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DISTRIBUTIVE p-ALGEBRAS AND OCKHAM ALGEBRAS: A TOPOLOGICAL APPROACH

MOSHE S. GOLDBERG

A *pseudocomplemented algebra* (or *p-algebra*), $\langle A; \wedge, \vee, {}^*, 0, 1\rangle$ is an algebra of type $\langle 2, 2, 1, 0, 0\rangle$ where $\langle A; \wedge, \vee, 0, 1\rangle$ is a bounded lattice, and every element x in A, has a pseudo-complement x^* in A; that is, $x \wedge y = 0$ if and only if $y \le x^*$. An *Ockham algebra* $\langle A; \wedge, \vee, \sim, 0, 1\rangle$ is an algebra of type $\langle 2, 2, 1, 0, 0\rangle$, where $\langle A; \wedge, \vee, 0, 1\rangle$ is a bounded lattice, and $\sim$ is a dual endomorphism of A; that is, $\sim(x \wedge y) = \sim x \vee \sim y$, $\sim(x \vee y) = \sim x \wedge \sim y$, $\sim 0 = 1$, $\sim 1 = 0$. In this thesis we make use of essentially non-algebraic techniques to study algebraic properties of the varieties of distributive p-algebras, distributive double p-algebras (those p-algebras whose order-theoretic duals are also p-algebras), and distributive Ockham algebras.

In 1969, Priestley [2] developed her duality theory for bounded distributive lattices, by identifying a bounded distributive lattice L, with the lattice of closed-open order-ideals of a certain ordered topological space. This representation has been extended to distributive p-algebras (Priestley [3]), distributive double p-algebras (Davey [1]) and distributive Ockham algebras (Urquhart [4]). Here, we obtain particularly useful information about the subidrectly irreducible, injective, and free algebras in the varieties mentioned above, as well as some of their subvarieties. In the infinite case we are actually doing topology rather than algebra. If L is finite, then 's duality coincides with G. Birkhoff's representation for finite distributive lattices; that is, L is isomorphic with the lattice of order-odeals of

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$J(L)$ - its poset of join-irreducible elements. Thus in the finite case, we are usually studying properties of finite posets; that is, we are doing combinatorics and graph theory.

En route, we prove several interesting results about posets and ordered topological spaces in general.

We have added, as an appendix, a set of reviews (from Mathematical Reviews - reprinted with the kind permission of the American Mathematical Society) of papers in the subject, covering the period 1955-1978, together with an author index. This should prove useful to other researchers in the field.

References

[1] Brian A. Davey, "Subdirectly irreducible distributive double p-algebras", *Algebra Universalis* 8 (1978), 73-88.

[2] H.A. Priestley, "Representation of distributive lattices by means of ordered Stone spaces", *Bull. London Math. Soc.* 2 (1970), 186-190.

[3] H.A. Priestley, "The construction of spaces dual to pseudocomplemented distributive lattices", *Quart. J. Math. Oxford Ser.* (2) 26 (1975), 215-228.

[4] A. Urquhart, "Distributive lattices with a dual homomorphic operation", *Studia Logica* (to appear).

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