capitalism is the coexistence of entrepreneurs and financiers. The financier class began to emerge under the modern monetary and financial systems of the early seventeenth century. An economy is able to exploit the tremendous financial resources supplied by financiers to complete large investment projects that would otherwise be impossible. Financiers lend their net worth to entrepreneurs to propagate their wealth in a financial market, and most financiers do not engage in production activities. Entrepreneurs, who trust in their own entrepreneurial talents, raise funds in the financial market and invest

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in projects that produce added value if they succeed. Even in an economy with an established financial market, however, agency problems remain, and accordingly, financial resources are not used in the most efficient manner. Acknowledging these situations in modern monetary and financial systems, we develop a dynamic general equilibrium model of a financially constrained economy by introducing financiers who only extract goods from producers (i.e., entrepreneurs) without engaging in any production activity. Employing this model, we investigate how the existence of financiers affects macroeconomic stability and derive endogenous boom–bust cycles in equilibrium. Moreover, we show that our model can produce countercyclical movements in total factor productivity (TFP) growth that cannot be explained by the standard real business cycle theory but were observed in the Great Recession of 2007–2008.

Over the past twenty years, the importance of financial constraints as a fundamental cause of business cycles has been emphasized by many researchers, such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Aghion et al. (1999), and Aghion et al. (2004).[1] Among those researchers, Woodford (1986) focuses on the interactive relationship between two distinct classes of agents, capitalists and workers, in a financially constrained economy and clarifies a mechanism that creates business cycles.[2] He assumes that capitalists and workers differ in their income sources and their access to credit and shows that endogenous business cycles driven by self-fulfilling rational expectations can emerge.[3] The present paper is along the same lines as Woodford (1986) in that we address the macroeconomic implications, specifically for business cycles, of the existence of two distinct classes of agents. Unlike Woodford (1986), we consider *entrepreneurs* and *financiers*. We investigate how endogenous business cycles arise in an economy in which the entrepreneur class interacts with the financier class through the financial market.

In our model, agents who have inherent entrepreneurial talents are called *entrepreneurs*. Entrepreneurs are able to access a production technology. However, because they receive uninsured idiosyncratic productivity shocks in each period, only highly productive entrepreneurs engage in production, borrowing financial resources in the financial market at an interest rate lower than their productivity. Less productive entrepreneurs lend their net worth in the financial market at an interest rate greater than their productivity. In other words, borrowing entrepreneurs and lending entrepreneurs endogenously arise in each period, depending on the idiosyncratic productivity shocks they face. *Financiers* are assumed to have no inherent entrepreneurial talents. Instead, they lend their net worth in the financial market. Financiers in our model are agents who acquire income only by lending their net worth in the financial market, and they never obtain labor income or engage in production. Accordingly, there are three types of agents in the economy in each period.

A number of recent studies explicitly focus on the role of financiers. For example, Takalo and Toivanen (2012) develop a static model that distinguishes the role of financiers from that of entrepreneurs.[4] Financiers in the model developed

by Laeven et al. (2014) can actively engage in financial innovation that promotes economic growth. Moreover, in the overlapping-generations framework, Myerson (2012, 2014) shows that dynamic interactions among financiers in different generations can produce boom–bust credit cycles. The role of financiers in our model is similar to that in Myerson (2012, 2014) in the sense that they play a significant role in the production of boom–bust cycles. However, unlike those in Myerson's model, financiers in our model, which has an infinite time horizon, do not explicitly solve agency problems but only acquire unearned income by lending their net worth in an imperfect financial market.[5] Although the setting of our model is simple, we can obtain rich dynamics of the model economy.

In our model, entrepreneurs who engage in production borrow in the financial market in equilibrium, but they face credit constraints. In such a situation, entrepreneurs who engage in production are not always the "most" productive but are "highly" productive entrepreneurs, implying that production resources are not used in the most efficient manner in each period. Under these circumstances, the existence of financiers has two-fold importance, given a certain extent of financial market imperfections. The highest growth rate is achievable in a financially constrained economy only if financiers coexist with entrepreneurs. However, if financiers coexist with entrepreneurs, the economy is very likely to exhibit endogenous business cycles and could face a financial crisis followed by a severe economic depression. That is, the existence of financiers boosts the growth rate, and simultaneously, the existence of financiers entails the risk that the economy could collapse.

The occurrence of boom–bust cycles in our model can be explained by the demand for and supply of financial resources. During a boom, because financiers propagate their net worth, the aggregate supply of financial resources grows at a higher rate than aggregate demand. As a result, the excess supply of financial resources exerts downward pressure on the interest rate. Although interest rates continue to decline during the boom, investment in unproductive projects increases because the population of less productive producers increases. Because the most productive entrepreneurs face financial constraints, an increase in investment, even in unproductive projects, boosts economic growth during a boom. At the end of a boom, a further increase in the excess supply of financial resources causes a steep decline in the interest rate, and the net interest rate could be negative. In the presence of a negative net interest rate, the financiers' net worth suddenly declines to a very low value, and the supply of financial resources is significantly reduced in the following period. Then credit contraction occurs in the financial market, and unproductive investment projects (which contribute to high growth during the boom) are no longer undertaken. Accordingly, the economy enters a recession. At the end of the recession, because the interest rate becomes sufficiently high, the financiers can begin to propagate their net worth, and another economic boom begins.[6] In our model, if one employs a single-peaked distribution (including a uniform distribution) regarding productivity shocks, the subjective discount factor

of entrepreneurs must be very small to give rise to endogenous business cycles. However, the dynamic behavior of an economy crucially depends on the shape of productivity distributions in our model. For example, a binary distribution that has two spikes in the probability density produces endogenous business cycles even though the subjective discount factor of entrepreneurs is nearly as large as that of financiers.

In addition to endogenous boom–bust cycles, our model can create counter-cyclical movements in TFP growth. This is because in the process of a boom, unproductive investment projects are undertaken, and when an economy goes into a recession, such unproductive investment projects are no longer undertaken. Fernald and Matoba (2009) demonstrate that the growth rate of utilization-adjusted TFP in the United States was negative until the mid-2000s, just before the financial crisis of 2007–2009, but TFP growth became positive approximately in 2007, with TFP increasing during the financial crisis. Whereas the standard real business cycle theory cannot explain Fernald and Matoba's evidence indicating countercyclical movements in TFP growth, Petrosky-Nadeau (2013) develops a model to explain the evidence provided by Fernald and Matoba (2009). Our model also generates countercyclical movements in TFP growth, as in Petrosky-Nadeau's model. The productivity of the aggregate production function derived from our model de-creases during a boom, and it is minimized just before the economy enters a depres-sion. When the economy enters a depression, productivity begins to increase. These countercyclical movements are consistent with Fernald and Matoba's empirical observations.

Many studies, including Cordoba and Ripoll (2004a) and Pintus and Wen (2013), assume the existence of heterogeneous agents who have different discount factors to endogenously obtain borrowers and lenders. In such a setting, agents with a lower discount factor become borrowers and agents with a higher discount factor become lenders in equilibrium. Although we also assume that the discount factor of financiers is greater than that of entrepreneurs, we do not make this assumption for the sake of creating borrowers and lenders. In our model, borrowers and lenders endogenously appear because productivity in general goods production differs across agents. The different discount factors in our model play a crucial role in deriving two steady states, and the presence of the two steady states enables us to obtain endogenous business cycles based on global dynamic behavior. In Cordoba and Ripoll (2004a) and Pintus and Wen (2013), however, a single steady state appears, and only the local propagation mechanism of exogenous shocks in the neighborhood of the steady state is analyzed. Moreover, Cordoba and Ripoll (2004a) and Pintus and Wen (2013) rely on numerical analyses to investigate business cycles, but our model provides simple phase diagrams to observe endogenous business cycles.

The remainder of this paper is organized as follows. In the next section, we present a growth model in which there are two classes of agents: entrepreneurs and financiers. In Section 3, we investigate equilibrium growth rates and the local and global stabilities of the economy. In Section 4, we discuss a severe

depression preceded by a contraction in the financiers' net worth. When we employ a single-peaked productivity distribution, endogenous business cycles occur only if entrepreneurs are very impatient relative to financiers. In Section 5, we demonstrate that a binary productivity distribution can produce endogenous business cycles despite entrepreneurs being nearly as patient as financiers. In Section 6, we discuss the literature related to our study, and in Section 7, we present concluding remarks.

2. MODEL

A closed economy consists of two classes: one unit measure of infinitely lived entrepreneurs and one unit measure of infinitely lived financiers. Time is discrete and goes from 0 to ∞. Entrepreneurs are ex ante homogeneous and ex post heterogeneous because they receive idiosyncratic productivity shocks in each period. No financiers engage in production, because they have no inherent entrepreneurial talents. Instead, financiers lend their net worth in the financial market to obtain income in each period.

2.1. Entrepreneurs

An entrepreneur has two methods of saving. One is lending her net worth in the financial market. If she lends one unit of general goods in the financial market at time $t-1$, she will acquire a claim to r_t units of general goods at time t, where r_t is the (gross) real interest rate. The other saving method is initiating an investment project. If an entrepreneur invests one unit of general goods in an investment project at time $t-1$, she will create $A\Phi_{t-1}$ units of general goods at time t. An idiosyncratic shock Φ_{t-1} with respect to productivity at time t is realized at time $t-1$, which implies that production requires one gestation period. Accordingly, an entrepreneur at time $t-1$ already knows her productivity at time t. Low productivity cannot be insured against because no insurance market exists for the idiosyncratic productivity shocks. An entrepreneur faces a credit constraint when borrowing financial resources because of agency problems in the financial market. In each period, entrepreneurs consume, lend their net worth, and/or invest in projects by borrowing financial resources in the financial market.

Productivity Φ_{t-1} is a random variable, implying that it is a function of a stochastic event ω_{t-1}, where $\{\omega_{t-1} \in \Omega \mid \Phi_{t-1}(\omega_{t-1}) \leq \Phi\}$ is an element of a σ-algebra $\mathcal{F}$ with the probability space $(\Omega, \mathcal{F}, P)$. As in Angeletos (2007), the stochastic events $\omega_0, \omega_1, ...$ [and the idiosyncratic productivity shocks $\Phi_0(\omega_0), \Phi_1(\omega_1), ...$] are assumed to be independent and identically distributed across both time and agents (the i.i.d. assumption). Φ has support over $[0, h]$, where $h > 0$ is finite. Φ's cumulative distribution function is given by $G(\Phi)$, where $G(\Phi)$ is continuous, differentiable, and strictly increasing in the support.

We define the histories of stochastic events and idiosyncratic productivity shocks until time $t-1$ so that $\omega^{t-1} = \{\omega_0, \omega_1, ...\omega_{t-1}\}$ and $\Phi^{t-1} = \{\Phi_0, \Phi_1, ...\Phi_{t-1}\}$.

Then there exists a probability space $(\Omega^t, \mathcal{F}^t, P^t)$, which is a Cartesian product of t copies of $(\Omega, \mathcal{F}, P)$, where $\Phi^{t-1}(\omega^{t-1})$ is a vector function of history $\omega^{t-1} \in \Omega^t$.

An entrepreneur at time t maximizes her expected lifetime utility, which is given by

$$U_t^e = E\left[\sum_{\tau=t}^{\infty} \beta_e^{\tau-t} \ln c_\tau(\omega^\tau) \middle| \Phi^t(\omega^t)\right],$$

subject to

$$k_\tau(\omega^\tau) + b_\tau(\omega^\tau) = A\Phi_{\tau-1}(\omega_{\tau-1})k_{\tau-1}(\omega^{\tau-1}) + r_\tau b_{\tau-1}(\omega^{\tau-1}) - c_\tau(\omega^\tau), \quad \textbf{(1)}$$

$$b_\tau(\omega^\tau) \geq -\theta a_\tau(\omega^\tau), \text{ where } a_\tau(\omega^\tau) := k_\tau(\omega^\tau) + b_\tau(\omega^\tau), \quad \textbf{(2)}$$

$$k_\tau(\omega^\tau) \geq 0, \quad \textbf{(3)}$$

for $\tau \geq t \geq 0$. $\beta_e \in (0, 1)$ is the subjective discount factor, which is common to all entrepreneurs, and $E[.|\Phi^t]$ is an expectation operator given the information set associated with Φ^t at time t. Equation (1) represents the flow budget constraints, where $c_\tau(\omega^\tau)$ is consumption, $k_\tau(\omega^\tau)$ is investment in a project, and $b_\tau(\omega^\tau)$ is a debt if negative and a credit if positive at time τ. $A\Phi_{\tau-1}(\omega_{\tau-1})k_{\tau-1}(\omega^{\tau-1})$ represents the general goods produced by the entrepreneur at time τ. We assume that the general goods are perishable in one period.[7] $a_\tau(\omega^\tau)$ is the entrepreneur's net worth and r_τ is the gross interest rate at time τ. Note that $a_\tau(\omega^\tau)$ is equal to her savings because $a_\tau(\omega^\tau) = A\Phi_{\tau-1}k_{\tau-1}(\omega^{\tau-1}) + r_\tau b_{\tau-1}(\omega^{\tau-1}) - c_\tau(\omega^\tau)$. We assume that at $t = 0$, the flow budget constraint is given by $k_0 + b_0 = w_0 - c_0$, where w_0 is the initial endowment that each entrepreneur holds at time 0, which is common to all entrepreneurs.

The credit constraint faced by each entrepreneur is given by equation (2). As in Aghion et al. (1999), Aghion and Banerjee (2005), Aghion et al. (2005), and Antrás and Caballero (2009), an entrepreneur is able to borrow financial resources in the financial market only up to θ times her net worth.[8] $\theta \in (0, \infty)$ represents the extent of credit constraints. As θ goes to infinity, the financial market approaches perfection, and as θ goes to zero, no entrepreneurs are able to borrow. $\theta \in (0, \infty)$ means that financial market imperfections are at an intermediate level. Finally, equation (3) is the nonnegativity constraint on investment.

Define $\phi_t := r_{t+1}/A$. From the entrepreneurs' maximization problem, it is optimal for entrepreneurs with $\Phi_t > \phi_t$ to invest in a project, borrow financial resources up to the limit of the credit constraint, and engage in general goods production. However, it is optimal for entrepreneurs with $\Phi_t < \phi_t$ to lend their net worth in the financial market and obtain the (gross) interest rate r_{t+1}. Note that ϕ_t is a cutoff for idiosyncratic productivity shocks that divides entrepreneurs into lenders and borrowers at time t. As a result, we obtain a lending–investment–borrowing plan for an entrepreneur who has net worth $a_t(\omega^t)$ at time t as follows:

$$k_t(\omega^t) = \begin{cases} 0 & \text{if } \Phi_t(\omega_t) < \phi_t \\ \frac{a_t(\omega^t)}{1-\mu} & \text{if } \Phi_t(\omega_t) > \phi_t \end{cases} \quad \textbf{(4)}$$

and

$$b_t(\omega^t) = \begin{cases} a_t(\omega^t) & \text{if } \Phi_t(\omega_t) < \phi_t \\ -\frac{\mu}{1-\mu} a_t(\omega^t) & \text{if } \Phi_t(\omega_t) > \phi_t, \end{cases} \tag{5}$$

where $\mu := \theta/(1 + \theta) \in (0, 1)$ also measures the extent of credit constraints. If there is no financial market, that is, $\mu = 0$, entrepreneurs cannot borrow and financiers cannot exist. Therefore, we focus on a case in which financial market imperfections are at an intermediate level, so that entrepreneurs are unable to borrow to the full extent that they desire but are able to borrow up to a certain proportion of their net worth. Under this lending–investment–borrowing plan, the flow budget constraint at time τ can be rewritten in intensive form as

$$a_\tau(\omega^\tau) = \tilde{R}_\tau a_{\tau-1}(\omega^{\tau-1}) - c_\tau(\omega^\tau), \tag{6}$$

where $\tilde{R}_\tau := \max\{r_\tau, \frac{A\Phi_{\tau-1}-r_\tau\mu}{1-\mu}\}$. We provide a derivation of the budget constraint, equation (6), in Appendix A. Given the lending–investment-borrowing plan in equations (4) and (5), an entrepreneur at time t maximizes her lifetime utility U_t^e subject to equation (6). The Euler equation is given by

$$\frac{1}{c_t(\omega^t)} = \beta_e E\left[\tilde{R}_{t+1} \frac{1}{c_{t+1}(\omega^{t+1})} \middle| \Phi^t(\omega^t) \right]. \tag{7}$$

The lifetime utility function is log-linear, and hence from equations (6) and (7) and the transversality condition, we obtain

LEMMA 1. *The law of motion of an entrepreneur's net worth $a_t(\omega^t)$ is given by*

$$a_{t+1}(\omega^{t+1}) = \beta_e \tilde{R}_{t+1} a_t(\omega^t). \tag{8}$$

Proof. See Appendix B.

2.2. Financiers

In this model, agents who exclusively acquire income by lending their net worth in the financial market and have no inherent entrepreneurial talents are called financiers. They never obtain labor income or engage in general goods production. In this analysis, we do not clarify how they brought their initial net worth into the economy. We may assume that they decided to earn their living exclusively by investing in the financial market when the modern monetary and financial system was established.[9] The important point regarding financiers in this model is that they never engage in any production activity, nor are they endowed with general goods in any period except for the initial one.

Each financier is endowed with an initial net worth $W_0 > 0$ at time 0. A representative financier at time t maximizes her lifetime utility as follows:

$$U_t^c = \sum_{\tau=t}^{\infty} \beta_c^{\tau-t} \ln c_\tau,$$

subject to

$$W_\tau = r_\tau W_{\tau-1} - c_\tau, \tag{9}$$

for $\tau \geq t \geq 0$, where W_τ is her net worth carried over from time τ to time $\tau + 1$ and $\beta_c \in (0, 1)$ is the subjective discount factor. We assume that $W_0 > 0$. Obtaining the Euler equation is straightforward:

$$\frac{1}{c_t} = \beta_c r_{t+1} \frac{1}{c_{t+1}}. \tag{10}$$

Similarly to the case of entrepreneurs, from equations (9) and (10) and the transversality condition, we obtain the law of motion for a representative financier's net worth in

LEMMA 2. *The law of motion for a representative financier's net worth W_t is given by*

$$W_{t+1} = \beta_c r_{t+1} W_t. \tag{11}$$

Proof. The proof is omitted because it is essentially the same as that of Lemma 1. ∎

2.3. Aggregation

We assume that the law of large numbers can be applied to entrepreneurs. Because $a_t(\omega^t) = \beta_e \tilde{R}_t a_{t-1}(\omega^{t-1})$ from equation (8), the net worth $a_t(\omega^t)$ of an entrepreneur who receives a stochastic event ω_t at time t and has history ω^{t-1} is given by

$$a_t(\omega^t) = \beta_e(A\Phi_{t-1}(\omega_{t-1})k_{t-1}(\omega^{t-1}) + r_t b_{t-1}(\omega^{t-1})), \tag{12}$$

where we should note from equations (4) and (5) that for an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) < \phi_{t-1}$, it follows that $k_{t-1}(\omega^{t-1}) = 0$ and $b_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})$. For an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) > \phi_{t-1}$, it follows that $k_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})/(1 - \mu)$ and $b_{t-1}(\omega^{t-1}) = -\mu a_{t-1}(\omega^{t-1})/(1 - \mu)$. The stochastic event ω_t and history ω^{t-1} are independent. Therefore, applying the law of large numbers to entrepreneurs, we aggregate the net worth of the entrepreneurs with the stochastic realization ω_t as follows:

$$\tilde{a}_t(\omega_t) := \int_{\Omega^t} a_t(\omega^t) dP^t(\omega^{t-1}) = \beta_e \int_{\Omega^t} (A\Phi_{t-1}(\omega_{t-1})k_{t-1}(\omega^{t-1})$$
$$+ r_t b_{t-1}(\omega^{t-1})) dP^t(\omega^{t-1}), \tag{13}$$

where we should note that ω^{t-1} is an element of Ω^t. From the financial-market-clearing condition at time $t - 1$, we have

$$W_{t-1} + \int_{\Omega^t} b_{t-1}(\omega^{t-1}) dP^t(\omega^{t-1}) = 0.$$

The aggregate output at time t is given by

$$Y_t := \int_{\Omega^t} A\Phi_{t-1}(\omega_{t-1})k_{t-1}(\omega^{t-1})dP^t(\omega^{t-1}).$$

Therefore, equation (13) is rewritten as

$$\tilde{a}_t(\omega_t) = \beta_e(Y_t - r_t W_{t-1}). \tag{14}$$

Equation (14) is the aggregate net worth across entrepreneurs who receive a stochastic event ω_t at time t. Equation (14) expresses the distribution of net worth in the economy with respect to ω_t, although the distribution is uniform over ω_t. Note that equation (14) is effective for $t \geq 1$. For $t = 0$, it follows that $\tilde{a}_0(\omega_0) = \beta_e w_0$.

As clarified in equations (4) and (5), entrepreneurs facing the stochastic event ω_t such that $\Phi_t(\omega_t) > \phi_t$ become producers at time t, whereas entrepreneurs facing the stochastic event ω_t such that $\Phi_t(\omega_t) < \phi_t$ become lenders. Therefore, from equations (4) and (5), the aggregate debt or credit $\tilde{b}_t(\omega_t)$ across the entrepreneurs with stochastic realization ω_t is given by

$$\tilde{b}_t(\omega_t) = \begin{cases} \tilde{a}_t(\omega_t) = \beta_e(Y_t - r_t W_{t-1}) & \text{if } \Phi_t(\omega_t) < \phi_t \\ -\frac{\mu}{1-\mu}\tilde{a}_t(\omega_t) = -\frac{\mu\beta_e}{1-\mu}(Y_t - r_t W_{t-1}) & \text{if } \Phi_t(\omega_t) > \phi_t. \end{cases} \tag{15}$$

Similarly, the aggregate investment $\tilde{k}_t(\omega_t)$ across the entrepreneurs facing stochastic realization ω_t is given by

$$\tilde{k}_t(\omega_t) = \begin{cases} 0 & \text{if } \Phi_t(\omega_t) < \phi_t \\ \frac{1}{1-\mu}\tilde{a}_t(\omega_t) = \frac{\beta_e}{1-\mu}(Y_t - r_t W_{t-1}) & \text{if } \Phi_t(\omega_t) > \phi_t. \end{cases} \tag{16}$$

From the financial-market-clearing condition, we have

$$W_t = -\int_E \tilde{b}_t(\omega_t)dP(\omega_t) - \int_{\Omega\backslash E} \tilde{b}_t(\omega_t)dP(\omega_t)$$

$$= -\beta_e(Y_t - r_t W_{t-1})\frac{G(\phi_t) - \mu}{1 - \mu}, \tag{17}$$

where $E = \{\omega_t \in \Omega \mid \Phi_t(\omega_t) \leq \phi_t\}$. Multiplying $A\Phi_t(\omega_t)$ by both sides of the second equation of (16) and aggregating the resulting equation across all entrepreneurs who engage in production, we obtain the total output Y_{t+1} as follows:

$$\int_{\Omega\backslash E} A\Phi_t(\omega_t)\tilde{k}_t(\omega_t)dP(\omega_t) = \int_{\Omega\backslash E} \frac{\beta_e A\Phi_t(\omega_t)}{1-\mu}(Y_t - r_t W_{t-1})dP(\omega_t)$$

$$\Longleftrightarrow Y_{t+1} = \frac{\beta_e AF(\phi_t)}{1 - \mu}(Y_t - r_t W_{t-1}), \tag{18}$$

where $F(\phi_t) := \int_{\phi_t}^{\infty} \Phi_t dG(\Phi_t)$.

3. EQUILIBRIUM DYNAMICS

3.1. Equilibrium

Defining $B := \beta_e/\beta_c$, we derive, from equations (11), (17), and (18), the dynamic equations for $t \geq 1$ with respect to the cutoff ϕ_t and the growth rate of aggregate output $\Gamma_{t+1}(\phi_t) := Y_{t+1}/Y_t$, respectively, as follows:

$$\frac{B(G(\phi_t) - \mu)}{1 - \mu - B(G(\phi_t) - \mu)} = \frac{\phi_{t-1}(G(\phi_{t-1}) - \mu)}{F(\phi_{t-1})} \tag{19}$$

and

$$\Gamma_{t+1}(\phi_t) = \frac{\beta_c ABF(\phi_t)}{1 - \mu - B(G(\phi_t) - \mu)}. \tag{20}$$

The net worth of the representative financier becomes

$$W_t = A\beta_c \phi_{t-1} W_{t-1}, \tag{21}$$

because $r_t = A\phi_{t-1}$.

$\phi_0 = r_1/A$ is a predetermined variable. To see this, we consider equation (17), which is effective for $t \geq 1$. Because we have $\tilde{a}_0(\omega_0) = \beta_e w_0$ for $t = 0$, equation (17) is modified for $t = 0$ such that $W_0 = -\beta_e w_0(G(\phi_0) - \mu)/(1 - \mu)$. Because W_0 and w_0 are predetermined, $\phi_0 = r_1/A$ is also predetermined.[10] In other words, ϕ_0 or r_1 is determined so that the financial market clears at time $t = 0$.

In a competitive equilibrium, the economy is recursively expressed by sequences $\{W_t, \phi_t, Y_{t+1}\}$, so that for all $t \geq 1$, these three sequences satisfy the difference equations (19), (20), and (21), given W_0, ϕ_0, and Y_1, where $Y_1 = \beta_e AF(\phi_0)w_0/(1 - \mu)$.

The dynamic behavior of ϕ_t associated with equation (19) provides information on both the dynamic behavior of W_t and the equilibrium growth rates of Y_t from equations (20) and (21). Therefore, we intensively analyze equation (19) in the following.

3.2. Steady States

We find from Lemma 2 that $W_t \geq 0$ for all $t \geq 0$, which implies that $G(\phi_t) \leq \mu$ for all $t \geq 0$ from equation (17), because $Y_t - r_t W_{t-1} > 0$ in equilibrium.[11] Therefore, we restrict the domain of the dynamical system (19) to $[0, G^{-1}(\mu)]$.

There are at most two steady states in the dynamical system (19). To examine the existence of the steady states in the dynamical system (19), we define ϕ^* and ϕ^{**} so that

$$G(\phi^*) = \mu,$$

$$\frac{1-\mu}{B} = \frac{F(\phi^{**})}{\phi^{**}} + (G(\phi^{**}) - \mu).$$

A unique value of ϕ^* must exist because $G(.)$ is a strictly increasing function over the support of Φ. To investigate the uniqueness of ϕ^{**}, we define a function

such that $H(x) := F(x)/x + (G(x) - \mu)$. $H(x)$ is strictly decreasing in $(0, h)$ because $H'(x) = -F(x)/x^2 < 0$ in $(0, h)$. In addition, $\lim_{x \to 0} H(x) = \infty$ and $\lim_{x \to h} H(x) = 1 - \mu$. Because ϕ^{**} is a solution of $H(x) = (1 - \mu)/B$, ϕ^{**} is uniquely determined in $(0, h]$ if and only if $B \leq 1$. ϕ^* and ϕ^{**} can be solved in terms of the parameters μ and B and the parameters of the distribution of Φ so that $\phi^*(\mu; \Theta)$ and $\phi^{**}(\mu, B; \Theta)$, where Θ is the parameter set of the distribution of Φ; however, we write ϕ^* and ϕ^{**} to save notation.

Because the domain of the dynamical system (19) is $[0, \phi^*]$, the system has a steady-state equilibrium ϕ^{**} in addition to ϕ^* if and only if $\phi^{**} < \phi^*$. Because $H(x) = F(x)/x + (G(x) - \mu)$ is a strictly decreasing function, the condition for $\phi^{**} < \phi^*$ is equivalent to $(1 - \mu)/B > H(\phi^*)$. In what follows, to focus on interesting cases, we assume this inequality. The inequality $(1 - \mu)/B > H(\phi^*)$ is rewritten

Assumption 1.

$$B < \frac{\phi^*(1 - G(\phi^*))}{F(\phi^*)} =: B^*. \tag{22}$$

Assumption 1 guarantees the existence of two steady-state equilibria in the dynamical system (19). Note that because $F(\phi^*) = \int_{\phi^*}^{h} \Phi dG(\Phi) > \int_{\phi^*}^{h} \phi^* dG(\Phi) = \phi^*(1 - G(\phi^*))$, Assumption 1 leads to $B = \beta_e/\beta_c < 1$. This finding implies that the entrepreneurs' subjective discount factor is strictly less than that of the financiers. In other words, for two steady-state equilibria to exist, financiers need to be more patient than entrepreneurs. This feature is absent in existing studies such as Kiyotaki and Moore (1997), Cordoba and Ripoll (2004a, 2004b), and Pintus and Wen (2013). To endogenously obtain borrowers and lenders, existing studies assume that agents have different discount factors. In our model, in contrast, borrowers and lenders appear endogenously because of their different productivity. The different discount factors in our model produce two steady-state equilibria. The presence of the two steady states allows us to investigate the global dynamic behavior and obtain endogenous business cycles. Because of the different discount factors, financiers play a crucial role in the current model, because otherwise the economy converges to ϕ^*, in which the presence of financiers is irrelevant.

We conclude this subsection with two remarks on the steady states of the economy. First, if μ is arbitrarily close to zero, ϕ^{**} does not exist, because the domain of the dynamical system $[0, G^{-1}(\mu)]$ shrinks to the origin. In this case, there is no financial market and financiers are unable to exist, because it is impossible for them to lend their net worth in the financial market. Second, if μ is arbitrarily close to one, ϕ^{**} and ϕ^* coincide and are equal to h. This can be verified from the definition of ϕ^* and ϕ^{**}. If μ is arbitrarily close to one, $G(\phi^*) = 1$ holds and thus $\phi^* = h$. Similarly, if μ is arbitrarily close to one, $F(\phi^{**}) = \phi^*(1 - G(\phi^{**}))$ holds. This equation holds if and only if $\phi^{**} = h$. In this case, the most talented entrepreneurs intensively and efficiently use all production resources, including financiers' resources.

3.3. Growth Rates

The growth rate Γ_{t+1} for $t \geq 1$ in equation (20) is a function of ϕ_t. In this section, we demonstrate that the steady state ϕ^{**} provides the highest growth rate in the economy.

PROPOSITION 1. *Suppose that Assumption 1 holds. Then, given the parameter values of β_e, β_c, and μ and the parameter values of the distribution $G(\Phi)$, the growth rate in the steady state ϕ^{**} is the highest for any $\phi_t \in [0, G^{-1}(\mu)]$.*

Proposition 1 is shown in the following. Differentiating Γ_{t+1} with respect to ϕ_t, we have

$$\Gamma'_{t+1}(\phi_t) = J(\phi_t) \frac{\beta_c ABG'(\phi_t)}{[1 - \mu - B(G(\phi_t) - \mu)]^2},$$

where $J(\phi_t) := BF(\phi_t) - \phi_t[1 - \mu - B(G(\phi_t) - \mu)]$. It can be verified easily that $J(\phi_t)$ is strictly decreasing and $J(\phi^{**}) = 0$. Therefore, Γ'_{t+1} is strictly greater than zero if $0 < \phi_t < \phi^{**}$ and is strictly less than zero if $\phi_t > \phi^{**}$. Therefore, the maximum of Γ_{t+1} is achieved at $\phi_t = \phi^{**}$.

We find from Proposition 1 that the existence of financiers has an important implication for the economy. Suppose that there are no financiers in this economy. In this case, $W_t = 0$ for all $t \geq 0$ and we only have the steady state $\phi^* = G^{-1}(\mu)$. This implies that given a certain extent of financial market imperfections, the economy can never attain the highest growth rate without financiers. The existence of financiers is necessary for the highest growth rate to be achievable, provided that the financial market is imperfect.

3.4. Local Stability

We investigate the local stability around the steady states of the dynamical system (19). Let us define two functions as $\Psi(\phi_t) := \frac{B(G(\phi_t) - \mu)}{1 - \mu - B(G(\phi_t) - \mu)}$, which is the left-hand side of equation (19), and $\Lambda(\phi_{t-1}) := \frac{\phi_{t-1}(G(\phi_{t-1}) - \mu)}{F(\phi_{t-1})}$, which is the right-hand side. $\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ are, respectively, approximated around the steady state of ϕ^* as follows:

$$\Psi(\phi_t) \approx \frac{BG'(\phi^*)}{1 - G(\phi^*)}(\phi_t - \phi^*)$$

and

$$\Lambda(\phi_{t-1}) \approx \frac{\phi^* G'(\phi^*)}{F(\phi^*)}(\phi_{t-1} - \phi^*).$$

From these approximations, we obtain the local dynamical system around the steady state of ϕ^* as follows:

$$\phi_t - \phi^* = \frac{\phi^*(1 - G(\phi^*))}{BF(\phi^*)}(\phi_{t-1} - \phi^*). \tag{23}$$

PROPOSITION 2. *Suppose that Assumption 1 holds. Then the steady state of ϕ^* is locally unstable.*

Proof. It follows from Assumption 1 that $\phi^*(1 - G(\phi^*))/BF(\phi^*) > 1$. ∎

$\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ are, respectively, approximated around the steady state of ϕ^{**} as follows:

$$\Psi(\phi_t) \approx \frac{\phi^{**}G'(\phi^{**})F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**}) - \mu)}{F(\phi^{**})^2}(\phi_t - \phi^{**})$$

and

$$\Lambda(\phi_{t-1}) \approx \frac{[(G(\phi^{**}) - \mu) + \phi^{**}G'(\phi^{**})]F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**}) - \mu)}{F(\phi^{**})^2}$$
$$\times (\phi_{t-1} - \phi^{**}).$$

Therefore, the local dynamical system around the steady state of ϕ^{**} is given by

$$\phi_t - \phi^{**} = \left[\frac{(G(\phi^{**}) - \mu)F(\phi^{**})}{\phi^{**}G'(\phi^{**})F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**}) - \mu)} + 1\right](\phi_{t-1} - \phi^{**}).$$

Because ϕ^{**} satisfies $(1 - \mu)/B = F(\phi^{**})/\phi^{**} + (G(\phi^{**}) - \mu)$, this equation is rewritten as

$$\phi_t - \phi^{**} = \left[\frac{(G(\phi^{**}) - \mu)(1 - \mu - B(G(\phi^{**}) - \mu))}{\phi^{**}G'(\phi^{**})(1 - \mu)} + 1\right](\phi_{t-1} - \phi^{**}). \quad \textbf{(24)}$$

PROPOSITION 3. *Suppose that Assumption 1 holds and define* $\bar{\Psi} := -\frac{(G(\phi^{**}) - \mu)(1 - \mu - B(G(\phi^{**}) - \mu))}{2\phi^{**}(1 - \mu)}$. *Then the steady state* ϕ^{**} *is locally stable if* $G'(\phi^{**}) > \bar{\Psi}$ *and unstable if* $G'(\phi^{**}) < \bar{\Psi}$.

Proof. Because $G(\phi^{**}) - \mu$ is negative and $1 - \mu - B(G(\phi^{**}) - \mu)$ is positive, the coefficient of $(\phi_{t-1} - \phi^{**})$ in equation (24) is less than 1. Note that $G'(\phi^{**})$ is in $(0, \infty)$. Thus if $-1 < \frac{(G(\phi^{**}) - \mu)(1 - \mu - B(G(\phi^{**}) - \mu))}{\phi^{**}G'(\phi^{**})(1 - \mu)} + 1$, which is equivalent to $G'(\phi^{**}) > \bar{\Psi}$, the steady state is locally stable, and if $\frac{(G(\phi^{**}) - \mu)(1 - \mu - B(G(\phi^{**}) - \mu))}{\phi^{**}G'(\phi^{**})(1 - \mu)} + 1 < -1$, which is equivalent to $G'(\phi^{**}) < \bar{\Psi}$, the steady state is locally unstable. ∎

We note from Proposition 3 and its proof that the stability of the steady state of ϕ^{**} depends on the distribution of Φ. In particular, a sufficiently small value of $G'(\phi^{**})$ ensures that the steady state of ϕ^{**} is unstable. In other words, if few entrepreneurs exist in the neighborhood of ϕ^{**} of the productivity distribution of Φ, the steady state of ϕ^{**} is unstable and the economy exhibits endogenous business cycles. If shocks that affect the parameter values of the distribution of Φ occur frequently and the configuration of the distribution is changed, the economy may frequently lose or regain stability around the steady state of ϕ^{**}.

If μ is arbitrarily close to zero, $\lim_{\mu \to 0} \phi^* = 0$, because $\lim_{\mu \to 0} G(\phi^*) = 0$. In this case, from Assumption 1, we do not have the steady state of ϕ^{**}, and thus the economy never exhibits endogenous business cycles. Alternately, if μ is arbitrarily close to one, it follows from the definition of ϕ^{**} that $\lim_{\mu \to 1} \phi^{**} = h$,

because $\lim_{\mu \to 1} G(\phi^{**}) = 1$. In this case, assuming that $\lim_{\mu \to 1} G'(\phi^{**}) \partial \phi^{**}/\partial \mu$ is bounded above, we are able to demonstrate, using L'Hospital's rule, that the coefficient of $(\phi_{t-1} - \phi^{**})$ in equation (24) is arbitrarily close to one, as μ is arbitrarily close to one. Again, in this case, the economy never exhibits endogenous business cycles. Therefore, the economy can exhibit endogenous business cycles when the extent of financial market imperfections is intermediate. This consequence is consistent with the existing literature [e.g., Aghion et al. (2004); Kunieda and Shibata (2011)].

3.5. Global Dynamics: Phase Diagram Analysis

From Propositions 2 and 3, we find that there are various patterns in the dynamic behavior of the economy, depending on the configurations of the functions of $\Lambda(\phi)$ and $\Psi(\phi)$ and particularly on the configuration of the distribution of Φ forming the functions of $\Lambda(\phi)$ and $\Psi(\phi)$. However, it is impossible to comprehensively analyze all of these patterns. In this section, we investigate two typical cases of the dynamic behavior of the economy, using phase diagrams.

Equation (19) can be rewritten as

$$G(\phi_t) = \frac{(1-\mu)\phi_{t-1}(G(\phi_{t-1}) - \mu)}{B[F(\phi_{t-1}) + \phi_{t-1}(G(\phi_{t-1}) - \mu)]} + \mu =: \Upsilon(\phi_{t-1}), \qquad (25)$$

or equivalently,

$$\phi_t = G^{-1}\left(\Upsilon(\phi_{t-1})\right).$$

We first consider the features of the function of $G^{-1}\left(\Upsilon(\phi_{t-1})\right)$. We easily obtain $G^{-1}\left(\Upsilon(0)\right) = G^{-1}(\mu)$ and $G^{-1}\left(\Upsilon(\phi^*)\right) = G^{-1}(\mu)$. Differentiating $G^{-1}\left(\Upsilon(\phi_{t-1})\right)$ yields

$$[G^{-1}\left(\Upsilon(\phi_{t-1})\right)]' = G^{-1'}\left(\Upsilon(\phi_{t-1})\right)\Upsilon'(\phi_{t-1}),$$

where $\Upsilon'(\phi) = (1-\mu)[(G(\phi) - \mu)(F(\phi) + \phi^2 G'(\phi)) + \phi G'(\phi) F(\phi)]/[B(F(\phi) + \phi(G(\phi) - \mu))^2]$. It is straightforward to show that

$$\lim_{\phi \to 0}[G^{-1}\left(\Upsilon(\phi)\right)]' < 0$$

and

$$\lim_{\phi \to \phi^*}[G^{-1}\left(\Upsilon(\phi)\right)]' > 0.$$

Therefore, from the continuity of $G^{-1}\left(\Upsilon(\phi)\right)$, there is a minimum value of $G^{-1}\left(\Upsilon(\phi)\right)$ in $(0, \phi^*)$. Let the value of ϕ that yields the minimum value be $\bar{\phi}$. Then $\bar{\phi}$ satisfies $[G^{-1}\left(\Upsilon(\bar{\phi})\right)]' = 0$, or equivalently $G(\bar{\phi}) - \mu = -\bar{\phi} G'(\bar{\phi}) F(\bar{\phi})/[F(\bar{\phi}) + \bar{\phi}^2 G'(\bar{\phi})]$. We define the minimum value of $G^{-1}\left(\Upsilon(\phi)\right)$ as $M := G^{-1}\left(\Upsilon(\bar{\phi})\right)$, which decreases as B increases. Note that $\lim_{B \to 0} \Upsilon(\bar{\phi}) = -\infty$. However, if $\Upsilon(\bar{\phi})$ becomes negative, we cannot obtain equilibrium. Therefore, we impose an assumption regarding $\Upsilon(\bar{\phi})$ as follows:

Assumption 2.

$$\Upsilon(\bar{\phi}) \geq 0.$$

Assumption 2 is equivalent to $M \geq 0$ and guarantees that any sequence of $\{\phi_t\}$ that is generated from the dynamical system (19) with an initial value of $\phi_0 \in [0, \phi^*]$ is an equilibrium path.[12]

Figure 1 provides two phase diagrams for two typical cases of the dynamic behavior of ϕ_t. In both cases, we assume that the initial value of ϕ_0 is close to ϕ^*. Panel A provides the case in which the steady state of ϕ^{**} is stable. Because ϕ^{**} yields the highest growth rate, as ϕ_t decreases from ϕ_0, the growth rate increases and the economy experiences a boom. Eventually, the economy converges to the steady state that yields the highest growth rate, because the steady state of ϕ^{**} is stable. Panel B provides the case in which the steady state of ϕ^{**} is unstable. As in the case of Panel A, the growth rate increases and the economy experiences a boom as ϕ_t decreases from ϕ_0. However, because the steady state of ϕ^{**} is unstable, ϕ_t does not converge to the steady state of ϕ^{**}. Accordingly, the economy fluctuates forever, and it may even exhibit complex dynamic behavior, depending on the configuration of $G^{-1}(\Upsilon(\phi_{t-1}))$.

A typical example is the case in which Φ follows a uniform distribution in $[0, a]$ ($a > 0$). In this case, $\phi_t = G^{-1}(\Upsilon(\phi_{t-1}))$ is reduced to

$$\frac{\phi_t}{a} = \frac{2(1-\mu)\left[\left(\frac{\phi_{t-1}}{a}\right)^2 - \mu\left(\frac{\phi_{t-1}}{a}\right)\right]}{B\left[\left(\frac{\phi_{t-1}}{a}\right)^2 - 2\mu\left(\frac{\phi_{t-1}}{a}\right) + 1\right]} + \mu.$$

Note from this equation that the dynamic behavior of ϕ_t is independent of the value of a. The case in which $\beta_c = 0.99$ and $\beta_e = 0.20$ with $\mu = 0.75$ yields endogenous business cycles. In contrast, in the case in which $\beta_c = 0.99$ and $\beta_e = 0.70$ with $\mu = 0.75$, the economy converges to the stable steady state of ϕ^{**}. For each case, Figure 2 provides time series of the cutoff ϕ_t for numerical examples. As shown in these numerical examples, if we employ a uniform distribution with regard to Φ, entrepreneurs must be extremely impatient to create endogenous business cycles. However, note that the dynamic behavior derived from the current model crucially depends on the shape of the distribution of Φ. As mentioned earlier, a sufficiently small value of $G'(\phi^{**})$ ensures that the steady state of ϕ^{**} is unstable. In particular, if Φ follows a binary distribution (although this departs from the assumption that $G(\Phi)$ is continuous and differentiable) or a continuous distribution close to a binary distribution, a very small difference in subjective discount factors can generate endogenous business cycles. We discuss this point in Section 5.

We have focused exclusively on two typical cases in this section. Even if ϕ^{**} is a stable steady state, the economy could experience endogenous business cycles, depending on the configuration of $G^{-1}(\Upsilon(\phi))$ and the initial value of ϕ_0. It is straightforward to graphically investigate the various patterns of the dynamic behavior of the economy.
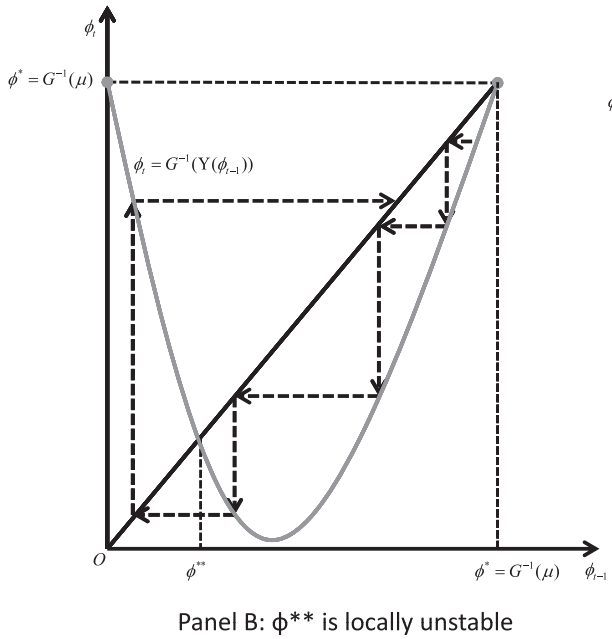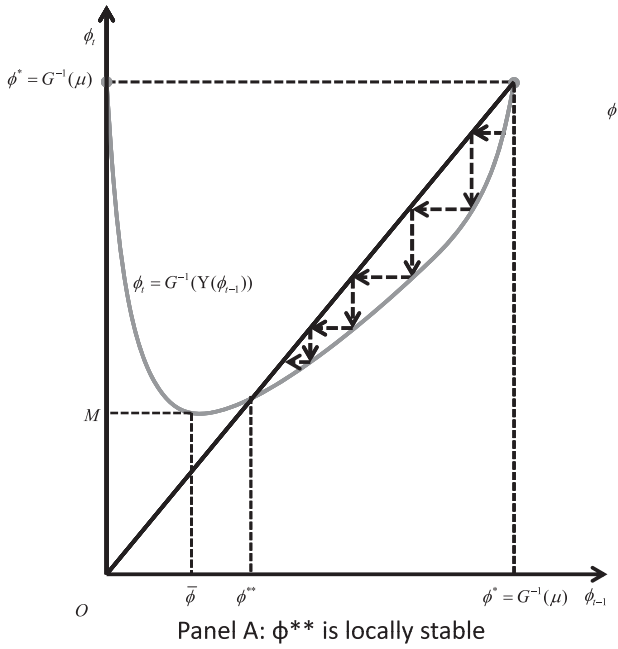
Panel A: φ** is locally stable

Panel B: φ** is locally unstable

FIGURE 1. Dynamic behavior of ϕ_t.

$\beta_c=0.99, \beta_e=0.20, \mu=0.75, \phi_0=0.10$

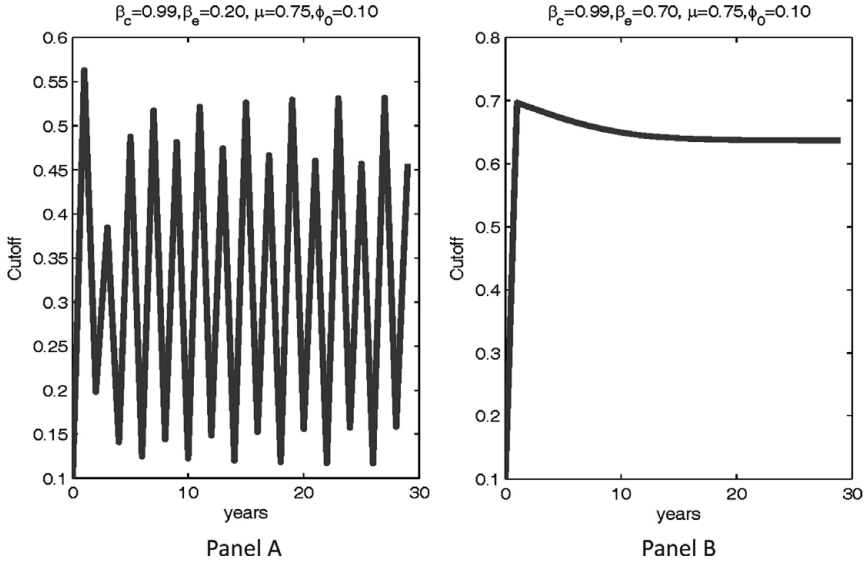$\beta_c=0.99, \beta_e=0.70, \mu=0.75, \phi_0=0.10$

Panel A

Panel B

FIGURE 2. Time series of the cutoff ϕ_t.

4. FINANCIAL CRISIS

In this section, we discuss a severe depression preceded by a contraction in the financiers' net worth, that is, a financial crisis, using the current model. Note that we do not aim to derive precise parameter conditions under which a financial crisis will occur with certainty. Instead, we discuss the potential for our model to generate a financial crisis. As we noted in the preceding subsection, there are various dynamic patterns in the economy in equilibrium, depending on the configuration of $G^{-1}(\Upsilon(\phi))$. On one hand, the economy could experience persistent business cycles of very small amplitude. On the other hand, it could exhibit significant economic swings. In this section, we study the latter case, using a phase diagram.

Let us suppose that M is very close to 0 under Assumption 2. Figure 3 illustrates an equilibrium path of the cutoff that leads to a financial crisis. As observed in Figure 3, the cutoff is assumed to begin with ϕ_{t_0}, which is greater than ϕ^{**}. As illustrated in Figure 4, the growth rate gradually increases and the economy experiences a boom. However, if ϕ_{t_3} is close to $\bar{\phi}$ at time t_3, the cutoff ϕ_{t_4} (equivalently, the interest rate) sharply declines to a very low value at time t_4, which is very close to zero. In the subsequent period, time t_5, ϕ_{t_5} suddenly increases and becomes very close to ϕ^*. Because ϕ^* deviates substantially from ϕ^{**}, which yields the highest growth rate, the growth rate suddenly decreases. In this case, the economy enters a severe depression because a significant amount of time is required for the cutoff to begin to increase steadily.

While a boom is under way, the cutoff ϕ_t (or the interest rate) continues to decrease, as observed in Figure 4. This decrease occurs because the financial
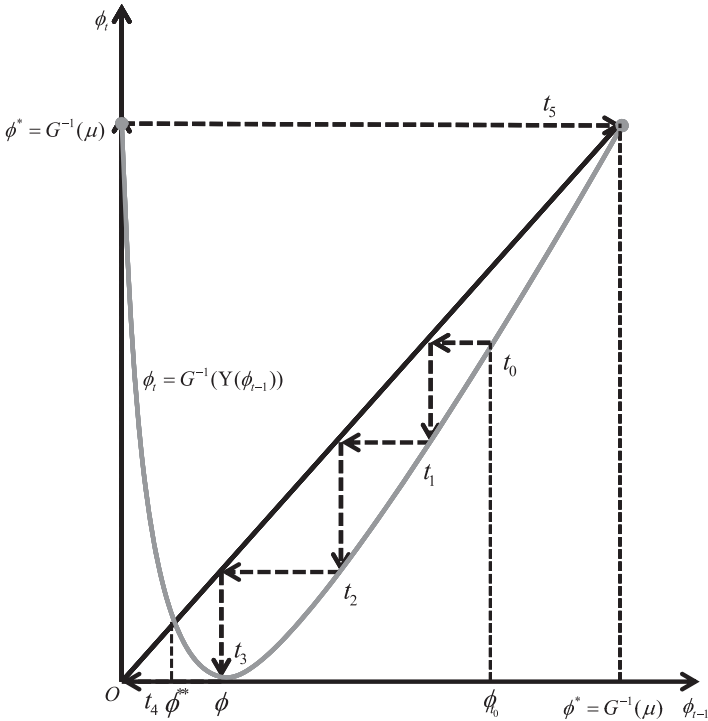
FIGURE 3. Financial crisis.

resources supplied by financiers continue to increase in the financial market throughout the economic boom. When the cutoff is relatively high, the equilibrium interest rate is also high and the financiers' net worth propagates following equation (21). The financiers' net worth grows until time t_3, provided that parameter A is sufficiently large. The increase in the supply of financial resources exerts downward pressure on the equilibrium interest rate. While interest rates continue to decrease during the boom, unproductive entrepreneurs engage in unproductive projects. Because the most productive entrepreneurs face financial constraints, increases in investment, even in unproductive projects, boost growth rates during the boom. At the end of the boom, a further increase in the supply of financial resources by the financiers causes a steep decline in the interest rate in the financial market.[13] The net interest rate becomes negative at the end of the boom when the cutoff is very close to zero at time t_4. Because of the negative net interest rate, the financiers' net worth suddenly declines to a very low value. Thus, the supply of financial resources is significantly reduced in the next period and the equilibrium interest rate rises substantially. Credit contraction occurs in the financial market, and the unproductive investment projects, which contribute to high growth during the boom, are no longer undertaken. Accordingly, the economy enters a depression. Note that in our model economy, if $\mu = 0$, that is, if there is no financial market in
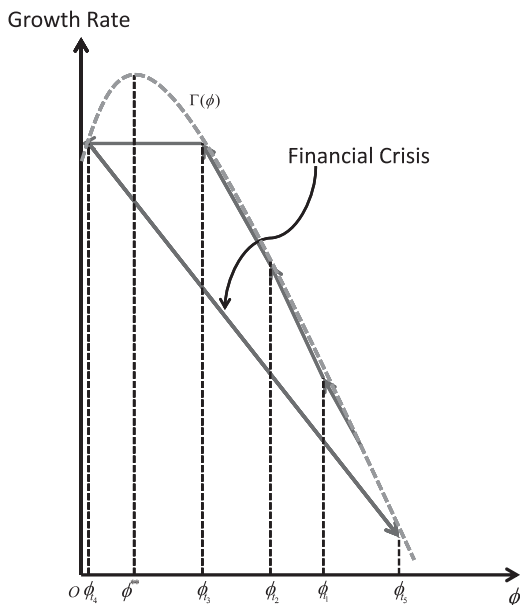
FIGURE 4. Example of financial crisis and growth rates.

the economy, financial crises never occur. In reality, we do not observe financial crises in economies without financial sectors or with the least developed financial sectors. The outcome of our model is consistent with this observation.

History has witnessed a large number of financial crises. Before a financial crisis occurs, a lending boom is observed, and financiers' net worth grows during a lending boom. At some point, however, their net worth suddenly declines and a severe depression follows from the sudden decline in net worth. Our model is consistent with this typical characteristic of a financial crisis. As a result of modern monetary and financial systems, a financier class has existed since the early seventeenth century. We investigated the macroeconomic implications of the coexistence of entrepreneurs and financiers. In Section 3.3, we demonstrated that the coexistence of entrepreneurs and financiers is likely to lead the economy to the highest possible growth rate, given a certain extent of financial market imperfections. This section, however, has clarified that the coexistence of financiers and entrepreneurs would cause a severe depression given certain parameter values. These two-sided implications of the coexistence of entrepreneurs and financiers explain why both instability and high growth are frequently observed in modern economies.

Fernald and Matoba (2009) demonstrate that the growth rate of utilization-adjusted TFP in the United States was negative until the mid-2000s, just before the financial crisis of 2007–2009, but TFP growth became positive in approximately 2007, with TFP increasing during the financial crisis. See Figure 2 in Fernald and

Matoba (2009). Whereas the standard real business cycle theory cannot explain Fernald and Matoba's evidence indicating countercyclical movements in TFP growth, Petrosky-Nadeau (2013) develops a model to explain their evidence. Similar to Petrosky-Nadeau's model, our model generates countercyclical movements in TFP growth. An aggregate production function is derived from our model. From equation (16), the aggregate investment K_t in the economy at time t can be computed as

$$K_t = \frac{1 - G(\phi_t)}{1 - \mu} \beta_e (Y_t - r_t W_{t-1}). \tag{26}$$

Equations (18) and (26) yield an aggregate production function as follows:

$$Y_{t+1} = \frac{A F(\phi_t)}{1 - G(\phi_t)} K_t. \tag{27}$$

The coefficient of K_t in equation (27) is the productivity of the aggregate production function and is an increasing function of ϕ_t. The economy therefore experiences diminishing productivity when a boom occurs (i.e., from time t_0 to time t_3), as a boom attracts investments by unproductive entrepreneurs, and productivity is minimized just before the economy enters a depression (i.e., from time t_3 to time t_4). When the economy enters a depression (i.e., from time t_4 to time t_5), it exhibits productivity growth, because unproductive entrepreneurs are compelled to exit production activities. This outcome regarding the productivity of the aggregate production function is consistent with Fernald and Matoba's (2009) finding. The outcome indicating that recessions have a positive effect on productivity is the "cleansing effect of recessions," which is formalized in the modern growth model developed by Caballero and Hammour (1994) and conceptually developed by Schumpeter (1939).[14]

In the current model, there is a tension between economic growth and TFP. The TFP in the current model is highest when financiers are absent. However, financiers cause the economy to achieve higher economic growth. This outcome is similar to that derived by Kaas (2014) in which low interest rates create lower TFP but lead to greater capital accumulation.

5. PATIENT ENTREPRENEURS AND ENDOGENOUS BUSINESS CYCLES

The numerical examples with a uniform distribution of idiosyncratic productivity shocks in Section 3.5 provide endogenous business cycles only when entrepreneurs are extremely impatient. However, the dynamic behavior derived from the current model depends on the shape of the distribution of Φ. In particular, when $G'(\phi^{**})$ is sufficiently small, endogenous business cycles occur as previously discussed. In this section, we provide an example in which a higher value of β_e, which is very close to β_c, can generate endogenous business cycles. To illustrate this situation, we depart from the assumption that $G(\Phi)$ is continuous, differentiable, and strictly increasing in the support and assume that Φ follows a binary distribution in which

$\Phi = a_s$ with probability α and $\Phi = a_l$ with probability $1 - \alpha$, where $0 < a_s < a_l$ and $0 < \alpha < 1$. Note that equation (25) can be derived despite our use of a binary distribution. In this case, equation (25) eventually yields a dynamical system with respect to $G(\phi_t)$, which is the population of lenders. Therefore, we define a new notation as $G_t := G(\phi_t)$.[15] Under the binary distribution, if $\alpha < G_t \leq 1$ at time t, all less productive entrepreneurs and a share of the highly productive entrepreneurs become lenders. Highly productive entrepreneurs are indifferent to whether they become lenders or producers, and hence, it follows that $\phi_t = a_l$. Because the population of highly productive entrepreneurs who become producers is equal to $1 - G_t$, $F(\phi_t)$ is computed as $F(\phi_t) = a_l(1 - G_t)$. In contrast, if $0 < G_t < \alpha$ at time t, a share of the less productive entrepreneurs and all highly productive entrepreneurs become producers. Less productive entrepreneurs are indifferent to whether they become lenders or producers, and hence, it follows that $\phi_t = a_s$. Because the population of less productive entrepreneurs who become producers is $\alpha - G_t$ and that of highly productive entrepreneurs is $1 - \alpha$, $F(\phi_t)$ is computed as $F(\phi_t) = a_s \times (\alpha - G_t) + a_l \times (1 - \alpha)$. If $G_t = \alpha$, ϕ_t takes a certain value in $[a_s, a_l]$ and $F(\phi_t) = a_l \times (1 - \alpha)$. Moreover, if $G_t = 0$, ϕ_t takes a certain value in $[0, a_s]$ and $F(\phi_t) = a_s \times \alpha + a_l \times (1 - \alpha)$, because all entrepreneurs become producers.

From equation (20), we obtain the economic growth rate at time t as follows:

$$\Gamma_t = \begin{cases} \frac{\beta_c AB[a_s\alpha + a_l(1-\alpha)]}{1-(1-B)\mu} & \text{if } G_{t-1} = 0 \\ \frac{\beta_c AB[a_s(\alpha - G_{t-1}) + a_l(1-\alpha)]}{1-\mu-B(G_{t-1}-\mu)} & \text{if } 0 < G_{t-1} < \alpha \\ \frac{\beta_c ABa_l(1-\alpha)}{1-\mu-B(\alpha-\mu)} & \text{if } G_{t-1} = \alpha \\ \frac{\beta_c ABa_l(1-G_{t-1})}{1-\mu-B(G_{t-1}-\mu)} & \text{if } \alpha < G_{t-1} \leq 1. \end{cases} \tag{28}$$

Note that Γ_t is continuous at $G_{t-1} = \alpha$ and maximized at this point. In other words, the growth rate is maximized when all less productive entrepreneurs become lenders and all highly productive entrepreneurs become producers. To focus on illustrative cases in which endogenous business cycles occur in equilibrium, we impose a parameter assumption as follows:

Assumption 3. $\mu > \alpha$, $1 - \alpha/\mu \leq B < 1$, and $a^l/a^s > 1 + (1 - \mu)\alpha/[(\mu - \alpha)(1 - \alpha)]$.

Under Assumption 3, equation (25) becomes

$$G_t = \begin{cases} \left[\frac{-\mu(1-\mu)}{B[1-\mu+(1-\alpha)(e-1)]}, \mu\right] & \text{if } G_{t-1} = 0 \\ \frac{(1-\mu)(G_{t-1}-\mu)}{B[1-\mu+(1-\alpha)(e-1)]} + \mu =: \Upsilon_1(G_{t-1}) & \text{if } 0 < G_{t-1} < \alpha \\ \left[\frac{\alpha-\mu}{B} + \mu, \frac{(1-\mu)(\alpha-\mu)}{B[1-\mu+(1-\alpha)(e-1)]} + \mu\right] & \text{if } G_{t-1} = \alpha \\ \frac{G_{t-1}-\mu}{B} + \mu =: \Upsilon_2(G_{t-1}) & \text{if } \alpha < G_{t-1} \leq 1, \end{cases} \tag{29}$$
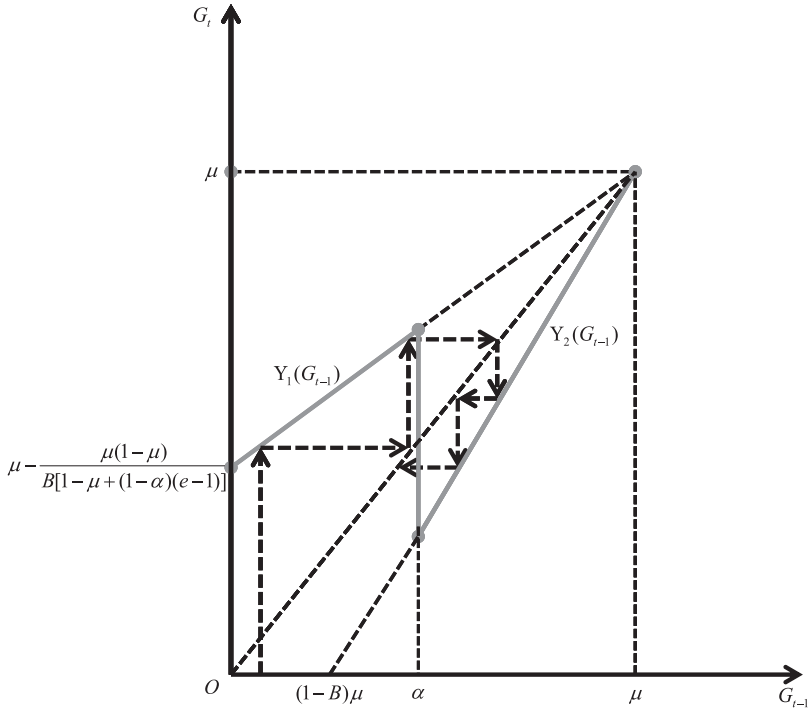
where $e := a_l/a_s$.

FIGURE 5. Endogenous business cycles with a binary distribution.

Equation (29) provides a simple piecewise linear dynamical system with respect to G_t. As in Section 3.2, we restrict the domain of the dynamical system (29) to $[0, \mu]$ to focus on the case in which $W_t \geq 0$. Under Assumption 3, $(1-\mu)/[B[1-\mu+(1-\alpha)(e-1)]]$, which is a slope of $\Upsilon_1(G_{t-1})$, is less than one and $1/B$, which is a slope of $\Upsilon_2(G_{t-1})$, is greater than one. Additionally, the intersection of $\Upsilon_2(G_{t-1})$ and the horizontal axis, which is $((1-B)\mu, 0)$, lies between the origin and $(\alpha, 0)$, and the intersection of $\Upsilon_1(G_{t-1})$ and the vertical axis, which is $(0, \mu - \mu(1-\mu)/[B(1-\mu+(1-\alpha)(e-1))])$, lies between the origin and $(0, \mu)$. Therefore, the phase diagram of the dynamical system (29) is drawn as in Figure 5. As seen in Figure 5, we can obtain endogenous business cycles under Assumption 3.[16]

Now, we consider two cases in Figure 6, in which the dynamic characteristics contrast with each other. First, suppose that $B = \beta_e/\beta_c$ is close to one. As B approaches one, both $\Upsilon_1(G_{t-1})$ and $\Upsilon_2(G_{t-1})$ rotate clockwise around (μ, μ) and $\Upsilon_2(G_{t-1})$ converges to the 45° line. We note that as long as $B = \beta_e/\beta_c$ is less than one, the economy exhibits endogenous business cycles, as depicted in Panel A of Figure 6. This outcome indicates that although entrepreneurs are
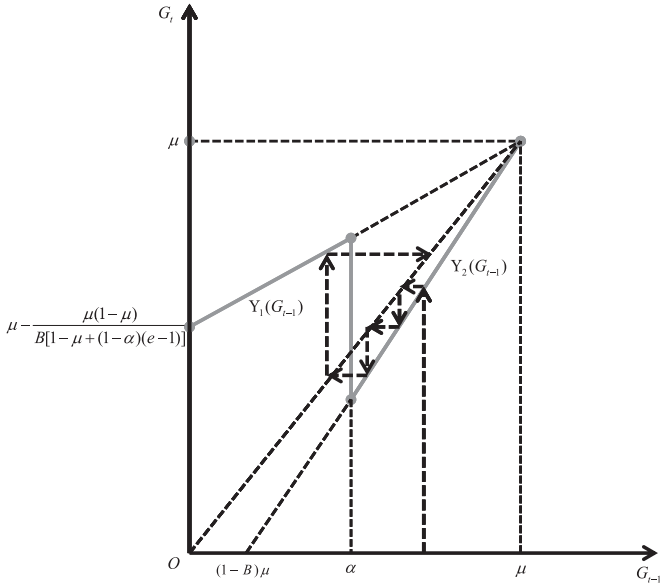
Panel A: Slow Economic Boom and Increasing Highly Productive Producers
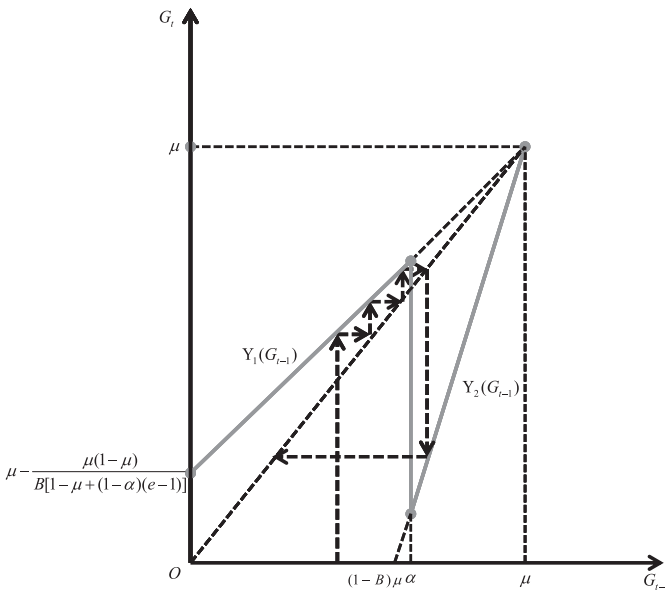
Panel B: Slow Economic Boom and Decreasing Less Productive Producers

FIGURE 6. Long-lasting booms.

nearly as patient as financiers, endogenous business cycles are obtained from the current model with a binary productivity distribution. In this case, a boom is very long-lasting because the economic growth rate is maximized at $G_t = \alpha$ and the economy experiences a sudden crash. Next, suppose that B is close to $1 - \alpha/\mu$ and $e = a_l/a_s$ is close to $1 + (1 - \mu)\alpha/[(\mu - \alpha)(1 - \alpha)]$. In this case, $\Upsilon_1(G_{t-1})$ is close to the 45° line and $(1 - B)\mu$ is close to α on the horizontal line in Panel B of Figure 6. In this case, a boom is very long-lasting and the economy experiences a sudden crash, as in the case of Panel A. The difference in dynamic behaviors between Panel A and Panel B is that in Panel A, a boom occurs as the population of highly productive producers increases, whereas in Panel B, it occurs as the population of less productive producers decreases.

A binary distribution has, as its name implies, two spikes in its probability density. Because of these two spikes, the mechanisms by which a boom occurs differ between an economy with a binary productivity distribution and an economy with a single-peaked continuous distribution. In both Panel A and Panel B, during the boom, the interest rate is constant, contrasting with a boom in an economy with a single-peaked continuous productivity distribution. During a boom in Panel A, the population of highly productive producers increases. In this case, the interest rate Aa_l is so high that the supply of financial resources continues to increase. At the end of the boom, the supply of financial resources becomes so large that the interest rate suddenly decreases to Aa_s and the financial resources available in the following period suddenly decrease. As such, the growth rate suddenly decreases and the economy enters a recession. This mechanism underlying a boom in Panel A is relatively similar to that in an economy with a single-peaked continuous productivity distribution, as studied in the preceding section. However, in the case of Panel B, a boom occurs at the low interest rate Aa_s, where the population of less productive producers continues to decrease. This boom occurs because the aggregate productivity in the economy continues to increase, because less productive entrepreneurs are excluded from production activities. The supply of financial resources continues to decline because of the low interest rate during the boom. At the end of the boom, the supply of financial resources is so small that the interest rate suddenly increases from Aa_s to Aa_l. However, the sudden increase in the interest rate induces a large increase in the financial recourses available in the following period. Therefore, the interest rate decreases and the population of less productive producers increases in one period. Then another economic boom begins. The mechanism through which a boom occurs in Panel B is entirely different from both that in Panel A and that in an economy with a single-peaked continuous productivity distribution.

Although we employ a binary distribution with support $\{a_s, a_l\}$ in this section, we can conjecture that a continuous productivity distribution in which a large population mass concentrates at productivity a_s and another large population mass concentrates at productivity a_l can yield results similar to those derived from an economy in which a binary distribution is employed.

6. DISCUSSION

The current paper is related to the literature on the financial accelerator and boom–bust cycles. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) demonstrate a mechanism termed the financial accelerator; that is, they show that collateral constraints amplify the effects of productivity shocks on macroeconomic activities. There are two approaches in this literature, depending on the modeling strategy employed. The first is the overlapping-generations modeling approach. Recent examples adopting this approach are Matusyama (2007, 2013), Kikuchi (2008), Orgiazzi (2008), Kikuchi and Stachurski (2009), Gokan (2011), Kunieda and Shibata (2011, 2014), Favara (2012), and Myerson (2012, 2014). All these studies derive endogenous business cycles in economies with financial frictions.[17] Our model belongs to the second approach, which employs infinitely lived agent models, examples of which are Cordoba and Ripoll (2004a), Pintus (2011), Gu et al. (2013), Pintus and Wen (2013), and Tomura (2013). By applying a similar setting to Kiyotaki and Moore's model, Cordoba and Ripoll (2004a) investigate how collateral constraints magnify and propagate small monetary shocks into aggregate output.[18] Pintus and Wen (2013) investigate an economy with the standard neoclassical production technology in which the borrowers' utility is affected by consumption habits. Tomura (2013) studies housing market boom–bust cycles in a credit-constrained economy with heterogeneous beliefs. Although Cordoba and Ripoll (2004a), Pintus and Wen (2013), and Tomura (2013) identify the propagation mechanism of exogenous shocks that creates boom–bust cycles, unlike the current study, they do not obtain endogenous business cycles.

The models of credit developed by Gu et al. (2013) produce various dynamic patterns in which debt limits exhibit deterministic cycles and chaos, as well as self-fulfilling stochastic equilibria driven by agents' beliefs. Pintus (2011) also derives endogenous business cycles caused by self-fulfilling expectations. He studies a financially constrained small open economy in which human and physical capital are incorporated into the production function but borrowing constraints are associated only with physical capital. Pintus shows that short-run boom–bust cycles and sunspot fluctuations occur if the credit multiplier is in the intermediate-to-high range.[19] The current study is along the same lines as Pintus (2011) in that we investigate the financial accelerator and endogenous business cycles, but in contrast to his study, we investigate a closed economy and derive *deterministic* endogenous business cycles.

Endogenous business cycles in economies *without* financial frictions have been studied for over twenty years by many researchers employing both overlapping-generations models and infinitely lived agent models.[20] In the extensive literature addressing these issues, it is widely known that deterministic endogenous business cycles are more likely to occur in overlapping-generations models than in infinitely lived agent models. Therefore, existing studies that derive deterministic endogenous business cycles with infinitely lived agents are far fewer than such studies with overlapping generations. The same holds for existing studies *with* financial

frictions. We derive endogenous business cycles by applying an infinitely lived agent model; this point is one of the contributions of the current paper.

7. CONCLUDING REMARKS

Over the past twenty years, many countries have suffered from financial crises followed by severe economic depressions. However, the reasons that such severe crises have repeatedly occurred in modern economies remain unclear. Is capitalism inherently unstable? Our dynamic general equilibrium model provides a possible answer to this question, generating endogenous business cycles and a financial crisis.

Our model has demonstrated that in a financially constrained economy, the coexistence of entrepreneurs and financiers has twofold importance. On one hand, economic growth is accelerated and the highest growth rate is only achievable when financiers coexist with entrepreneurs. On the other hand, because of the coexistence of financiers and entrepreneurs, a severe depression preceded by a contraction in the financiers' net worth, namely a financial crisis, is highly likely to occur given certain parameter values. If a financial market becomes perfect, no financial crises occur in our model. Therefore, it is important to consider a policy to establish a financial market that is close to perfection. However, it seems very difficult to enact a complete policy to obtain a perfect financial market because of potential agency problems remaining in a financial market. As such, it is also important to consider a policy to avoid financial crises given a certain extent of financial market imperfections. This topic is left for future research.

NOTES

1. Agency problems that generate financial market imperfections have long been investigated by researchers, such as Townsend (1979) and Williamson (1986, 1987). Many macroeconomists, such as Khan (2001), Matsuyama (2004, 2007), Aghion et al. (2005), Ho and Wang (2005, 2012), and Azariadis and Kaas (2008), study economic growth and development while accounting for financial market imperfections. Galor and Zeira (1993) and Kunieda et al. (2014) investigate the relationship between inequality and financial market imperfections.

2. Research on the macroeconomic implications of business cycles in an economy with two distinct classes dates back to Kalecki (1937).

3. See also Woodford (1988).

4. Because Takalo and Toivanen's (2012) model is static in nature, it cannot be applied to business cycle problems. Azariadis et al. (2015) also develop a dynamic general equilibrium model in which unproductive entrepreneurs lend capital to productive entrepreneurs at equilibrium, as in Takalo and Toivanen's model. Azariadis et al. derive endogenous fluctuations induced by extrinsic uncertainty with respect to credit market conditions, whereas we derive deterministic endogenous business cycles without any extrinsic uncertainty.

5. In this sense, financiers in our model can be regarded as *rentiers* à la Robinson (1956).

6. Typical examples of boom–bust cycles in our model are asymmetric in that a slow expansionary process of output leads to a sudden crash. As in our model, a savings glut plays an important role in the creation of asymmetric boom–bust cycles in the models of Matsuyama and (2013) and Boissay et al. (in press). However, in contrast to our model, their models do not assume the presence of financiers.

7. In equation (1), it is unnecessary to assume full depreciation of capital if one assumes that $A\Phi_{\tau-1}(\omega_{\tau-1})k_{\tau-1}(\omega^{\tau-1})$ includes both output and the nondepreciated stock of capital.

8. Many researchers employ this type of credit constraint. See also Antràs and Caballero (2010). Readers are referred to the appendices in Aghion et al. (2005) and Kunieda and Shibata (2012) regarding the microfoundations of the credit constraint.

9. One might argue that we should call these agents "savers." In many macroeconomic models, however, savers often engage in production, for instance as workers.

10. For $G(\phi_0)$ to be well defined, it must follow that $\mu - (1-\mu)W_0/(\beta_e w_0) > 0$. We assume this parameter condition.

11. Otherwise, Y_{t+1} becomes negative in equation (18).

12. Assumption 2 is need to obtain an economically meaningful equilibrium. More precisely, if Assumption 2 does not hold, for almost every initial value of the cutoff ϕ_0, the sequence $\{\phi_t\}$ cannot be an equilibrium because $G(\phi_t)$ becomes negative in finite time. The remaining subset of initial values of the cutoff ϕ_0 that create equilibrium sequences form a Cantor set, the Lebesgue measure of which is equal to zero. To the best of our knowledge, in the economics literature, only Boldrin et al. (2001) address equilibria on a Cantor set. The investigation of such equilibria is beyond the scope of this paper.

13. One might argue that there may be another pressure that reduces the equilibrium interest rate. The burden of repayment facing producing entrepreneurs becomes increasingly heavy as the financiers' net worth evolves during the boom. We find from equations (19)–(21) and the function $J(\phi_t)$ that when $\phi_t > \phi^{**}$, the growth rate of W_t is higher than the growth rate of Y_t. This finding implies that the total net worth held by entrepreneurs is likely to decline, provided that ϕ_t is larger than ϕ^{**}. As a result, the demand for borrowing would decrease because of the financial constraints associated with the entrepreneurs' net worth. Although this decreased demand for borrowing may also exert downward pressure on the equilibrium interest rate, the effect of the decreased demand for borrowing on a steep decline in the interest rate is limited because ϕ_t is close to ϕ^{**} at the end of the boom.

14. There has been a debate over the relationship between business cycles and productivity growth. Saint-Paul (1993), Malley and Muscattelli (1999), and Basu et al. (2006) provide evidence indicating that recessions have a positive effect on productivity, whereas Field (2010) finds evidence of a negative effect. Interested readers are referred to a survey of this issue by Aghion and Saint-Paul (1998).

15. This notation is convenient because even if, for instance, $\phi_t = a_s$, $G(\phi_t = 1)$ can take various values with a binary distribution.

16. Azariadis et al. (2015) calibrate $a_l = 1.08$ and $a_s = 0.779$ using U.S. data. In the business cycle literature, the subjective discount factor of consumers is often calibrated as 0.99. Azariadis et al. (2015) calibrate the subjective discount factor of producers as 0.89, targeting the capital–output ratio in the U.S. economy. Therefore, we assume that $\beta_c = 0.99$ and $\beta_e = 0.89$ in Assumption 3. Under these parameter settings, when $\mu = 0.85$ and $\alpha = 0.5$, the parameter conditions in Assumption 3 are satisfied.

17. Early influential papers including Suarez and Sussman (1997) and Azariadis and Smith (1998) carefully study the microfoundations of credit market imperfections, explicitly taking into consideration agency problems in the financial market. Subsequent studies cited here, except Favara (2012) and Myerson (2012, 2014), focus more on the macroeconomic effects of credit constraints, taking the microfoundations of credit market imperfections as given. Favara (2012) explicitly models agency problems in the financial market in an overlapping-generations economy. Myerson (2012, 2014) constructs a dynamic moral hazard model of an overlapping-generations economy consisting of financial agents and entrepreneurs.

18. Utilizing stylized models that incorporate the primary mechanism employed in the Kiyotaki and Moore (1997) model, Cordoba and Ripoll (2004b) demonstrate that, under orthodox parameter assumptions regarding preferences and technologies, collateral constraints alone may not sufficiently amplify productivity shocks to explain actual large business fluctuations. Aoki et al. (2004) and Iacoviello (2005) show that housing plays a critical role in generating financial accelerator effects on monetary shocks when the borrowing limit depends on the value of housing.

19. As shown in Pintus (2011), endogenous business cycles are also induced through self-fulfilling rational expectations as sunspot equilibria. For sunspot equilibria in growth models of infinitely lived agents, see Woodford (1986), Benhabib and Farmer (1994, 1996), Boldrin and Rustichini (1994), and Gali (1994), among others.

20. Examples that employ an overlapping-generations model are Benhabib and Day (1982), Grandmont (1985), Farmer (1986), Reichlin (1986), Benhabib and Laroque (1988), Kitagawa and Shibata (2001), Rochon and Polemarchakis (2006) and Nourry and Venditti (2012), all of which derive competitive equilibrium without financial frictions. Examples that employ an infinitely lived agent model include Benhabib and Nishimura (1985), Boldrin and Denecker (1990), and Nishimura and Yano (1995). These studies develop a model of an infinitely lived representative agent with two production sectors and derive deterministic endogenous business cycles. As in our study, Futagami and Mino (1993) develop a Ramsey-type one-sector growth model with nonconvexities arising from threshold production externalities and derive deterministic cycles.

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APPENDIX A: DERIVATION OF EQUATION (6)

We should note that when making a lending–investment–borrowing decision at time $t-1$, an entrepreneur has information about her productivity at time t, which is given by $\Phi_{t-1}(\omega_{t-1})$. From equations (4) and (5), the lending–investment–borrowing plan at time $t-1$ of an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) > \phi_{t-1} := r_t/A$ is such that $b_{t-1}(\omega^{t-1}) = -\mu k_{t-1}(\omega^{t-1})$ and $k_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})/(1-\mu)$. Therefore, her budget constraint at time t is given by

$$k_t(\omega^t) + b_t(\omega^t) = (A\Phi_{t-1}(\omega_{t-1}) - r_t\mu)k_{t-1}(\omega^{t-1}) - c_t(\omega^t),$$

or equivalently,

$$a_t(\omega^t) = \frac{A\Phi_{t-1}(\omega_{t-1}) - r_t\mu}{1-\mu} a_{t-1}(\omega^{t-1}) - c_t(\omega^t). \tag{A.1}$$

Similarly, from equations (4) and (5), the lending–investment–borrowing plan at time $t-1$ of an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) < \phi_{t-1} := r_t/A$ is such that $b_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})$ and $k_{t-1}(\omega^{t-1}) = 0$. Therefore, her budget constraint at time t is given by

$$k_t(\omega^t) + b_t(\omega^t) = r_t b_{t-1}(\omega^{t-1}) - c_t(\omega^t),$$

or equivalently,

$$a_t(\omega^t) = r_t a_{t-1}(\omega^{t-1}) - c_t(\omega^t). \tag{A.2}$$

From equations (A.1) and (A.2), the flow budget constraints for $\tau \geq t$ are given by equation (6).

APPENDIX B: PROOF OF LEMMA 1

From the flow budget constraint (6), we have

$$E\left[\frac{a_{t+1}(\omega^{t+1})}{c_{t+1}(\omega^{t+1})}\bigg| \Phi^t(\omega^t)\right] = a_t(\omega^t)E\left[\frac{\tilde{R}_{t+1}}{c_{t+1}(\omega^{t+1})}\bigg| \Phi^t(\omega^t)\right] - 1. \tag{B.1}$$

Substituting equation (7) into equation (B.1), we have

$$\frac{a_t(\omega^t)}{c_t(\omega^t)} = \beta_e E\left[\frac{a_{t+1}(\omega^{t+1})}{c_{t+1}(\omega^{t+1})}\middle|\Phi^t(\omega^t)\right] + \beta_e.$$

From this equation and the law of iterated expectations, we obtain

$$\frac{a_t(\omega^t)}{c_t(\omega^t)} = \beta_e^\tau E\left[\frac{a_{t+\tau}(\omega^{t+\tau})}{c_{t+\tau}(\omega^{t+\tau})}\middle|\Phi^t(\omega^t)\right] + \beta_e + \beta_e^2 + \ldots + \beta_e^\tau.$$

From the transversality condition, we have $\lim_{\tau\to\infty} \beta_e^\tau E[a_{t+\tau}(\omega^{t+\tau})/c_{t+\tau}(\omega^{t+\tau})|\Phi^t(\omega^t)] = 0$. Therefore, $a_t(\omega^t)/c_t(\omega^t) = \beta_e/(1-\beta_e)$ for all $t \geq 0$ and thus $a_{t+1}(\omega^{t+1}) = \beta_e \tilde{R}_{t+1} a_t(\omega^t)$ from equation (6). ∎