"Substituting these values of $1+a_x$ and $\cdot75+a_x$ in (1), we get

$$\mathrm{H}_x=\frac{1-\dfrac{d}{\pi_x+d}}{2\left(\dfrac{1}{\pi_x+d}-\cdot25\right)}=\tfrac{1}{2}\frac{\pi_x}{1-\cdot25(\pi_x+d)}."$$

This latter equation may be again reduced to

$$\frac{\pi_x}{2-\frac{1}{2}d-\frac{1}{2}\pi_x},$$

which, when the rate of interest is 3 per cent., becomes

$$\mathrm{H}_x=\frac{\pi_x}{1\cdot9854-\frac{1}{2}\pi_x},$$

which is extremely simple.

The example worked in my last letter will, by this formula, be as follows:—

$$\pi_x=\cdot0410 \therefore 1\cdot9854-\cdot0205=1\cdot9649,$$
$$\text{and } \cdot0410\div1\cdot9649=\cdot02087, \text{ as before.}$$

The general formula will be found readily from the foregoing to be

$$\frac{1}{m}\cdot\frac{\pi_x}{1-\dfrac{m-1}{2m}(\pi_x+d)}.$$

Apologizing for the length of this communication,

I am, Sir,

Your obedient servant,

Eagle Insurance Company, SAMUEL L. LAUNDY.
6th September, 1864.

ON A PARTICULAR ARRANGEMENT OF ELEMENTARY VALUES.

To the Editor of the Assurance Magazine.

SIR,—1. The values of annuities and assurances of all kinds consist of certain elements variously combined. These elements are not usually exhibited in detail, their combinations being otherwise attainable.

2. But an intimate knowledge of details will enable its possessor to surmount such difficulties as occur in the treatment of complex questions involving many lives. Something may also be gained by particular arrangements of elementary values. For these reasons the following brief exposition is offered.

3. Our elementary table contains the logarithms, for the Carlisle life table, from age 90 upwards, of the following quantities:—

l_x =number that complete age x;
d_x=number that die in the $(x+1)$th year of age;
v =an unit of money, discounted for one year (3 per cent.);

and combinations of these, as shown in the headings.

4. The present value of £1, to be received in event of a life now aged x completing the $(x+1)$th year of age, is $\frac{vl_{x+1}}{l_x}=vp_x$. This is an endowment.

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The logarithms of a series of deferred endowments may be formed by continuous addition of $\log vp_x$, from the youngest age upwards.

5. The present value of £1 to be received at the end of the year, provided that a life aged x fail within the $(x+1)$th year, is $\frac{vd_x}{l_x}=vr_x$. This is an assurance for one year. If $\frac{d_{x+1}}{d_x}=q_x$, then the logarithms of a series of deferred assurances for one year may be formed by continuous addition of $\log vq_x$ to the initial value of $\log vr_x$.

6. The present value of £1 to be received in event of two lives x and y jointly surviving one year, is $\frac{vl_{(x.y)}}{l_{(x.y)}}=vp_{x.y}$; and the present value of £1 to be received at the end of the year, provided that one or both lives fail within the next year, is $\frac{vd_{x.y}}{l_{x.y}}=vr_{x.y}$. These symbols are under the same laws as those for single lives.

7. The present value of £1 to be received at the end of the year, provided that a life x fails in the $(x+1)$th year, and that a life y survives the instant at which x dies, is $\frac{vd_x l_{y+\frac{1}{2}}}{l_{x.y}}=vr_x p_{y\frac{1}{2}}$. This is a survivorship assurance for one year. Taking this as an initial value, we may, by adding to its logarithm continuously $\log vq_x+\log p_{y+\frac{1}{2}}$, obtain those of a deferred series.

8. It thus appears that the construction of the logarithmic values of the foregoing elements is uniform. But not only this: there is a relation which connects them. When the logarithms of successive deferred endowments have been formed, then, by adding to these the corresponding logarithms of a series $\frac{d_x}{l_{x+1}}=S_x$, we may derive those of deferred yearly assurances; and if to these again we add corresponding terms of the series $\log\frac{l_{y+n+\frac{1}{2}}}{l_y}$, we obtain the logarithms of deferred yearly survivorship assurances on x against y. This will be clearly seen in the following arrangement. Carlisle 3 per cent. $y=90$.

x.	1. Endowments. $\Sigma(\log vp_x)$.	2. $\text{Log}\,\frac{d_x}{l_{x+1}}=S_x$.	3. Assurances. (1)+(2).	4. $\text{Log}\,\frac{l_{y+n+\frac{1}{2}}}{l_y}$.	5. Survivorship Assurances. (3)+(4). x against y.
95	9·87177	9·48337	9·35514	9·93938	9·29452
6	·75248	·44370	·19618	·80195	·99813
7	·63050	·45593	·08643	·65727	·74370
8	·51292	·43573	·94865	·51981	·46846
9	·41294	·34679	·75973	·39178	·15151
100	·29095	·45593	·74688	·27096	·01784
1	9·13199	·60206	·73405	·15947	·89352
2	8·89730	9·82391	·72121	·05183	·77304
3	·40734	0·30103	·70837	·94462	·65299
4	..		·39451	·84771	·24222

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1. This is the sum of $\log vp_x$ from age x (inclusive) upwards.
2. This is $\log S_x$, from the elementary table.
3. This is the sum of the two preceding columns. See para. 5.
4. In this the initial value is $\log p_{y\frac{1}{2}}$, and the successive differences are $\log p_{y+\frac{1}{2}}$. These two symbols must be carefully distinguished. One is the probability of living half a year, the other that of living from the middle of one year to the middle of the next year.
5. This is the sum of the two preceding columns. See para. 7.

9. If the numbers corresponding to columns 1, 3, and 5, be summed from the bottom upwards, they will possess all the properties of the symbols N, M, and $M_{\overline{x.y}}^{1}$ in commutation tables. The writer does not mean to propose this as a model arrangement, yet for ease of calculation it seems to be very convenient. What he does propose is to sum the numerical values from the youngest age downwards; and he conceives that by this arrangement some remarkable advantages would be gained. For illustration of this, the following specimen table may suffice. It exhibits assurances, ages 47 and 52; Carlisle 3 per cent.

Assurances. $x=52$; $y=47$. *Diff.* 5 *years.*

Term of Assurance. Years. n.	(1) A_x.	(2) A_y.	(3) $A^{1}_{\overline{x.y}}$.	(4). $A_{\overline{x.y}}^{\ 1}$.
1	·01476	·01418	·01465	·01407
2	·02975	·02712	·02932	·02671
3	·04473	·03929	·04378	·03840
4	·05990	·05071	·05822	·04919
5	·07523	·06237	·07261	·05999
6	·09129	·07424	·08746	·07076
7	·10897	·08629	·10356	·08146
8	·12854	·09833	·12108	·09187
9	·15041	·11053	·14032	·10209
10	·17234	·12285	·15925	·11206
11	·19379	·13576	·17741	·12212
12	·21429	·14998	·19437	·13278
13	·23420	·16571	·21041	·14411
14	·25337	·18329	·22538	·15626
15	·27184	·20092	·23930	·16794
16	·28976	·21817	·25232	·17887
17	·30716	·23466	·26448	·18884
18	·32420	·25066	·27592	·19806
19	·34074	·26607	·28658	·20649
20	·35809	·28092	·29729	·21416
21	·37644	·25933	·30813	·22114
22	·39548	·30932	·31886	·22742
23	·41515	·32301	·32942	·23305
24	·43356	·33631	·33880	·23801
25	·45098	·35026	·34719	·24270
Life	·57599	·52544	·38344	·26834

$A_{xy}=(3)+(4)$. $A^{2}_{\overline{xy}}=(1)-(3)$; $A_{\overline{xy}}^{\ 2}=(2)-(4)$.

$A^{2\,2}_{\overline{x.y}}=(1)+(2)-\{(3)+(4)\}$.

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Deferred assurances are obtained by subtracting the temporary from the whole life values.

10. This table scarcely requires explanation. The value of an assurance on the joint lives 47 and 52 is ·38344+·26834=·65178. An assurance on this status for ten years is ·15925+·11206=·27131, and the difference of these, =·38047, is an assurance on 47 and 52 deferred for 10 years. An assurance on the last survivor of lives now aged 47 and 52 would be ·57599+·52544—·38344—·26834=·44965. In like manner, any other conditions within the limits tabulated may be determined by simple addition and subtraction.

11. A complete set of such tables would be exhaustive. But the uses of them lie within moderate limits. Up to $n=25$, and $x-y$ or $y-x=10$, they have already been computed, and the writer would willingly complete them to such extent as may be thought needful.

12. It is possible, with small trouble and with sufficient exactness, to pass from the elementary numerical values at one rate of interest to those at another rate. The following is an example of this. The values are those of survivorship assurances, 30 against 35.

n	3 per Cent.		Subtract.		4 per Cent.	
	Single Years.	Temporary.	S	t	Single Years.	Temporary.
1	·00976	·00976	9	9	·00967	·00967
2	·00938	·01914	18	27	·00920	·01887
3	·00885	·02799	25	52	·00860	·02747
4	·00834	·03633	32	84	·00802	·03549
5	·00801	·04434	38	122	·00763	·04312
6	·00768	·05202	43	165	·00725	·05037
7	·00749	·05951	49	214	·00700	·05737
8	·00729	·06680	54	268	·00675	·06412
9	·00710	·07390	59	327	·00651	·07063
10	·00715	·08105	66	393	·00649	·07712

Here the column marked S is a quantity depending on the single year values, and t is the sum of that column. These being taken from the 3 per cent. columns give the 4 per cent. columns. S is taken from a small subsidiary table. In like manner, we may proceed from 4 to 5 per cent., and so on. The work is very much less than if logarithms were used.

I am, Sir,

Yours truly,

W. H. OAKES,

London, August, 1864. *Lieut.-Col.*