being affected by aspect, and by distance in so far as detail is concerned, it is also dependent upon the season, the weather, and the time of day. A coastline in winter, covered with snow, on a dull overcast day or with long shadows cast by a low sun, will be very different in everything save silhouette when seen on a bright summer's day. Such drawbacks have not, however, deterred publication of such views in the Pilots, even when there is little choice of quality.

As to the position from which each radar photograph is taken, this will almost settle itself. For pictures intended as an aid to landfall, the range should perhaps be that at which just sufficient land will paint on a normal set under normal conditions for identification to be positive. If super-refraction conditions exist when the landfall is being made, so much the better for identification; while sub-refraction of any material consequence is unlikely. The bearing of the position may be stated as well as the range—thus conforming to the practice established on so many of the existing views.

Accuracy Contour Maps of a Ship's Position

from N. Sameshima
(*Nautical Society of Japan*)

The writer has drawn the accuracy contour maps of a ship's position fixed by cross bearing, horizontal sextant angle method and Loran aid. The accuracy of the fixes has been evaluated in terms of the probability density.

In Fig. 1, P is the ship's position fixed from cross bearings of the objects A and B, and θ is the angle between two bearings. d_1 and d_2 are the distances of the objects. The displacement probable error in each of the lines of bearing at P is

$$r = d \sin \Delta Z \tag{1}$$

where ΔZ is the probable error in observed bearing.

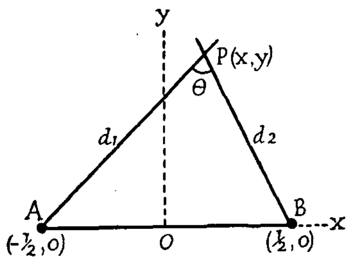

Fig. 1.

The probability density at P is given by

$$p = (1/2\pi\sigma_1\sigma_2) \sin \theta = (0 \cdot 0724/r_1 r_2) \sin \theta \tag{2}$$

where σ_1, σ_2 are the displacement standard errors in each line of bearing at P.

From equations (1) and (2)

$$p = (k/d_1 d_2) \sin \theta \tag{3}$$

where $k = 0 \cdot 0724/\sin^2 \Delta Z$.

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Consequently

$$p = ky/\{(x+\tfrac{1}{2})^2 + y^2\}\{(x-\tfrac{1}{2})^2 + y^2\} \tag{4}$$

Assuming $k = 1$, equation (4) will give the contour map shown in Fig. 2.

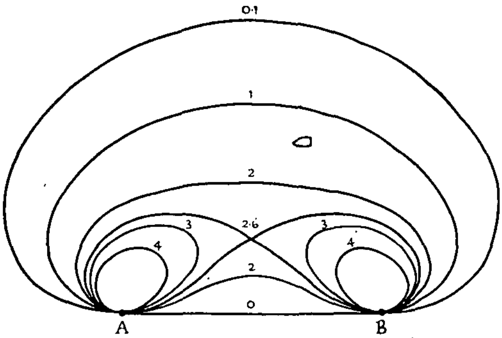

Fig. 2. The accuracy contour map for the cross bearing method (values of p).

In Fig. 3, ϕ_1 and ϕ_2 are the horizontal sextant angles between the objects A, B

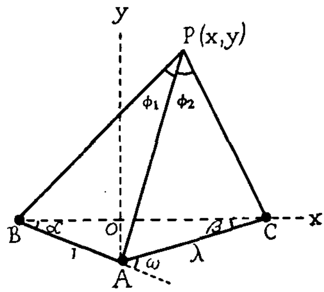

Fig. 3.

and A, C. The displacement probable errors at P due to the error in sextant angles are shown as follows[2]:

$$\left.\begin{aligned} r_1 &= \Delta\phi_1 \text{AP(BP)/AB} \\ r_2 &= \Delta\phi_2 \text{AP(CP)/AC} \end{aligned}\right\} \tag{5}$$

where $\Delta\phi_1$ and $\Delta\phi_2$ are the probable errors in sextant angles ϕ_1 and ϕ_2.

The cutting angle of two position circles at P is

$$\theta = 180^\circ - (\phi_1 + \phi_2 - \omega) \tag{6}$$

From equations (2), (5) and (6), the probability density at P is

$$k(\text{AB})\text{AC} \sin(\phi_1 + \phi_2 - \omega)/\text{AP}^2(\text{BP})\text{CP}$$

where $k = 0{\cdot}0724/\Delta\phi_1\Delta\phi_2$.

If AB $= 1$, AC/AB $= \lambda$, $a = \text{BC} = \sqrt{(1 + \lambda^2 + 2\lambda \cos\omega)}$ then

$$p = k\frac{\lambda\,|\,ay\cos\omega - \{x^2 + y^2 + (1-\lambda^2)x/a - \lambda\cos a\cos\beta\}\sin\omega\,|}{\{x^2 + (y + \sin\alpha)^2\}\{(x + \cos\alpha)^2 + y^2\}\{(x - \lambda\cos\beta)^2 + y^2\}} \tag{7}$$

Assuming $k = 1$ and $\lambda = 1$, equation (7) gives the contour maps shown in Figs. 4 and 5.

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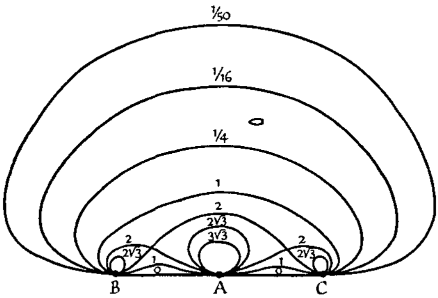

Fig. 4. When the angle between base lines is 180°

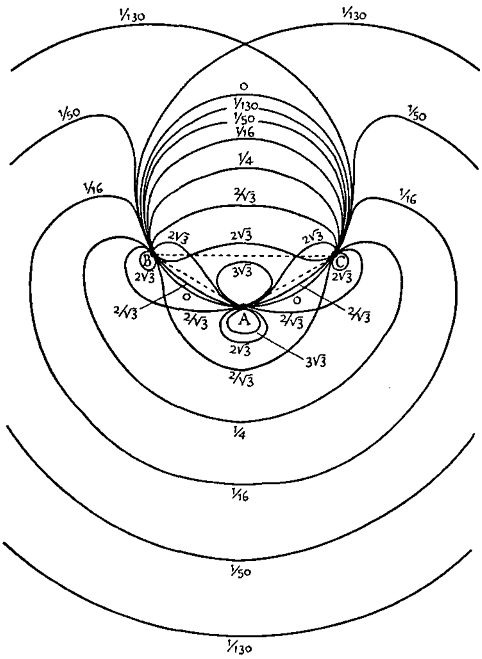

Fig. 5. When the angle between base lines is 120°.

The displacement probable error in a Loran position line at P is given by[3]

$$r = b \cos \tfrac{1}{2}\phi \tag{8}$$

where b is the error on the base line due to the probable error in the measurement of time difference and ϕ is the difference in azimuth of the two stations from the ship (Fig. 3).

The angle between two Loran position lines at P is

$$\theta = \tfrac{1}{2}(\phi_1 + \phi_2) \tag{9}$$

From equations (2), (8) and (9), the probability density at P is given by

$$\begin{aligned} p &= (0{\cdot}0724/b^2) \sin \tfrac{1}{2}\phi_1 \sin \tfrac{1}{2}\phi_2 \sin \tfrac{1}{2}(\phi_1+\phi_2) \\ &= k\{\cos \tfrac{1}{2}(\phi_1-\phi_2) - \cos \tfrac{1}{2}(\phi_1+\phi_2)\} \sin \tfrac{1}{2}(\phi_1+\phi_2) \end{aligned} \tag{10}$$

where $k = 0{\cdot}0362/b^2$.

Assuming $k=1$ and AB = AC, equation (10) gives the contour maps in Figs. 6, 7, 8.

The author is grateful to Mr. H. Uranishi, of the University of Mercantile Marine, for help with the mathematical solutions.

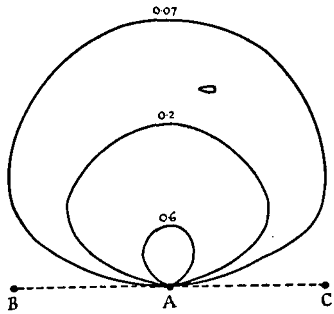

Fig. 6. When the angle between base lines is 180°.

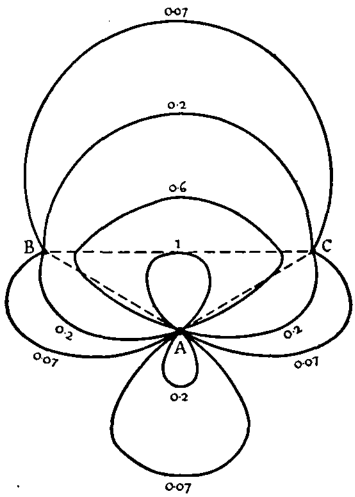

Fig. 7. When the angle between base lines is 120°.

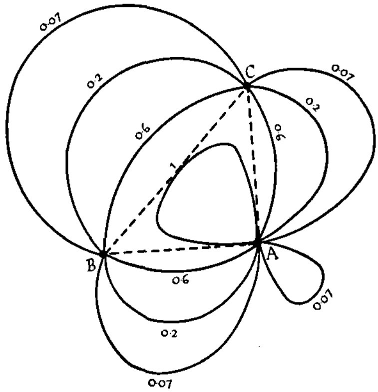

Fig. 8. When the angle between base lines is 90°.

Mr. J. B. Parker comments:

Mr. Sameshima has developed a series of fixing accuracy contours by using the concept of probability density. The probability density at a given point is defined as the ratio 'probability of position lines intersecting in a small area around the given point: size of the small area' when the small area tends to zero.

Jessell and Trow [4] based their accuracy contours on the 95 per cent radial error method. This is the radius of the circle, centred at the fix, which will contain the true position on 95 per cent of occasions.

In Sameshima's notation, p, the probability density, is given by

$$p = \frac{\sin \theta}{2\pi\sigma_1\sigma_2}$$

Jessell and Trow use D, the 95 per cent radial error, given by

$$D = \frac{2\sqrt{(\sigma_1^2 + \sigma_2^2)}}{\sin \theta}$$

Since Sameshima draws his contours in terms of p, and Jessell and Trow theirs in terms of D, corresponding diagrams will not be the same. In particular, a large p (Sameshima) means a good fix, whereas a large D (Jessell and Trow) indicates a large error. Sameshima's Fig. 2 and Jessell and Trow's Fig. 8 both refer to the same problem; yet the accuracy diagrams, being based on different formulae, appear quite different.

Obviously the crux of the problem is 'How do we measure accuracy?' The difficulty arises because, in the general navigational problem, the area of uncertainty is not circular, but elliptical. To define an ellipse, three quantities are necessary: (*a*) the length of the major axis, (*b*) the length of the minor axis, (*c*) the orientation of the major axis.

Any simple accuracy diagram can only record the value of one quantity at any one point, so that either two of the three above quantities must be jettisoned, or else accuracy diagrams must be drawn in terms of some compromise quantity. Sameshima has chosen the probability density, and Jessell and Trow the 95 per cent radial error.

Comparing Fig. 2 (Sameshima) with Fig. 8 (Jessell and Trow), both diagrams show a steady increase in accuracy along the perpendicular bisector of the line joining the two objects, or stations, up to a point, after which there is a steady deterioration. This is intuitively correct; on the base line, angle of cut is 180° (D infinite; $p = 0$), and accuracy improves as the angle of cut gets more reasonable. At great distances the angle of cut decreases to zero, so again D tends to infinity, and p to zero. Near one of the stations, however, the two methods give different results. In Sameshima's method p becomes very large (provided the angle of cut is not 180°) because in the vicinity of station A σ_1 becomes vanishingly small. But this state of affairs does not occur in Jessell and Trow's diagram.

What really happens near station A is that the area of uncertainty becomes a long, thin ellipse. Since A is so close, a position line from it, even if slightly in error, will not lead to any important linear errors about it; but the position line from B will have an appreciable, though not too large, band of error. It seems unrealistic, navigationally, to identify such a state of affairs with perfect fixing accuracy, even if the probability density tends to infinity. A second disadvantage is that the concept of probability density, while intelligible, and indeed in many ways preferable for the mathematician, has no simple and direct appeal to the practical navigator. Admittedly, the 95 per cent radial error suffers from the disadvantage of not giving any information about the shape of the error ellipse, but in view of the impossibility of conveying all the desirable information by means of a single quantity, it is not a bad compromise.

In spite of this, Sameshima's diagrams have great theoretical interest, and compel attention to a familiar and evergreen problem: 'What is meant by accuracy?'

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Radar and Collision at Sea

from Commander P. C. H. Clissold

CAPTAIN G. C. FORREST's letter (this *Journal*, Vol. VII, p. 203) raises some interesting points well worth discussion:

'(1) That radar-using ships, taking broad evasive action, shall alter course to starboard only, and not at all once they are within five miles of the other vessel, though they may reduce speed or stop.'

If we consider ship A in fog which sees another, B, some eight or nine miles on her starboard side, what should her action be? After plotting observations of B, A (we will assume) finds that B is crossing her own course and that if each ship maintains her course and speed they will be dangerously close together at the crossing. A is not yet bound by any rule to take any action (for the Rule of the Road, devised long before radar was thought of, clearly assumes ships to be in sight of one another when laying down the correct action for crossing steam vessels), but if she continues as she is going she will eventually arrive in such a position as to be compelled by the rules to take avoiding action. Prudence dictates that she should take some action to avoid the dangerous close-quarter situation: what action should that be? She can reduce speed, alter course or do both, and before deciding what she should do we must consider ship B.

If B has no radar she will continue at her present course and speed; if she has radar, she will become aware of the situation at about the same time as A. Not yet bound to any course of action, B will, if the situation develops unchanged, be in the position of the standing-on ship, directed to keep her course and speed until collision cannot be avoided by action of the giving-way vessel, A, alone. She may not relish this prospect and desire to avoid close quarters. Should she slow down or alter course to pass under A's stern? She cannot tell if A is using radar, but will guess that if she is A may alter course to pass under *her* stern. So an alteration of course to port by B may not achieve the desired effect and will in any case increase the relative speed of approach and reduce the time available for avoiding action before the danger point is reached. Reduction of speed will not do this; but will keep her clear of A should A not have radar, and will not embarrass A should A alter course sufficiently to pass under B's stern; while if A slows down the situation remains as before but with more time to negotiate the crossing. It seems definitely to be the safer plan.

If B then reduces speed, A can safely alter course to pass under her stern and, since she may expect B to reduce speed (if B has radar), A should allow for this and alter course until B is fine upon her port bow. When this alteration is bold, B, if using radar, will soon be aware of it.

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