AMPLIFICATION OF MAGNETIC FIELDS BY COMPRESSIBLE CONVECTION

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Abstract: Previous work on the concentration of magnetic field by cellular convection in a Boussinesq fluid is extended to a perfect gas, so that the gas pressure is reduced in the presence of a strong magnetic field. Attention is focussed on 2-dimensional flows and on the case of small layer depths, so that the pressure is effectively uniform in field free regions. It is shown that if the field is sufficiently strong, flux sheets with significantly reduced pressures and densities may form. Criteria are established which measure the relative importance of this effect to the more familiar 'magnetic drag' which acts to prevent the concentration of field by generating counter vorticity.

In recent years, high resolution observations of the solar photosphere have shown the existence of vertical magnetic fields confined to thin flux tubes, with peak fields of up to 2000G (see e.g. Beckers 1981). Near the surface, these fields approach the maximum permitted by the requirement that the gas pressure must be small inside the tubes. If the fluid is regarded as a perfect conductor, so that the field lines are 'frozen in', it is difficult to produce such high fields by dynamical means (see e.g. Parker 1979). A complete theory must involve the action of diffusion (either molecular or turbulent) which allows flow across field lines. In this view, the size of a flux tube is determined by a balance between advective effects (due to the convective motions) and diffusion (Galloway et al. 1977). Previous theories of the dynamical effects of flux concentration (Galloway et al. 1978) have ignored the compressibility of the fluid, supposing it to be Boussinesq. Thus the evacuation of the tube cannot be modelled. In this paper the latter effect is investigated in the simple case of 2-dimensional convection, making use of the 'magneto-Boussinesq' approximation (Spiegel & Weiss 1982). It is shown that the effects of compressibility are less than might have been expected and that although the tubes can become evacuated, the amplification mechanism is essentially the same as for the Boussinesq case, at least in the parameter range considered.

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ANALYSIS

We consider steady 2-dimensional convection in a perfect gas in the presence of an imposed vertical magnetic field $B_0 z$; the convecting layer has depth d. The physics is simplified by supposing large thermal conductivity, so that the temperature T can be taken as fixed, and the energy equation dispensed with. Motion is then caused by horizontal density and pressure gradients, and this confines the magnetic field to thin boundary layers (see e.g. Proctor & Weiss 1982). If we write the magnetic field $\underline{B} \equiv \nabla_\wedge (A\underline{y})$, where $\underline{y}$ is at right angles to the convection rolls, the dimensionless equations describing the system are,

$$\nabla.(\rho\underline{u}) = 0 \qquad (1) \qquad R_m(\underline{u}.\nabla A) = \nabla^2 A \qquad (2)$$

$$Re(\underline{u}.\nabla\underline{u}) + \frac{1}{\varepsilon\beta}\nabla A\nabla^2 A + \left[\nabla^2\underline{u} + \frac{1}{3}\nabla(\nabla.\underline{u})\right] - \frac{1}{\varepsilon}\nabla(\rho T) + \rho\underline{z} = 0 \qquad (3)$$

where $\underline{u}$ is velocity, ρ density, and the dimensionless parameters are

$$\varepsilon = gd/RT_0, \qquad \beta = \frac{\mu_0 R T_0 \rho_0}{B_0^2}, \qquad R_m = \rho_0 g d^3/\mu\eta, \qquad Re = \rho_0{}^2 g d^3/\mu^2 \qquad (4)$$

if g is gravitational acceleration, R the gas constant, μ_0 permeability, μ viscosity and ρ_0, T_0 typical values of ρ, T. R_e is the Reynolds and R_m the magnetic Reynolds number, β is the ratio of initial gas pressure to magnetic pressure and ε the ratio between the layer depth and a pressure scale height. Previous work (Galloway et al. 1977, 1978) has concentrated on the case $\varepsilon \to 0$ (Boussinesq fluid). In this case the pressure is effectively uniform and the role of the Lorentz forces is to generate vorticity opposite in sense to that of the flow in the absence of field. If $R_m \gg 1$, the field is confined to a boundary layer of (scaled) width $O(R_m^{-1/2})$, and in this case the curl of (3) may be used to show that this 'magnetic drag' becomes significant (reduces the amplification) if $\beta \lesssim R_m^{1/2}\varepsilon^{-1}$. If R_m is so large that $R_m^{1/2}\varepsilon \gg 1$, however, then β can be $O(R_m)$ or less before the former inequality can be satisfied, and in this case the gas pressure in the flux sheet will be reduced below its ambient value since the total pressure must be constant, and the peak field B* is $O(R_m^{1/2})B_0$. If we suppose Re not too large, the effects of dynamic pressure may be ignored. Then because of continuity the velocity field will change to accommodate the horizontal density contrast, and this will change the shape of the flux sheet, which is Gaussian for very large β. There is also a generation of vorticity due to horizontal density gradients, but this is always small compared with the original vorticity. We can therefore ignore the effect of magnetic drag (which requires curved field lines) and consider a simple model in which $u = (R_m^{-1/2}\hat{u}(\xi), z)$, where $\xi = R_m^{1/2} x$ is the boundary layer coordinate for a flux sheet near x=0 (Fig.1). This causes a vertical sheet in which $A = A(\xi)$, $|\underline{B}| = R_m^{1/2}\hat{B}(\xi) = R_m^{1/2} dA/d\xi$. The boundary layer equations are then (if $T \simeq 1$ near x=0 and $\rho = 1$ outside the boundary layer)

$$\rho + \frac{R_m}{2\beta}\hat{B}^2 = 1; \qquad \frac{\partial}{\partial\xi}(\rho\hat{u}) = -\rho; \qquad \hat{u}\hat{B} = d\hat{B}/d\xi \qquad (5)$$

with $\partial\rho/\partial\xi = \partial\hat{B}/\partial\xi = \hat{u} = 0$ at $\xi = 0$, and $\rho \to 1$, $\hat{B} \to 0$, $\hat{u} \to -\xi$ as $\xi \to \infty$, with $\int_0^\infty \hat{B} d\xi = \frac{1}{2}$,

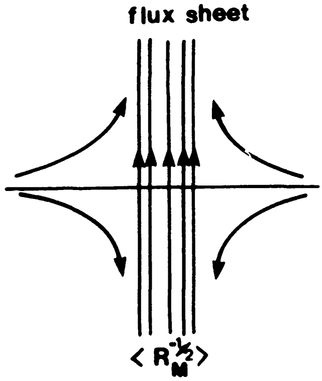

Figure 1. Geometry of the model problem: converging flow creates a thin flux sheet

say, since the flux is fixed. This system can be reduced to a first order o.d.e. which has been solved numerically. Fig.2 shows $\hat{B}$ and ρ as a function of ξ for 3 values of $\gamma \equiv \beta / R_m$. The sheet becomes flattened as γ decreases, with much more flux in the wings; it seems that the central density $\rho_{centre} \to 0$ as γ tends to a finite value (Fig.3) (though of course the assumptions of the theory are no longer valid then.)

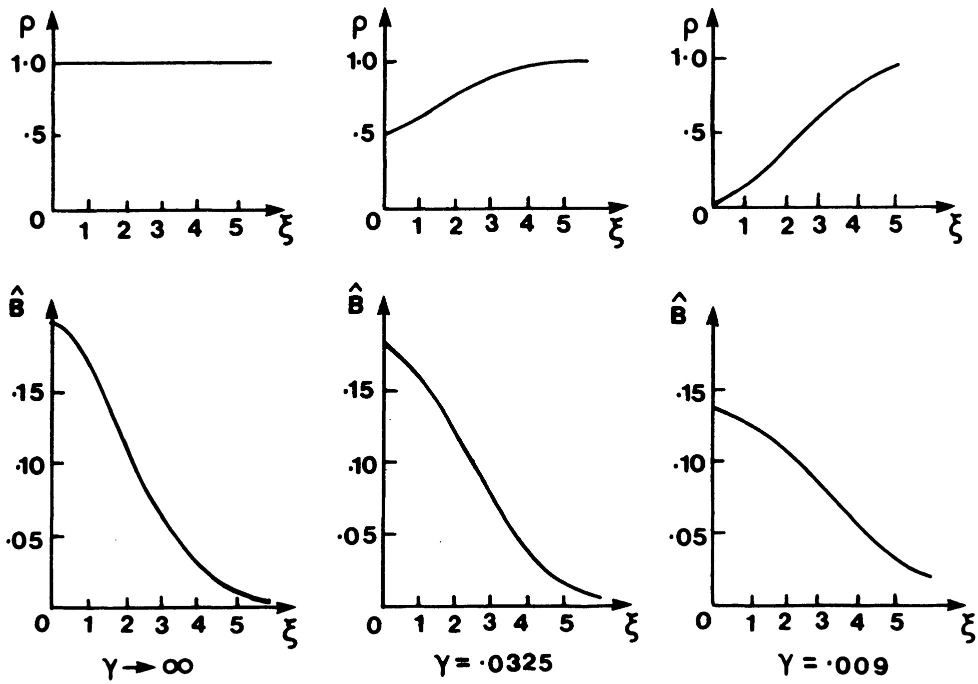

Figure 2. ρ and $\hat{B}$ as functions of ξ for three different values of γ.

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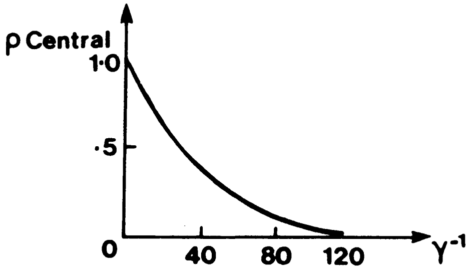

Figure 3. Central density as a function of γ^{-1}.

DISCUSSION

A start has been made in assessing the effect of compressibility on the concentration of magnetic fields by convection. The small ε results here complement those for large ε obtained numerically by Cattaneo (1982), which show qualitatively similar effects. They show that provided diffusion plays a role in fixing the size of the flux tubes (itself a controversial matter, see e.g. Spruit 1981), evacuated tubes can be supported almost as easily as non-evacuated ones. Further work is in progress to include the effects of an energy equation, and to consider the interaction between evacuation and magnetic drag effects.

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DISCUSSION

GIOVANELLI: (1) Can you give me any idea of the depth of a granule? One suggestion has been 1/4 granule diameter. Others have spoken of 1500 km to perhaps 10 000 km. If they are shallow, then of course the idea of a structure small compared with the scale height is OK, but otherwise this is not so. (2) The observations have shown increasing downflow velocities *inside* magnetic elements. Most of the downflow observations have been made by my colleagues and myself. These observations were done before we realized that the fields splayed out so rapidly with height. I believe that the downflow observations should now be redone taking cogniscance of our present improved knowledge on tube structure. Jack Harvey and I are attempting this, but I hope that it will be possible to interest others in similar studies. It is obviously of great theoretical interest.

PROCTOR: (1) A few hundred km is the most commonly accepted level for granules, and one imagines that the appropriate depth is one scale height (though if the granulation takes place in strongly stably stratified regions the vertical scale may be rather less). A full theory should certainly take account of larger layer depths. However, Cattaneo's numerical work seems to show similar features, even though layer depths there are large. (2) I heartily concur with the need for new downflow observations.

CRAM: Recent calculations of the electrical conductivity of the solar photosphere give a minimum value of about 1 Siemens m^{-1}. This value is too large to lead to significant Joule dissipation. If "classical" diffusivity is not operative, what is the picture of "turbulent" diffusivity that we should think about in relation to magnetic structures that are only a few hundred km across?

PROCTOR: "Turbulent diffusion" acts at both scalar and vector fields both to reduce scales below the resolution threshold and, more importantly, to such scales that molecular or radiative effects can operate on the time scale available. For scalars the effective diffusivity does not depend significantly on the molecular value, but there are certainly problems in understanding the diffusion processes when they are strongly affected by the anisotropy introduced by the Lorentz force. I would expect the existence of very small-scale fluctuating flows down to the scale of a few (perhaps a few tens) of km, as the end product of the cascade due to the turbulent convection.

SPICER: I have a comment with respect to Cram's remark on the electrical conductivity of the plasma at the temperature minimum. As there are flow fields which appear to flow perpendicular to the ambient magnetic field, perpendicular currents will result. Hence the Pedersen conductivity can lead to significant dissipation via $\eta_{Peder} J_{\perp}^2$.

PROCTOR: In highly conducting fluids the field lines move with the fluid, so there is no perpendicular flow! One does not want *too much* dissipation anyway, since otherwise the very intermittent magnetic field structure would not be seen.

WEISS: In modelling fluxtubes, we poor theoreticians have to rely on what the observers tell us, and it is confusing when even Australian observers seem to disagree with each other. As I understand it, Dr. Giovanelli still maintains that there is a flow of gas across the field lines and downwards into the fluxtube. Dr. Cram insists that ohmic diffusion will not permit this flow. So we have to postulate the existence of a different diffusion process.

PROCTOR: It is certainly important not to be content with "kinematic" turbulent diffusivity theories but to include the effects of Lorentz forces. Perhaps we can then understand how we can obtain flows "down under" the tubes.