

Corrigenda
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‘Finite Bol loops: III’

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It was affirmed in [1] that, for any odd prime p , a Moufang loop of order $2p^2$ is a group and there is exactly one non-associative right Bol loop of order $2p^2$. The first assertion is correct, but the second is wrong: there are, in fact, exactly two non-associative right Bol loops of order $2p^2$.

Construction of a second Bol loop of order $2p^2$. Let $D_p = \langle a, b \rangle$ with $a^p = b^2 = (ab)^2 = 1$, and let G be the subgroup of $C_p \times D_p \times D_p$ generated by $\alpha = (a, 1, 1)$, $\beta = (1, a, 1)$, $\theta = (1, 1, a)$ and $\gamma = (1, b, b)$. Then if $P = \langle \alpha, \beta \rangle$ and $Q = \{\alpha^i \theta^j \gamma \mid i, j \in \mathbb{Z}\}$, $P \cup Q$ acts regularly and transitively on the right cosets of the subgroup $H = \langle \beta \theta \rangle$ in G . $P \cup Q$ is Bol closed and not closed, and so $P \cup Q$ (like $A \cup B$ in Theorem 1) is the set of right multiplications of a non-associative Bol loop. An argument similar to that of Theorem 2 establishes that this loop is isomorphic to all its loop isotopes.

The mistake in the original paper occurs in case (iii) of Theorem 5, where it was correctly said that from $\eta\alpha\eta = \alpha$, $\eta\beta\eta = \beta^{-1}$ and $\eta^2 = 1$, η is determined uniquely from the image of one loop element. Thus if η overlaps $\beta^i\gamma$, $\eta = \beta^i\gamma$ and $\Pi = \langle \alpha, \beta, \gamma \rangle$ as was claimed. But if η overlaps $\alpha^i\beta^j\gamma$ when $i \neq 0$, then since $\alpha^i\beta^j\gamma$ has order $2p$, $\eta \neq \alpha^i\beta^j\gamma$ and $\eta \notin \langle \alpha, \beta, \gamma \rangle$. In this case $\langle \alpha, \beta, \gamma, \eta \rangle$ has defining relations

$$\alpha^p = \beta^p = (\gamma\eta)^p = \gamma^2 = \eta^2 = (\gamma\beta)^2 = (\eta\beta)^2 = 1$$

and

$$\alpha\beta = \beta\alpha, \quad \alpha\gamma = \gamma\alpha, \quad \alpha\eta = \eta\alpha,$$

and is isomorphic to the group G above (putting $\gamma\eta = \theta$).

The argument of theorem 6 of [1] (about Moufang loops) is not affected by this mistake, as it only uses hypotheses common to the two Bol loops.

The identification of the error in [1] followed the study of [2].

REFERENCES

- [1] R. P. BURN. Finite Bol loops: III. *Math. Proc. Cambridge Philos. Soc.* **97** (1985), 219–223.
- [2] B. L. SHARMA. Classification of Bol loops of order 18. *Acta Univ. Carol., Math. Phys.* **25** (1984), no. 1, 37–44.