The Schuler Pendulum's Fatal Flaw

Frank Coffman Bell

SCHULER himself genially but clearly warned here and there in his 1923 paper that he had not proved his principle, and indicated that the general proof was not within the range of his own limitations, thus inviting his readers to be critical of what amounted to interesting suggestions cheerfully offered. Professor Stratton's commentary (*Journal*, 21, 507), 'The Schuler pendulum and inertial navigation', by its scope, clearness and authenticity, could make effectual the remark that Schuler's calculation of acceleration is not correct except in one and the same great circle and possibly a few other special cases. Schuler jumps to the conclusion that it holds for any spherical motion, thus neglecting much of kinematics. The following standard analysis uses the moving trihedral and the intrinsic equations of ordinary differential geometry. There are many other methods having their own advantages.

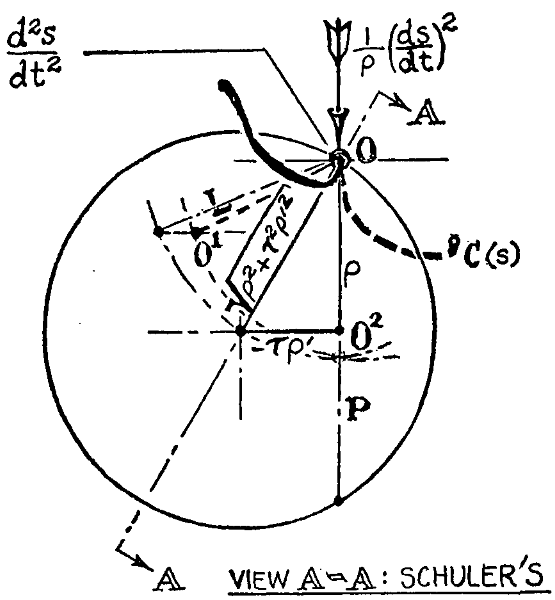

FIG. 1. The osculating sphere (shown in the current normal plane)

Let L be the presumably constant distance between the pendulum's point of restraint o and its centre of oscillation o^1. Let o, a moving point, follow any curve C or arc s and having the varying osculating plane P current at o. Then as has been well known for near a century, the current acceleration of o is a variable vector lying in P and passing through o. Its current components on the current tangent and current principal normal of C at o are, respectively:

$$\frac{d^2s}{dt^2}$$

and

$$\frac{1}{\rho}\left(\frac{ds}{dt}\right)^2$$

where ρ, a variable, is the radius of the current circle of curvature osculating C at o. Let the centre of this circle be the variable point o^2 in P. If the pendulum is to point the current principal normal to C at o, o^1 must coincide with o^2

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The Institute of Navigation

A VISIT TO THE ROYAL GREENWICH OBSERVATORY

By courtesy of the Astronomer Royal, it has been arranged for Members of the Institute to visit the Royal Greenwich Observatory at Herstmonceux Castle, near Hailsham, Sussex, during the afternoon of Saturday, 27 September 1969

Numbers have had to be limited to 100 and for that reason, initially at least, only Members will be eligible for tickets; should a relaxation of this rule be possible nearer the date, guests may be allowed.

The party will meet for a buffet lunch near the Observatory. There will be a bus at Eastbourne station for those who travel by rail.

Members who wish to attend should complete the form below and return it to the Executive Secretary before the end of July. There will be a charge of 15s. to cover the cost of lunch.

at the Royal Geographical Society
1 Kensington Gore
London SW7

By order of the Council
M. W. Richey
Executive Secretary

To the Executive Secretary
The Institute of Navigation
at the Royal Geographical Society
London SW7.

Kindly send me a ticket for the visit to the Royal Greenwich Observatory on Saturday, 27 September 1969, for which I enclose 15s. to cover the cost of lunch.

Should the opportunity arise I should like to bring a guest. (Tick if applicable.) ☐

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identically, $\rho \equiv L$ must hold, and the pendulum must remain always in the current osculating plane P of C at o. One curve which meets these requirements is a circle of radius L, and as far as I can see, this is the only such curve. Now it has also been well known for near a century (1) that a necessary and sufficient condition that a curve C lie upon a sphere is given by

$$\frac{\rho}{\tau} + (\tau\rho')' = 0$$

where $1/\tau$, a variable, is the torsion of C current at o; and (2) that for a curve C drawn on a sphere, o^2, the centre of curvature current at o, is the foot of the perpendicular let fall from the centre of the sphere upon the osculating plane P of the curve C at o. Clearly there are many motions of o on a sphere of radius R for which a pendulum having $L \equiv R$ does not point the 'vertical', among which may be mentioned as examples motions in small circles and motions in loxodrome curves. In fact, Schuler's pendulum in which $L \equiv R$ seems to be valid only in one and the same great circle as an 'artificial horizon'.

Although this last is the only case he actually considers, Schuler's paragraph 12 reads:

'Ein derartiges Pendel, wenn es einmal zur Ruhe gekommen ist, bleibt immer in seiner Gleichgewichtslage und zeigt die Lotlinie an, die an der betreffenden Stelle der Erdoberflaeche herrscht, gleichgueltig wie das Fahrzeug sich bewegt.'

The American translators laboriously render this as the first sentence, in Fisher's paragraph 15 (1956) and Slater's paragraph 12 (1960)—but there is no need to consult them. The German simply says what has to be said if the next 25 paragraphs are to appear at all, viz. that the Schuler pendulum is always and everywhere valid on Earth as an 'artificial horizon' no matter what the motion of o in the sphere of Earth—a thing much to be hoped for, indeed. Further hope seems to spring eternal that his paragraph 32 (Slater's paragraph 29) and his pendulous top may be saviours of Schuler and his principle from fatal flaw, but this also is quite vain, as the reader can easily prove by the methods here indicated. For a counter example, one has only to analyse the top horizon in carriage in a small circle on the sphere of Earth.